

Hole-filling Algorithm for High-curvature Region in Triangular Mesh Models

Liu Xumin*, Li Cailing, Wang Xiaojun

College of Information Engineering, Capital Normal University
Beijing 100048, china

*corresponding author, e-mail: clcxsl@163.com

Abstract

As the problem of hole-filling in the high-curvature region of the triangular mesh isn't satisfactory, a new algorithm is proposed. In the mesh subdivision process, judge the mean curvature of the triangles that have a common edge, if the edge in the high curvature area, inserting new vertices in different ways according to the different situations of the edge length to achieve the mesh subdivision. The algorithm can make holes with neighboring mesh transit more natural and the repairing effect is satisfactory by many examples.

Keywords: hole-filling; triangular mesh; Gaussian curvature; high-curvature; mesh subdivision

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1. Introduction

Reverse engineering is a model that transforms the existing product model or physical model into an engineering design model or conceptual model. With the development and application of computer technology especially the theories and techniques of computer aided geometric design, reverse engineering has become an important means to obtain three-dimensional model. Reverse engineering including data acquisition, data processing[1] and surface reconstruction, and surface reconstruction is one of the key technologies in the reverse engineering. At present, there are two types[2] in the model of surface reconstruction: one is discrete grid model, another is parametric surface model. For the discrete grid model, because the definition of the triangular mesh is relatively simple, and the description ability of the complex topology is strong, and related algorithms are more mature, thus the triangular mesh become a more common model representation. However, due to the limitations of measurement tools or parts itself reason the triangular mesh often has the missing data, thus holes form. In this paper, we study the problem of hole-filling in the high-curvature region in the triangular mesh.

At present, many scholars at home and abroad have done research on the problem of hole-filling in the triangular mesh. Zhang Liyan etc. [3] proposed to project the triangulation problem of the space polygon on the plane, and then generated new triangles according to the angles of the polygon boundaries. This algorithm is very suitable for the situation when the hole is close to the plane. If the hole-surface is more curved, the concealment is poor. Davis J, etc. [4] proposed the method of voxel diffusion to patch holes. This method has good concealment to the holes of complex shape, but time complexity of the whole algorithm is too high, and the repaired mesh may change the original structure. Liepa[5] used Pfeifle's[6] algorithm to refine and adjust triangles, add new triangles, and finally used the mesh smoothing method to adjust the new triangles vertices' position, but the algorithm's time complexity is high. Zhao[7] from Zhejiang university used the AFM (Advancing Front Mesh) technology to fill the holes, then calculate the normal vector of each new triangle, and solve a Poisson equation according to the normal vector and hole boundary points, and then adjust the position of the new points. But when the points were too much, the solution of Poisson equation would be more time-consuming. Wu[8] from HIT proposed an automatic hole-filling algorithm based on RBFs. The algorithm projected the 3D to 2D coordinate plane, and the holes couldn't contain any overlap in the projection direction. The above algorithms repaired holes from different perspectives, and most of the algorithms are not very satisfactory for non-flat areas.

This article researches the problem of hole-filling in non-flat areas in the triangular mesh. At first, time-costly triangulation is replaced by a simple initial triangulation. Then, refine the triangular mesh according to the edge-length and the curvature of the triangles that contain the edge. Finally, the refined triangular mesh is adjusted by Delaunay algorithm, and then smoothed by Laplacian operator. The information of the new vertices and triangles are applied to the refinement process, thus receive a triangular mesh of better geometric properties and make holes with neighboring mesh transit more natural.

2. Hole-filling

2.1 Hole-polygons detection

In a triangular mesh with the closed structure, if a point is a vertex of a triangle, this triangle is the point's adjacent triangle. If the edge only connects one triangle, it is boundary edge. The points on the boundary edge are boundary points. The triangle that has one or two boundary points is boundary triangle. If the edge has two adjacent triangles, it is internal edge. The hole-polygon is connected by boundary edges from end to end. As shown in Figure 1.

2.1.1 The extraction of hole-polygons

The above definition shows that the edges of the mesh model are divided into the internal edges and boundary edges. Thus, the vertices of the mesh model are divided into the internal vertices and boundary vertices. The process of determining the boundary vertices is to determine whether the vertex is on the boundary edge. The judgment algorithm is as follows:

- 1) Establish a container `_vertVec`, and be empty.
- 2) The current vertex is processed as follows: if the number of the adjacent edges and the number of adjacent triangles are different, it's a boundary vertex, and then put it into `_vertVec`.
- 3) Judge the adjacent point is a boundary point, and if so, add it to `_vertVec`.

2.1.2 Classification and sorting of the hole-polygons

After boundary vertices are extracted, the set of the boundary points is disordered and there may exist more than one hole, so it's necessary to classify and sort the set to get a set of boundary points for each hole. For this situation, the vertices of the container `_vertVec` are deposited in different vector object according to whether belong to a hole. The algorithm is as follows:

- 1) Traverse the container `_vertVec`, put the first vertex into a new container `vertOne`.
- 2) Judge the follow-up vertices and the vertices in `vertOne` belong to a hole, and if so, put it into `vertOne`, otherwise put it into a new container `vertTwo`.
- 3) Loop through the container until it is empty.

2.2. Hole-polygon Triangulation

2.2.1 Hole-polygon triangulation

After the boundary of the hole-polygon is identified, the polygon needs triangulation. Five methods to repair the initial triangles are the minimum area method, the minimum edge length method, the minimum angle method, the wave-front method and boundary elimination method. But this algorithm does not take these costly and optimal triangular method, it takes a simple initial triangulation. To achieve the process of the initial triangulation, we define a recursive function `Triangulate (i, j)`, where `i` and `j` are the label of the boundary points. If `j=i+2`, increase the triangle $v_i v_{i+1} v_j$; otherwise, let `N` is an integer between `i` and `j`, if `N ≠ i+1`, recursive call `Triangulate (i, N)`, increase the triangle $v_i v_N v_j$. If `N ≠ j-1`, recursive call `Triangulate (N, j)`. Call `Triangulate (0, n-1)` and triangulate the hole-polygon. The vertices of each triangle from triangulation belong to the boundary vertices, and any one boundary edge belongs to a triangle. As shown in Figure 2.

2.2.2 Classification and sorting of the hole-polygons

The quality of the triangular mesh after initial triangulated is poor and largely different from the original mesh, and the edge length is longer and the hole may be highly curved, so, this part need subdivision.

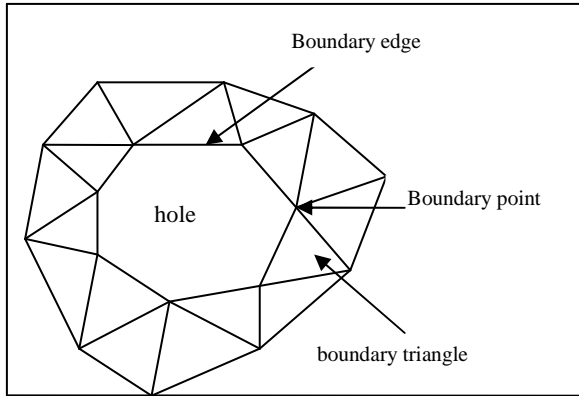


Figure 1 hole-polygon

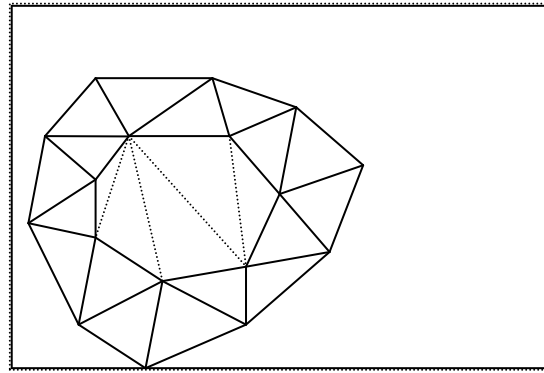


Figure 2 hole-polygon triangulation

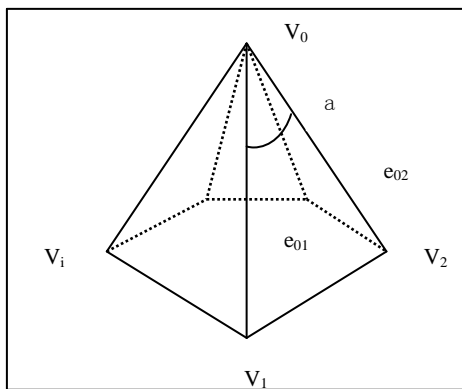


Figure 3 the neighborhood information of V_0

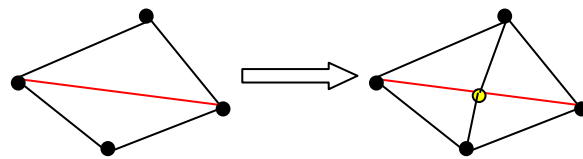


Figure 4 the subdivision of the triangular meshes

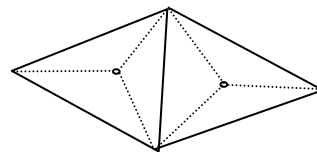


Figure 5 the subdivision of the triangle

First, calculate the average length of each edge $aver_l$; and then calculate the Gaussian curvature of each vertex. For the vertex V_0 that its costs are n , its adjacent vertices are V_1, V_2, \dots, V_n , as shown in figure 3, some geometric quantities are defined as follows:

- 1) the adjacent edges of V_0 are $e_{01}, e_{02}, \dots, e_{0n}$
- 2) the angle between the two adjacent edges $e_{0i}, e_{0(i+1)}$ is:

$$\alpha_i = \text{ang}(e_{0i}, e_{0(i+1)})$$

According to the definitions, the formula of the vertex's (V_0) Gaussian curvature (K) is [9]:

$$K(v_0) = \frac{2\Pi - \sum_{i=1}^n \alpha_i}{3 * \text{area}(S)}$$

In this formula, S is the summation of the areas of all triangles that are adjacent to the vertex V_0 . According to this formula, we can calculate the Gaussian curvature of each boundary points and their average $aver_k$, and then calculate the average curvature of the triangles T_i, T_j which each edge belongs to. For T_i , calculate its three vertices' Gaussian curvature k_{i1}, k_{i2}, k_{i3} , and the curvature of T_i is the average curvature of the three vertices:

$$K_i = \frac{k_{i1} + k_{i2} + k_{i3}}{3}, \text{ so, the average curvature of } T_i, T_j \text{ is } K_{ij} = \frac{k_i + k_j}{2}.$$

The process of the subdivision is: first, judge whether the average curvature K_{ij} is bigger than $w \cdot \text{aver_k}$, where w is the weight. And then, judge whether each edge length l_{ij} is longer than $2 \cdot \text{aver_l}$, if so, insert a new vertex at the midpoint of the edge, and connect it with the opposite vertices of the edge to achieve the subdivision, as shown in figure 4; else, subdivide the triangles according to Figure 5, and the new vertex is the geometric center of gravity of the triangle.

For the subdivision, we can comprehend it intuitively like this: when the Gaussian curvature of the triangles which the edge belongs to is bigger, that is the edge in highly curved surface; the algorithm uses different ways to insert new vertices according to the edge length to achieve the subdivision. If the surface where the edge is in is relatively flat, it does not need to be subdivided, and this improves the speed.

2.3 Mesh Form Adjustment and Surface Smoothing

After the triangulation and subdivision, in order to ensure the quality of the mesh, the mesh needs the geometric state adjustment. To ensure the mesh has local Delaunay nature, we need Delaunay adjustment[10]. As shown in figure 6, for the two adjacent triangles ΔABC and ΔABD , ΔABC can define a circle, and if the projection point D' of D in this circle is in outside of the circle, the edge AB has the property of local Delaunay and needn't the adjustment, else, the edge AB is replacement by the edge CD .

After the above steps, we adjust the new vertices by Laplacian operator[11] for the purpose of smoothing the mesh. For each vertex v_i , all the triangles that contain v_i are the 1-ring triangles of v_i ; all the edges that share v_i are the 1-ring edges of v_i ; all the vertices that are on v_i 's the 1-ring edges are v_i 's 1-ring vertices(except v_i), as shown in figure 7. $v_{i1} v_{i2} \dots v_{in}$ are v_i 's 1-ring vertices, so v_i 's new position is adjusted by the polygon $v_{i1} v_{i2} \dots v_{in}$.

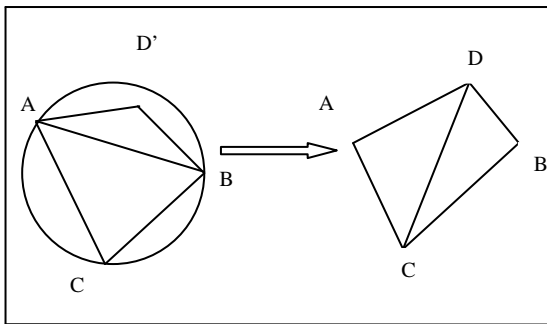


Figure 6 Delaunay adjustment

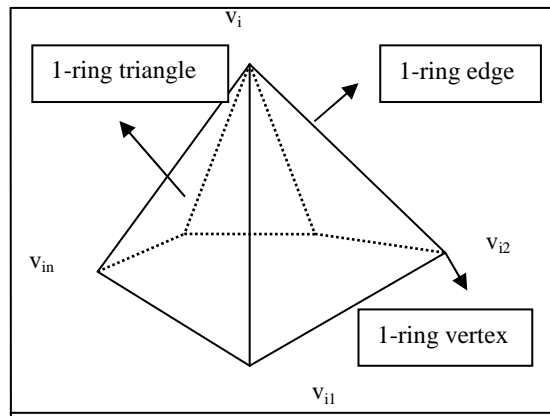


Figure 7 Laplacian operator in mesh

$NV(i)$ is the 1-ring vertices set of v_i , the new position of v_i is calculated as follows:

$$v'_i = v_i + \lambda \frac{1}{n} \sum_{k \in NV(i)} (v_k - v_i)$$

n is the number of v_i 's 1-ring vertices, and λ is a small positive constant.

Laplacian operator is equal to pulling the vertex to the geometric mean of its surrounding vertices in the triangular mesh, like a low-pass filter[12], by restraining high frequency noise to ensure the low frequency original data points for the surface smoothing.

3. Application examples

In this paper, the algorithm is used for triangular mesh. Figure 8 (a)(c)(e) are the models with holes, and (b)(d)(f) are the models after repairing.

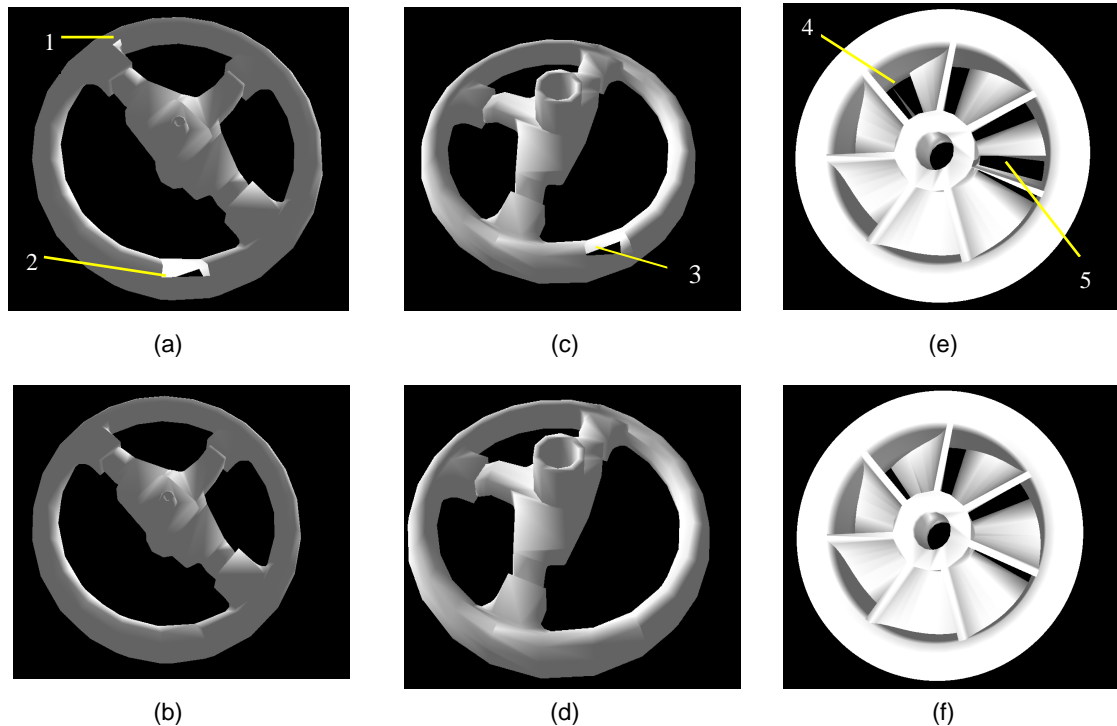


Figure 8 models with holes and the results after repairing

Figure 8 (a)(e) are a steering wheel model with holes, and it is clear that holes are in non-flat areas (the curvature changes relatively large), (b)(d) are the models after repairing. We can see that the repaired surface is not only smoothly linked with original surface, but also stays in the original shape. (e) is a turbine model with holes, and (f) is the model after repairing. We can see that the geometry around the hole changes relatively large, but we can see that the repairing effect is satisfactory compared with the original fans from (f). The algorithm can make holes transit naturally with neighboring mesh.

4. Conclusions

As the problem of hole-filling in the high-curvature region in the triangular mesh isn't satisfactory, this article proposes an efficient, robust algorithm. In this algorithm, time-costly hole triangulation is replaced by a simple initial triangulation. Then, it uses different ways to insert new vertices according to the edge length to achieve the subdivision. After subdivision, the mesh needs Delaunay adjustment to adjust the positions of new vertices. The new triangles are well distributed after the triangulation and subdivision. Finally, it uses the Laplacian to smooth the mesh. The new vertices and triangles are applied to the process of the subdivision, which receives the mesh with better geometric properties, and the hole can smoothly connect with the surrounding mesh. The examples prove this algorithm is efficient.

Acknowledgments

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