Gamma Stirling Engine for a Small Design of Renewable Resource Model

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Abstract
This paper presents a research on designing a heat engine known as the Stirling engine. The first task is to study on the background of Stirling engine including its robustness, advantages and disadvantages, history and its ability to produce useful energy. Gamma type Stirling engine will be the main focus for this paper. Thus, an effort has been made in determining a suitable formulation that will be used to design a functioning Gamma Stirling engine. This formulation can be divided into several criteria, the Stirling cycle method used to find the p-V diagram of Stirling engine, the 0th order calculation method used as a preliminary system analysis on the efficiency and performance of the engine and lastly, the Schmidt Analysis whereby used in dealing with the design and development of the engine. This formulation is then arranged accordingly into Excel programming software. As for the hardware analysis, it will be on the performance of the Stirling engine model in term of its electrical power production based on different heat source. At the end of this project, it shows that the obtained formulations can be used in designing the Gamma Stirling engine and are capable to produce an output power from the Stirling engine.

Keywords: Stirling engine; Gamma Stirling engine; formulation; Stirling cycle; Schmidt analysis; power production.

1. Introduction

The development of advanced technologies that help to provide energy is one of the challenges that people nowadays still try to solve, from the use of fire and animal's power to the present use of electricity and engines. Engines have been powering our world since the beginning of time as we know todays. Starting from steam engines that used coal to power up, and then come a much cleaner and more efficient type of engine know as gasoline engines [1].

Also, the quest of human being to develop engine that capable to produce a high power, high torque and most essentially no pollution and environmental friendly is still on since the discovery and development of the first engine. Stirling engine is just one step forwards in the creation of a noise free and pollution less engine.

Stirling engine is also known as external combustion engine in which it works in a closed cycle known as Stirling cycle. It was first invented by Robert Stirling in the year 1816 [2]. This Stirling engine was used back in the days as a rather low power water pumping engine from nineteenth century to 1920 before a more reliable internal combustion engine and electric motor replaced it [3].

A working process of Stirling engine consist of two zones known as hot and cold sides maintained at different temperatures and the working gas inside the engine will travelled between it to extract work similar to other heat engines. However, the differences are that this engine does not require any valves because the working fluid never leaves the engine and can be used over and over again. The working fluid can be air, hydrogen or helium [4]. This engine is basically an external combustion engine that capable to operate using almost any type of heat source [2].

Some of the advantage of Stirling engine is that it can be designed using very few moving parts and required less maintenance than other engine. It is more reliable and less harmful for environment compare to an internal combustion engine [5].

To develop this type of engine, some parameter will need to be specified. Therefore, in this project, the main focus is to determine and acquired a list of formulations that can be used to estimate the Stirling engine process and its development. It also has to prove that this formula can achieve the same parameter values similar to an actual Stirling engine. By using the formulations, it will design and build a working Gamma type of Stirling engine and followed by analyzing the model based on its performance using different heat source.

2. Gamma Stirling Engine

A Gamma Stirling engine is one of the simplest and easiest types of Stirling engine compare to its Alpha and Beta counterpart. It is simply a Beta Stirling engine in which the displacer and power piston is mounted in a separate cylinder but still connected to the same flywheel. This type of engine work will with low heat source compare to its others counterpart that required much higher heat for it to work [2].

The working process of a Stirling engine starts by heating the engine with an external heat source at the hot side of the engine. The gas inside the engine will expand upon heating thus push out the displacer piston inside the cylinder and provide power stroke by the displacer piston. By doing so, the gas will flow around the displacer piston and move toward the cold side of the engine through the regenerator, before moving to cold side of the engine, the heat exchanger or regenerator will cool down the hot gas thus causing it to condense. Flywheel is then turns and powering a second sealed piston known as power piston to recycle the gas back to the hot side where the same process is repeated over and over again [6].

3. 1st Order Calculation (Schmidt Analysis)

Schmidt Analysis was first published in the year 1871 by Gustav Schmidt of German Polytechnic institute in Prague [7]. It is one of the standard methods used in dealing with Stirling engine design and useful during Stirling engine development [4,8]. This method basically used to calculate the isothermal process of the Stirling engine that refers to the isothermal expansion and compression of an ideal gas [8]. This analysis is reasonably accurate in predicting the cycle power. However, it is not suitable in predicting the efficiency and heat transfer of a Stirling engine [7].

3.1 Assumption made Schmidt analysis [7];
1) Working gas act as an ideal gas located inside the engine.
2) Three dead volumes are taken into account.
3) Temperatures at each gas space are constant during the cycle.
4) Sinusoidal motion occurs between the expansion and compression space (displacer and power piston).
5) Constant speed and pressure throughout the engine.

3.2 Calculation for Schmidt Analysis [7, 8]
1) Swept Volume for Displacer, V_{SE}, and Power Piston, V_{SC}

The swept volume for the displacer and power piston can be calculated using equation (1) and (2) respectively.

$$V_{SE} = \frac{\pi}{4} \times \left(B_{dp}\right)^2 \times S_{dp} \tag{1}$$

$$V_{SC} = \frac{\pi}{4} \times \left(B_{pp}\right)^2 \times S_{pp} \tag{2}$$

B_{dp} and B_{pp} in equations (1) and (2) are the diameter of piston known as bore, respectively. For the S_{dp} and S_{pp}, it is the distance in which the piston will travel from TDC to BDC known as stroke. Both the displacer and power piston have its own bore and stroke.

2) Expansion space volume, V_E

Expansion volume, V_E is the volume located inside the expansion space. This volume represents by the Equation (3).

$$V_E(\alpha) = \frac{V_{SE}}{2} \cdot (1 - \cos\alpha) \tag{3}$$

The swept volume of displacer piston, V_{SE} can be obtained using equation (1).

3) Compression space volume, V_C

Compression space is defined as the space between the power piston and displacer. Thus, for the calculation of compression space volume, V_C, it will includes the swept volume of displacer piston, V_{SE}.

$$V_C(\alpha) = \frac{V_{SE}}{2} \cdot (1 + \cos\alpha) + \frac{V_{SC}}{2} \cdot (1 - \cos(\alpha - \varphi)) \tag{4}$$

The swept volume for both the displacer, V_{SE}, and power piston, V_{SC}, that can be obtained in equations (1) and (2), respectively.

4) Crank angle, α, and Phase angle, φ

Crank angle, α will be used to determine the expansion and compression volume during different crank angle, α. This crank angle, α, is represented by equation (5).

$$\alpha = 0° \ to \ 360° \tag{5}$$

As for the phase angle, φ, it is the angle difference between the displacer and power piston. this phase angle, φ, will always be 90° which mean that the displacer and power piston will move sinusoidal to each other.

$$\varphi = 90° \tag{6}$$

5) Total dead volume, V_D

The total dead volume, V_D represent by Equation (7) is the sums of dead volume inside the expansion and compression cylinder and the regenerator space.

$$V_D = V_{DE} + V_R + V_{DC} \tag{7}$$

6) Total space volume, V

The total volume, V at certain crank angle, α is determined using Equation (8).

$$V(\alpha) = V_E + V_D + V_C \tag{8}$$

7) Dead volume space

Equation (9) shows the mass of working gas in regards to dead space. Using equations (10) and (11), the volume and temperature of the dead space can be determined, respectively.

$$m = \frac{p}{R} \cdot \left[\frac{V_{DE}}{T_E} + \frac{V_R}{T_R} + \frac{V_{DC}}{T_C} \right] = \frac{p}{R} \cdot \frac{V_D}{T_D} \tag{9}$$

$$V_D = V_{DE} + V_R + V_{DC} \tag{10}$$

$$T_D = \frac{V_{DE} + V_R + V_{DC}}{\frac{V_{DE}}{T_E} + \frac{V_R}{T_R} + \frac{V_{DC}}{T_C}} \tag{11}$$

8) Regenerator space

Regenerator temperature obtained using Equation (12).

$$T_R = \frac{T_E - T_C}{\ln\left(\frac{T_E}{T_C}\right)} \tag{12}$$

9) Total mass of working gas, *m*

      The Equations (13) and (14) show the total mass of the working gas in regards to the mass of the expansion, compression and regenerator space, respectively.

$$m = \sum m_i = m_E + m_R + m_C \tag{13}$$

$$m = \frac{p}{R}.\left[\frac{V_E}{T_E} + \frac{V_R}{T_R} + \frac{V_C}{T_C}\right] \tag{14}$$

      Equations (15) until equation (21) are substitute into equation (14) to form a new equation for the mass of working gas, *m*.

$$t = \frac{T_C}{T_E} \tag{15}$$

$$v = \frac{V_{SC}}{V_{SE}} \tag{16}$$

$$X_D = \frac{V_D}{V_{SE}} \tag{17}$$

$$s = \frac{V_D}{V_{SE}}.\frac{T_C}{T_D} \tag{18}$$

$$B = t + 1 + v + 2.s \tag{19}$$

$$A = \sqrt{t^2 - 2.t + 1 + 2.(t-1).v.cos\,\varphi + v^2} \tag{20}$$

$$\delta = arc\,tan\left(\frac{v.\sin\varphi}{t-1+v.\cos\varphi}\right) \pm (0; \pi; 2\pi; 3\pi; \dots) \tag{21}$$

Thus, the mass of working gas, *m* is given as,

$$m = \frac{p.V_{SE}}{2.R.T_C}.[B - A.\cos(\alpha - \delta)] \tag{22}$$

10) Engine pressure, *p*

      Equation (23) for engine pressure is obtained based on Equation (22).

$$p = \frac{2.m.R.T_C}{V_{SE}.[B - A.\cos(\alpha - \delta)]} \tag{23}$$

By substitute the Equation (24) into equation (23), new Equation for the pressure is obtained.

$$c = \frac{A}{B} \tag{24}$$

$$p = \frac{2.m.R.T_C}{V_{SE}.B.[1 - c.\cos(\alpha - \delta)]} \tag{25}$$

11) Mean average pressure, p_m

$$p_m = \frac{2.m.R.T_C}{V_{SE}.B.\sqrt{1 - c^2}} \tag{26}$$

      Equation (26) represents the mean pressure, p_m for the engine. Equation (26) is then substituted into equation (25) to obtained equation (27) for pressure, *p* with respect to the mean average pressure, p_m.

$$p = \frac{p_m.\sqrt{1-c^2}}{1-c.\cos(\alpha-\delta)} \tag{27}$$

12) Maximum, p_{max}, and minimum engine pressure, p_{min}

The maximum and minimum pressure can be obtain using equation (28) and (29) with respect to the mean average pressure, p_m, respectively.

$$p_{max} = p_m.\frac{\sqrt{1+c}}{\sqrt{1-c}} \tag{28}$$

$$p_{min} = p_m.\frac{\sqrt{1-c}}{\sqrt{1+c}} \tag{29}$$

4. Result and Discussion

In conjunction with the comparison between the calculated parameters with the actual parameters of Gamma Stirling engine, the ensuing step is to design a Gamma Stirling engine based on the calculated parameters. An excel programming installed can be used for this process to easily calculate and determine the parameters of Gamma Stirling engine at any desired value. Prior to that, determine the initial parameters for the Gamma Stirling engine similar to the steps taken for comparison purposes.

Table 1. Initial design parameters for Gamma Stirling model

Initial parameters	Estimation value
Engine power, P	0.2 watt (0.08 watt)
Engine speed, n	1200 rpm (20 Hz)
Mean average pressure, p_m	1.013 bar (101325 Pa)
Hot space temperature, T_E	473 K
Cold space temperature, T_C	323 K
Working gas	Air
Gas constant (based on working fluid)	286.9 J/kg.K
Fluid constant (based on working fluid)	1.402

The calculation used to determine the volume, bore and stroke for both pistons and the efficiency of the engine is similar to the previously discussed comparison process. For this model of Gamma Stirling engine, the bore is similar in terms of its dimension.

Table 2. Engine parameters for the piston

Piston parameters	Calculated value
Displacer bore, B_{dp}	$0.012m$
Displacer stroke, S_{dp}	$0.0213m$
Swept volume displacer, V_{SE}	$2.620e^{-6}m^3$
Power piston bore, B_{pp}	$0.0125m$
Power piston stroke, S_{pp}	$0.0142m$
Swept volume power piston, V_{SC}	$1.75e^{-6}m^3$

Based on the parameters obtained in Table 2, the dimension of cylinder and piston can now be obtained by referring to Figure 1. The parameter is then tabulated in Table 3.
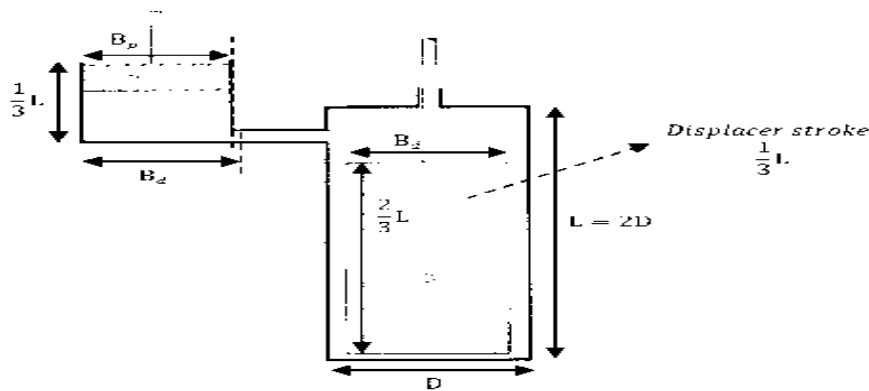
Figure 1. Basic structure of Gamma Stirling engine

Table 3. Parameters for the engine's cylinder and piston

Expansion Space	Symbol	Unit	Value
Displacer Piston			
Bore	B_dp	m	0.0125
Stroke	S_dp	m	0.0213
Length	L_dp	m	0.0427
Expansion Cylinder			
Length	L_e	m	0.0640
Diameter	D_e	m	0.0320
Compression Space	Symbol	Unit	Value
Power Piston			
Bore	B_pp	m	0.0125
Stroke	S_pp	m	0.0142
Compression Cylinder			
Length	L_c	m	0.0213
Diameter	D_c	m	0.0125

For the dead volume of this Gamma Stirling engine model, it is referring in [5-7]. The expansion, compression and total volume of the engine are also obtained by using Equations in [5-7].

Table 4. The expansion, compression and total volume of Gamma model

α, deg	V_E, m^3	V_C, m^3	V, m^3
0	0	$3.49e^{-6}$	$1.37e^{-5}$
90	$1.31e^{-6}$	$1.31e^{-6}$	$1.28e^{-5}$
180	$2.62e^{-6}$	$8.73e^{-7}$	$1.37e^{-5}$
270	$1.31e^{-6}$	$3.06e^{-6}$	$1.45e^{-5}$
360	0	$3.49e^{-6}$	$1.37e^{-5}$

The next task is to determine the constant mass and Cv discusseb in [5-7]. The preloaded temperature, T_L, and pressure, p_L, is considered to be the atmospheric value. This is due to the Gamma Stirling engine model size that does not require a large amount of pressure to push the working gas inside the engine. For the Cv constant of this model, it is based on the working gas used that is the air. The value for this parameters is shown in Table 5.

Table 5. Mass of working gas, m_g, and C_v

	Symbol	Unit	Value
Preloaded Pressure	p_L	Pa	101325
Preloaded Temperature	T_L	K	293
Maximum Volume	V_max	m^3	1.45E-05
Gas Constant	R	J/kg.K	286.9
Mass of Working Gas	m_g	kg	1.75E-05

	Symbol	Unit	Value
Gas Constant	R	J/kg.K	286.9
Working Fluid	k		1.402
	Cv		713.68

In order to construct the p-V diagram of the Gamma Stirling engine model, this requires executing equations given in [5-7]. The result of this calculation is shown in Table 6.

Table 6. The volume and pressure of Gamma Stirling model at each stage

State	Volume	Pressure
x	V(x), m^3	p(x), Pa
1	1.45E-05	1.12E+05
2	1.28E-05	1.27E+05
3	1.28E-05	1.86E+05
4	1.45E-05	1.64E+05

Lastly, the heat and work done by the Gamma Stirling engine model is obtained using equation in [5-7]. This is based on the Stirling cycle process of a Stirling engine.

Table 7. Heat and work done by Gamma Stirling model

	State 1 - 2	State 2 - 3	State 3 - 4	State 4 - 1
	Q_{12}	Q_{23}	Q_{34}	Q_{41}
Heat Done, Q (J)	-0.2077	1.877	0.3042	-1.877

			Work Done
	Symbol	Unit	W_net
Heat Rejection to Cold Side	Q_12	J	-0.2077
Heat Addition from Hot side	Q_34	J	0.3042
Work Done	W_net	J	0.5119

4.1   Analysis of Gamma Stirling engine model

In relation with the parameters obtained for designing the Gamma Stirling engine model. The next task is to analyze the performance of the small scale of Gamma Stirling engine model. Before proceed to the analysis, there are several reasons to choose the small scale of Gamma Stirling engine model instead of a large size high performance Gamma Stirling engine. One of the reasons is because of budget limitation to fabricate a large size of Gamma Stirling engine model. In fact, this research activity is conducted mostly to focus on searching the accurate formulation required for designing the Gamma Stirling engine model inclusive with the operating process involved and also to analyze the performance of this model. Finally, this small scale of

Gamma Stirling engine model is mainly used to show and highlight the concept of Stirling engine cycle and its capability to produce power.

Table 8. Analysis of the model using different heat source

	Alcohol Lamp	Butane Burner
Speed, rpm	950	1200
Voltage, V	3.5	6
Current, mA	11	20
Power, Watt	0.04	0.12

From the results obtained in Table 8, by applying a butane burner to the Gamma Stirling engine, the speed and power generated by the engine is higher compared to output obtained based on the heat source of alcohol lamp. However, the power generated from the Gamma Stirling engine is not very satisfying due to the current produced is quite low compared to the voltage. Hence, this will not be enough to supply power to a load that required a large current range. In addition to that, the Gamma Stirling engine is not suitable to be linked with the small type of DC motor that produce low amount of output current. Usually, a small size of DC motor has the limitation in its current rating due to the small number of coil winding inside the DC motor.

5. Conclussion
From the comparison obtained between designed formula and an actual Gamma Stirling engine shows that it is possible for the formulation obtained from this research to be used as preliminary design process for Gamma Stirling engine. Thus, based on the formulation and theory that acquired over the course of this research, it can now be understand why and how this formula was acquired and the function of it during each cycle of the Stirling engine. As for the analysis made on the Gamma Stirling model, it shows that it is capable to produce electrical power. However, the power produces by this model is insufficient enough to supply to a load due to its low current output. This is mainly due to the type of DC motor used in conjunction to the Stirling engine. This DC motor has low current rating that are not suitable for a power production and it is only intended to show that Gamma Stirling engines are capable to produce an electrical energy.

Nomenclature
α :Crank angle, deg
$\varphi = 90°$:Phase angle, deg
ε :Compression ratio or volume ratio
ω :Rotational velocity or angular velocity, rad/s
n :Engine speed, rps or Hz
η_{th} :Thermal efficiency
B_e :Beale number
B_{dp} :Bore of displacer piston, m
B_{pp} :Bore of power piston, m
E_f :Rotational kinetic energy, J or Watt
f :Engine speed cycle frequency, Hz
F :Factor F
I :Moment of inertia, kgm^2
k :Specific heat ratio

m, m_g :Total mass of working gas, kg
m_f :Mass of the flywheel, kg
m_C :Mass of gas at compression space, kg
m_E :Mass of gas at expansion space, kg
m_R :Mass of gas at regenerator, kg
N :Engine speed, rpm
p :Engine pressure, Pa
p_m :Mean average pressure, Pa
p_{max} :Maximum engine pressure, Pa
p_{min} :Minimum engine pressure   , Pa
p_L :Preloaded pressure, Pa
P :Engine performance, Watt
Q :Heat done by the engine, J
Q_{12}, Q_{out}:Heat rejection to compression space, J
Q_{23} :Heat addition from regenerator, J
Q_{34}, Q_{in}:Heat addition to expansion space, J
Q_{41} :Heat rejection from regenerator, J
r_f :Radius of the flywheel, m
R :Gas constant, $Jkg^{-1}K^{-1}$
s :Reduced dead space
S_{dp} :Stroke of displacer piston, m
S_{pp} :Stroke of power piston, m
t, t' :Temperature ratio
T_C, T_{min} :Compression space temperature, K
T_D :Dead space temperature, K
T_E, T_{max} :Expansion space temperature, K
T_L :Preloaded temperature, K
T_R :Regenerator temperature, K
v :Swept volume ratio
V :Total dead volume, m^3
V_C :Compression space volume, m^3
V_D :Total dead volume, m^3
V_{DC} :Dead volume in compression space, m^3
V_{DE} :Dead volume in expansion space, m^3
V_E :Expansion space volume, m^3
V_R :Regenerator space volume, m^3
V_{SC}, V_o :Swept volume of power piston, m^3
V_{SE} :Swept volume of displacer piston, m^3
W, W_{net} :Total work done by the engine, J
W_f :Work done by the flywheel, J
X_D :Dead volume ratio

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