

## Position Synchronization of Electronic Virtual Line Shafting with Sliding Mode Variable Structure Control

Zhang changfan<sup>\*1</sup>, Xiao Yuanyuan<sup>1</sup>, Wen Long<sup>1</sup>, He Jing<sup>1,2</sup>

<sup>1</sup>College of Electrical and Information Engineering, Hunan University of Technology,  
Zhuzhou Hunan 412007, China

<sup>2</sup>College of Mechatronics and Automation, National University of Defense Technology,  
Hunan, 410000, China

\*Corresponding author, e-mail: 35113479@qq.com

### Abstract

*The degree of precision for positioning of adjustable rollers during pre-registering stage has direct influences on register precision of the entire printing. The paper puts forward control strategy of sliding mode variable structure-based electronic virtual line shafting based on the influences of low-speed pre-register process of shaft-less drive printing press on register precision and the influences of nonlinearity and disturbance on synchronous precision of printing process. It also proves the stability of the control algorithm using Lyapunov function. The experimental results demonstrate that the control strategy proposed by this paper can realize synchronous control of shaft-less drive printing press, and also can inhibit effectively the influences on synchronization of system position due to change of parameters and friction during pre-registering stage.*

**Keywords:** position synchronization, electronic virtual line shafting, sliding mode variable structure

**Copyright © 2014 Institute of Advanced Engineering and Science. All rights reserved.**

### 1. Introduction

Shaft-less drive technology is widely used in printing, textile, paper-making, printing and dyeing, steel rolling and other production processes [1]. The key point of the application of shaft-less drive in production process is how to obtain good synchronous control strategy via design so that high-precision synchronization can be ensured in production process with nonlinearity and disturbance.

The conventional synchronization control strategy mainly contains cross coupling control, relative coupling control, master control, electronic virtual line shafting control and etc. [2]. Koren proposes cross coupling synchronization strategy which solves effectively synchronization control problem of twin motor [3]. Turl and others introduce expansion form of cross coupling structure based on this [4]. However, the proposed method has the disadvantage that the compensation law is difficult to define when the quantities of motor are more than 2. Therefore, Perez et al propose relative coupling synchronization control [5]. This method has good synchronous performance, but the control structure will become complicated [5] with increase of number of motors. Master control is a normal synchronization control strategy, for which the basic control concept is to realize synchronization by the way that making one state-variable motor as guide shaft and the remain shafts to follow the guide shaft. The synchronization error after combination of each shaft may not be the minimum because there isn't coupling between shafts in the event that disturbance is occurred on a shaft, although the tracking error can be minimal via control strategy. Lorenz and Meyer proposed the control method of electronic virtual line shafting in 1999 in order to compensate the deficiencies of master control, based on which further development is made by Valenzuela and Lorenz [6]. The control method introduces restoring torque feedback process that not included in master control to simulate physical properties of mechanical shafting based on master control. It has inherent synchronization property similar to that of mechanical shafting. Therefore, it's widely applied on actual engineering [7].

Synchronous coordinating running of each printing roller of shaft-less drive printing press is mainly by independent drive servo motor. Register control will be used in case there's chromatic aberration, which results in that higher multi-shaft synchronization control

requirements are needed by the system. Drive motor is a complicated nonlinear object with time-varying parameters in servo system of shaft-less drive printing press. The robust performance on disturbance and parameter variation resistance of normal PID control is not adequate. Thus it's difficult to obtain satisfied speed adjustment and positioning performance. Therefore, how to find a multi-shaft synchronization method of high performance for shaft-less drive printing press has been a topic with very good application property. Sliding mode variable structure is a discontinuous nonlinear control [8]. The system will have more superior invariance than robust when moves on sliding surface [9]. In addition, it has the properties of simple algorithm and good real-time ability and easy engineering realization. Thus its application in high-precision tracking control has attracted great attention.

There's friction in all motion systems, especially that friction has prominent influences on high-performance servo system. As for shaft-less drive printing press, friction is an important factor affecting performance of system when it runs at low speed. It will cause steady-state error of system and affect register precision. It can also result in oscillation in system. Reference [10] and [11] apply electronic virtual line shafting control strategy into printing press to reduce the influences of disturbance to system, which make the system have good robust. However, the author didn't take the influences of friction on register precision of system at low-speed pre-register stage of printing press into consideration. The paper proposes control strategy of sliding mode variable structure-based electronic virtual line shafting based on the adverse effects caused by friction of pre-register process of shaft-less drive printing press and system disturbance normally occurred during running of printing press. It also proves the stability of the algorithm using Lyapunov function. Simulation and verification is carried out for four-shaft intaglio printing system including electronic virtual line shafting on Matlab experimental platform. The experimental results verify the effectiveness of sliding mode variable structure-based electronic virtual line shafting synchronization strategy.

## 2. Mathematical Model of System

Synchronous coordinating running of each printing roller of shaft-less drive printing press is mainly by independent drive servo motor. Register control will be carried out in case of occurrence of chromatic aberration. All of these will result in fast, accurate and steady control of the system with respect to position and speed of objects. There're many models of servo drive motor. It's needed to emphasize disturbance torque in motor model in order to demonstrate performance of electronic virtual line shafting-based control strategy when system parameters are variable and at the time when there's external disturbance in system. In the paper it selects DC motor as servo drive motor. The schematic of single DC motor is as follows:

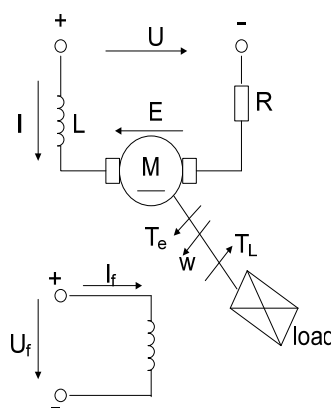


Figure 1. DC Motor Circuit and Mechanical Connection Schematic

Friction shall be taken into consideration of mathematic model of motor at low speed in order to improve precision of register and to reduce register error. Following motor voltage and torque balance equation can be obtained combining with Figure 1:

$$\begin{cases} U = RI + L \frac{dI}{dt} + E \\ E = c_e \omega \\ T_e - T_L - F_f(t) = J \frac{d\omega}{dt} \\ T_e = k_m I \end{cases} \quad (1)$$

Where  $U$  is armature voltage,  $R$  is total resistance of armature circuit,  $L$  is armature inductance,  $\omega$  is turning speed of DC motor,  $I$  is armature current of motor,  $E$  is induced voltage of armature,  $c_e$  is voltage feedback coefficient,  $J$  is inertia,  $k_m$  is electromechanical torque coefficient,  $T_L$  is load torque,  $T_e$  is electromechanical torque generated by motor,  $F_f(t)$  is friction, and it's mathematic model can be expressed as follows [12]:

When  $|\dot{\theta}(t)| < \alpha$ , the static friction is:

$$F_f(t) = \begin{cases} F_m & F_t > F_m \\ F_t & -F_m < F_t < F_m \\ -F_m & F_t < -F_m \end{cases} \quad (2)$$

When  $|\dot{\theta}(t)| > \alpha$ , the kinetic friction is:

$$F_f(t) = [F_c + (F_m - F_c)e^{-\alpha_1|\dot{\theta}(t)|}] \text{sgn}(\dot{\theta}(t)) + k_v \dot{\theta}(t) \quad (3)$$

$$F(t) = J \ddot{\theta}(t) \quad (4)$$

Where  $F(t)$  is driving force,  $F_m$  is the maximum static friction,  $F_c$  is coulomb friction,  $k_v$  is viscous friction torque proportional coefficient,  $\dot{\theta}(t)$  is angular velocity of rotation,  $\alpha$  and  $\alpha_1$  is very small positive constant.

### 3. Design of Single Shaft Tracking System

Since that multi-shaft synchronization of shaft-less drive printing press contains contents of two layers (i.e. tracking control and cooperative control), in the paper the author designs sliding mode variable structure controller for single shaft system firstly in order to reduce tracking error of single shaft and to improve precision of synchronization control of multi-shaft. The control structure of single shaft system is as shown in Figure 2.

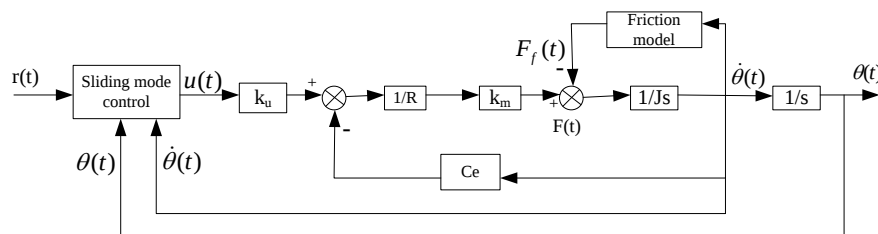


Figure 2. Control Structure Diagram of Single Shaft System

The state equation of system can be obtained based on control architecture diagram of single shaft system and Equation (2):

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\frac{k_m c_e}{JR} x_2(t) + k_u \frac{k_m}{JR} u(t) - \frac{1}{J} F_f(t) - d(t) \end{cases} \quad (5)$$

where  $x_1(t)$  is angle  $\theta(t)$ ,  $x_2(t)$  is speed  $\omega(t)$ , i.e.  $\dot{\theta}(t)$ ,  $K_u$  is PWM coefficient of amplification of power amplifier,  $u(t)$  is control output. It's assumed that  $\bar{a}_1 = -\frac{k_m c_e}{JR}$ ,  $\bar{b} = \frac{k_u k_m}{JR}$  and  $\bar{a}_2 = -\frac{1}{J}$  are three nominal values. When parameters are known and there isn't external load disturbance, Equation (5) can be changed to be:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \bar{a}_1 x_2(t) + \bar{b} u(t) + \bar{a}_2 F_f(t) \end{cases} \quad (6)$$

Considering parameter drifting and disturbance of external load of system, the dynamic Equation (6) can be changed to be:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = a_1 x_2(t) + b u(t) + a_2 F_f(t) - d(t) \end{cases} \quad (7)$$

Where  $a_1 = \bar{a}_1 + \Delta a_1$ ,  $b = \bar{b} + \Delta b$ ,  $a_2 = \bar{a}_2 + \Delta a_2$ .  $\Delta a_1$ ,  $\Delta b$  and  $\Delta a_2$  are parameter variation uncertain items.

$r(t)$  is defined as command signal, control error  $e = r(t) - x_1(t)$ , and then  $x_1(t) = r(t) - e$ ,  $x_2(t) = \dot{x}_1(t) = \dot{r}(t) - \dot{e}$ . Make  $e_1 = e$ , and the position state equation can be changed to be error state equation:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = \ddot{r}(t) - \bar{a}_1(\dot{r}(t) - \dot{e}_1) - \bar{b}u(t) - \bar{a}_2 F_f(t) + d(t) - \Sigma\Delta \end{cases} \quad (8)$$

Where  $\Sigma\Delta = \Delta a_1(\dot{r}(t) - \dot{e}_1) + \Delta b u(t) + \Delta a_2 F_f(t)$  are uncertain items,  $M$  is upper limit of  $\Sigma\Delta$ , for which  $|\Sigma\Delta| \leq M$ ,  $d$  is external disturbance,  $D$  is upper limit of  $d$ , for which  $|d| \leq D$ .

Select sliding surface as follow:

$$s(t) = c e + \dot{e} = c e_1 + e_2 \quad (c > 0) \quad (9)$$

Design control law is:

$$u(t) = \frac{1}{b} \left[ (c + \bar{a}_1) \dot{e}(t) + \ddot{r}(t) - \bar{a}_1 \dot{r}(t) - \bar{a}_2 F_f(t) - \beta(x, t) \operatorname{sgn}(s(t)) \right] \quad (10)$$

Where  $\beta(x, t)$  is control gain:

$$\beta(x, t) = M + D + \eta \quad (\eta > 0) \quad (11)$$

Stability of system proves that:

Take the derivative of sliding surface

$$\dot{s}(t) = c \dot{e}_1 + \dot{e}_2 = c \dot{e}_1 + \ddot{r}(t) - \bar{a}_1 \dot{r}(t) - \bar{b}u(t) - \bar{a}_2 F_f(t) - d(t) - \Sigma\Delta \quad (12)$$

Introduce  $u(t)$  into above equation, and then the following can be obtained.

$$\dot{s}(t) = d(t) + \Sigma\Delta - \beta(x, t) \operatorname{sgn}(s) \quad (13)$$

Select Lyapunov function  $V_T(t) = \frac{1}{2}s^2(t)$ , and then take the derivative with respect to time  $t$  the following can be obtained:

$$\begin{aligned} \dot{V}_T &= s(t)\dot{s}(t) \\ &= s(t)[d(t) + \Sigma\Delta - \beta(x, t) \operatorname{sgn}(s)s(t)] \\ &= s(t)[d(t) + \Sigma\Delta - \beta(x, t)|s(t)|] \\ &\leq |s(t)|[|d(t) + \Sigma\Delta| - \beta(x, t)|s(t)|] \\ &\leq |s(t)|[|d(t)| + |\Sigma\Delta| - \beta(x, t)|s(t)|] \\ &\leq |s(t)|[M + D + \eta] \\ &= -\eta|s(t)| \end{aligned} \quad (14)$$

i.e.  $\frac{1}{2} \frac{d}{dt}(s^2(t)) \leq -\eta|s(t)|$ , it can be known that index of system is stable based on stability criterion of Lyapunov.

In the paper, the method of relay characteristics continuity is adopted to reduce chattering since that there's chattering on sliding surface, i.e. replace  $\operatorname{sgn}(s)$  with continuous function  $\lambda(s)$  [8].

$$\lambda(s) = \frac{s}{|s| + \delta} \quad (15)$$

Where  $\delta$  is very small positive constant.

#### 4. Design of Multi-Shaft Synchronous System

In addition to design of effective synchronous control algorithm to ensure tracking precision and robust performance of single shaft servo system in shaft-less drive printing press, accurate synchronization requirements between shafts shall also be considered, i.e. to ensure speed synchronization and position synchronization between shafts (namely cooperative control).

Cooperative control of most domestic printing presses adopts shafting drive control. Shafting drive control (it's also referred as mechanical shaft drive) refers to activating drive elements of each printing press unit using one main motor of the system, but it's uneasy to modify since damping gain, stiffness gain, attenuation gain and other parameters are completely determined by mechanical shaft itself. The parameters of coupling shaft of electronic virtual line shafting system can be realized by software, which is not the same as mechanical shaft system (parameters of the system depend on structure of mechanical shaft itself while the structure is difficult to modify). Therefore, it has relatively big flexibility. Furthermore, the system can have appropriate damping gain by adjusting of parameters so that the dynamic performance of system can be improved. Structure diagram of electronic virtual line shafting system is shown as follow.

It can be known from Figure 3 that electronic virtual line shafting control is to introduce restoring torque feedback process based on master control while master control doesn't have restoring torque feedback. Thus it can simulate physical features of mechanical shaft and make the system have inherent synchronous feature similar to that of mechanical shaft. Each shaft can keep good synchronization performance with electronic virtual line shafting when the system is at steady state. When one of the shafts or more shafts are disturbed, the electronic virtual line shafting can feel the disturbance by restoring torque feedback process and it will then coordinate movement of other shafts in order to achieve the objective of synchronization.

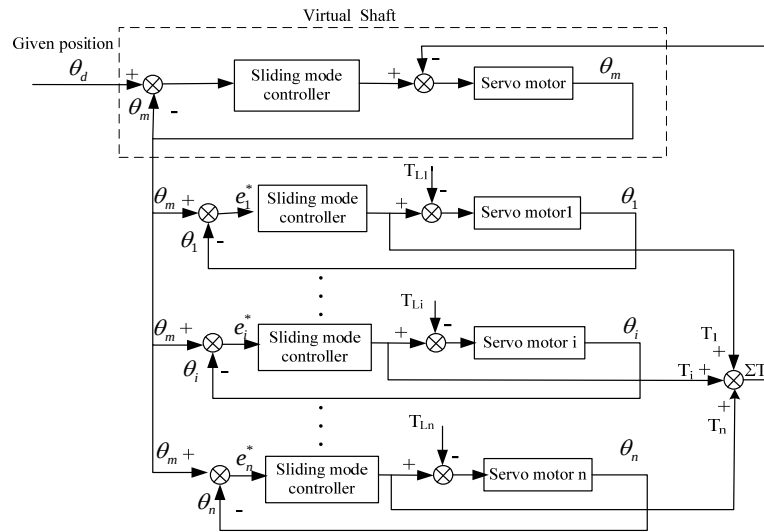


Figure 3. Structure Diagram of Electronic Virtual Line Shifting Control-Based Multi-shaft Coordinating Control System

Feedback can be made to main shaft with respect to restoring torque applied on each moving shaft via synchronous mechanical device as well to make it keep balance with shaft drive torque, and its balance equation is as follow:

$$T - \sum T_i = J_m \ddot{\theta}_m \quad (16)$$

Where  $T_i$  ( $i=1,2,\dots,n$ ) is the torque made by feedback of each moving shaft, i.e. restoring torque.  $T$  is drive torque of main shaft,  $J_m$  is inertia of main shaft,  $\theta_m$  is displacement of output angle of main shaft.

Multi-shaft of shaft-less drive printing press refers to speed and position of each drive motor keep bisynchronous at steady state or transient state without mechanical spindle. Angular speed  $\omega$  is differential of angular displacement  $\theta$  based on mathematic relation:

$$\omega = \frac{d}{dt} \theta \quad (17)$$

While difference of angular speeds is differential of difference of angular displacement:

$$T_i = b_r \frac{d}{dt} \Delta \theta + k_r \Delta \theta + k_{ir} \int \Delta \theta dt \quad (18)$$

Where  $\omega_m$  is angular speed of output of main shaft (i.e. reference angular speed),  $\omega_i$  is angular speed of output of any one shaft,  $\theta_m$  is angular displacement of main shaft (i.e. reference angular displacement),  $\theta_i$  is angular displacement of output of any one shaft.

Tracking error of No.  $i$  shaft is  $e_i^* = \theta_m - \theta_i$ ; the synchronous error between No.  $i$  shaft and No.  $i+1$  shaft is  $E_i = e_i^* - e_{i+1}^*$ . When load  $T_{Li}$  of any shaft (for example the No.  $i$  shaft) is disturbed,  $e_i^*$  can converge to zero by adjusting of controller in order to obtain synchronous performance at steady state. However, desynchronization of system at the transient process of disturbance will result in generation of corresponding stiffness torque (i.e. restoring torque) by shaft of the unit. Feedback of the restoring torque is made to electronic virtual line shifting by

electronic virtual line shafting control strategy, and the shafting forces other shafts to track the variation to make synchronous error  $E_i$  keep zero.

**Lemma** Tensile stress  $\sigma$  is proportional to tensile strain  $\varepsilon$  within elastic limits of object, and its proportional coefficient is referred as modulus of elasticity  $E$ . The expression is  $\sigma = E\varepsilon$ .

The restoring torque of each drive shaft deduced from Hook's Law is:

$$T_i = b_r \frac{d}{dt} \Delta\theta + k_r \Delta\theta + k_{ir} \int \Delta\theta dt \quad (19)$$

Where  $\Delta\theta$  is difference of accumulated movement angular displacements of each shaft at input and output ends;  $b_r$  is damping gain of each shaft coupling device;  $k_r$  is stiffness gain of each shaft coupling device;  $k_{ir}$  is integrated stiffness gain of each shaft coupling device.

Multi-shaft synchronous control is the core topic of shaft-less drive printing press, and the concept of electronic virtual line shafting control strategy is just to ensure synchronization between shafts. Torque integration and feedback is one of the main features of electronic virtual line shafting control strategy. The combined action is to make each shaft as a whole but not to consider difference between shafts. Therefore, change of any one shaft or more shafts will change the comprehensive torque, which will then be transferred to electronic virtual line shafting via feedback process. The electronic virtual line shafting will modify current output value according to change of torque so that the entire system will return to synchronous state and the dynamic response of torque feedback will become fast as well.

## 5. Examples

### 5.1. Setting of Experimental Parameters

Establish simulation model in Simulink environment taking four shafts of shaft-less drive printing press as an example. Strong nonlinearity can only be reflected by the system when the printing press is at pre-register stage of low speed. Therefore, low-amplitude and low-frequency sine signal is used as given position signal during test, i.e.  $r = 0.1\sin(2\pi t)\text{rad}$ , nominal value of

system  $R = 7.77\Omega$ ,  $k_u = 11$ ,  $J = 0.06\text{kgm}^2$ ,  $C_e = 1.2\text{V}/(\text{rad}/\text{s})$ ,  $F_c = 15\text{Nm}$ ,  $k_v = 2.0\text{Nms}/\text{rad}$ ,  $k_m = 6\text{Nm}/\text{A}$ ,  $J_m = 8 \cdot 10^{-4}\text{kgm}^2$ ,  $F_m = 20\text{Nm}$ ,  $\alpha = 0.01$ ,  $\alpha_1 = 0.1$ ,  $b_r = 0.5$ ,  $k_r = 0.8$ ,  $k_{ir} = 0.025$ , parameters of control system are  $k_t = 5$ ,  $c = 30$  and  $\delta = 0.01$ , respectively. The initial state of system is  $[-0.5; 1]$ .

### 5.2. The Experimental Results

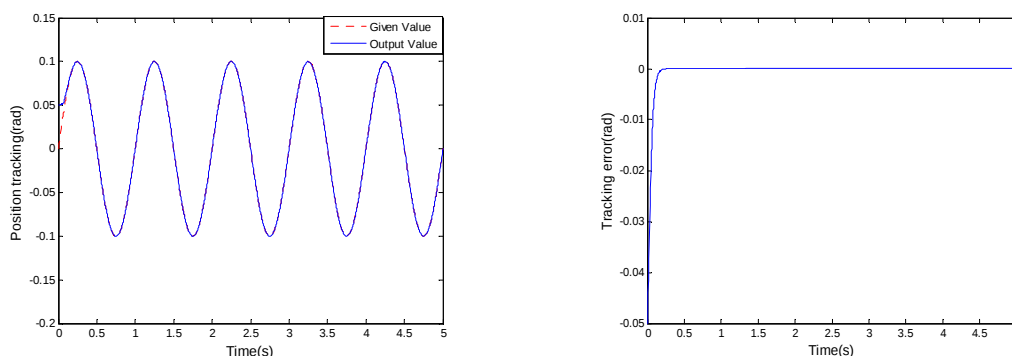


Figure 4. Single-shaft Position Tracking and Tracking Error when there's no External Load Disturbance

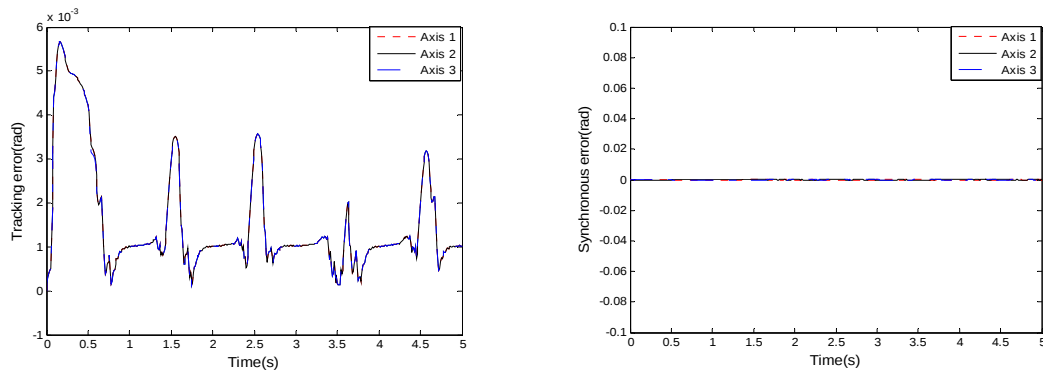


Figure 5. Multi-shaft Tracking Error and Synchronous Error when there's no External Load Disturbance

### (1) Case1

The single shaft tracking and multi-shaft synchronous simulation wave shape are shown in Figure 4 and Figure 5 when there's no external disturbance. It can be known from Figure 4 that actual value of system can track the given value within a short time with the help of slide mode variable structure. The tracking error is converged to be zero after  $t > 0.2s$ . It can be known from Figure 5 that tracking error and synchronous error of multi-shaft are basically converged to be zero.

### (2) Case 2

Add another load torque of  $5 N \cdot m$  from shaft No. 2 at 1s, tracking error and synchronous error of each shaft are shown in Figure 6. Sliding mode variable structure controller has fast tracking performance for external disturbance of single shaft and it can make synchronous error keep stable when there's disturbance. It meets the requirements of high performance of shaft-less drive printing press.

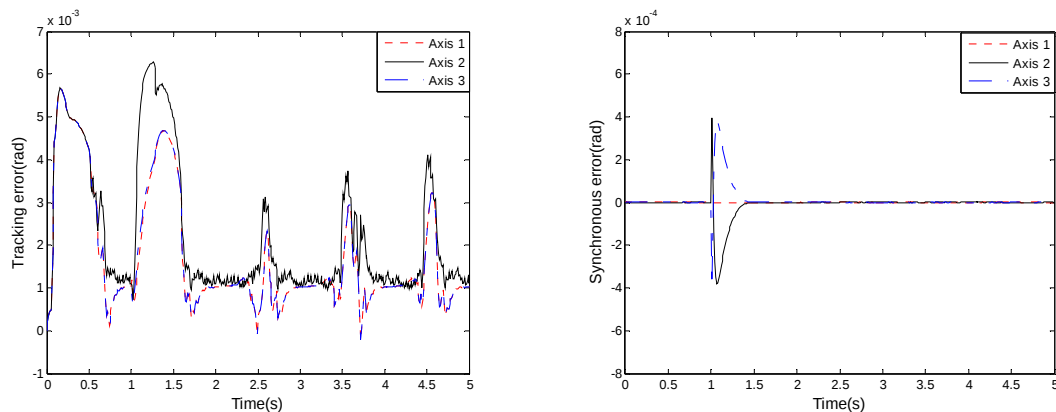


Figure 6. Tracking Error and Synchronous Error of each Shaft when there's Load Disturbance on Single Shaft

### (3) Case 3

Add another load torque of  $5 N \cdot m$  from shaft No. 1 and No. 2 respectively at 1s, and apply another load torque of  $3 N \cdot m$  from shaft No. 3 at 2s, tracking error and synchronous error of each shaft are shown in Figure 7. Sliding mode variable structure controller still has fast tracking performance when there're external disturbances on multi-shaft and the convergence speed of synchronous error is fast. Thus it can still meet the requirements of high performance of shaft-less drive printing press.

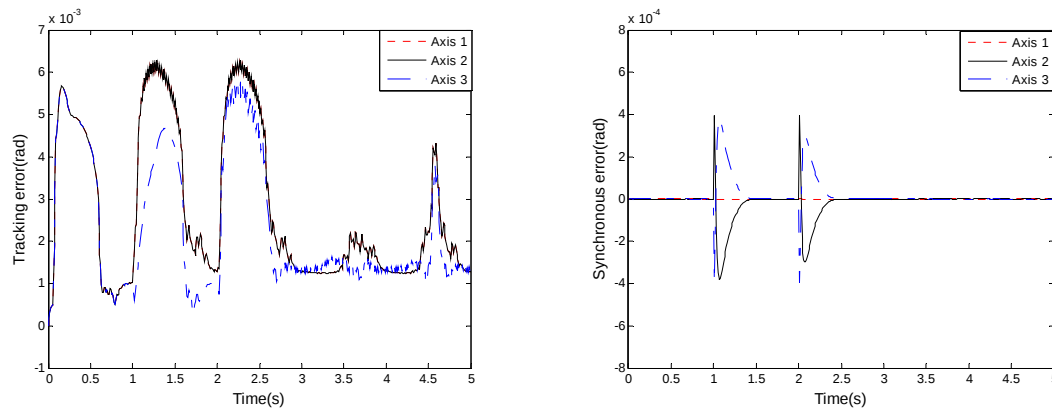


Figure 7. Tracking Error and Synchronous Error when there're Load Disturbances on Multi-shaft

Above simulation results demonstrate that: electronic virtual line shafting control system can provide the system with good synchronous performance although it makes the system lose tracking performance. As for printing equipment, synchronization between shafts is more important. In addition, sliding mode variable structure controller can make the system keep steady operation when there's disturbance, which improves robust performance, steady state performance and synchronous performance of system.

## 6. Conclusion

1) The paper proposes control strategy of sliding mode variable structure-based electronic virtual line shafting, improves register precision of system and reduces register error based on the influences of friction on register precision during low-speed pre-register running process of shaft-less drive printing press.

2) Sliding mode variable structure controller is designed for each shaft based on the influences of nonlinearity and disturbance on synchronous precision of printing process so that synchronous error and tracking error can be converged to be zero within limited time.

3) The experimental results demonstrate that the control strategy of sliding mode variable structure-based electronic virtual line shafting can effectively improve the influences of friction on register precision during pre-registering stage of low speed. Meanwhile, the control strategy also effectively improves the influences of parameter variation of system and external disturbance on synchronous performance of system.

## Acknowledgements

This work is supported by Natural Science Foundation of China (No. 61273157 and 61104024)

## References

- [1] Chen C, Liu X, Liu G, et al. Multi-motor synchronous system based on neural network control[C]. *Proceedings of the 11th International Conference on Electrical Machines and Systems*. 2008; 3: 1231-1236.
- [2] Perez-Pinal FJ, Nunez C, Alvarez R, et al. Comparison of Multi-motor Synchronization Techniques. *The 30th Annual Conference of the IEEE Industrial Electronics Society*, Korea. 2004: 1670-1675.
- [3] Sun D, Shao X, Feng G. A Model-Free Cross-Coupled Control for Position Synchronization of Multi-Axis Motions: Theory and Experiments. *IEEE Trans. Control Systems Technology*. 2007; 15(2): 306-314.
- [4] Sun D. Position synchronization of multiple motion axes with adaptive coupling control. *Automatica*, 2003; 39(6): 997-1005.
- [5] Chen C, Chen L. Robust crossing-coupling synchronous control by shaping position commands in multi-axes system. *IEEE Trans. Industrial Electronocs*. 2012; 59(3): 2320-2314.

- [6] Perez-Pinal FJ, Calderon G, Araujo-Vargas I. Relative Coupling Strategy. *IEEE International Electric Machines and Drives Conference. Madison*. 2003: 1162-1166.
- [7] Valenzuela A, Lorenz R. Electronic line-shafting control for paper machine drives. *IEEE Transaction on Industry Applications*. 2001; 137(1): 15-19.
- [8] Xiao Y, Zhu K, Liaw HC. Generalized synchronization control of multi-axis motion systems. *Control Engineering Practice*. 2005; 13: 809-819.
- [9] Utkin VI. Sliding mode and their application in VSSs. Moscow. 1978.
- [10] Sabanovic A. Variable structure systems with sliding modes in motion control a survey. *IEEE Trans. Industrial Informatics*. 2011; 7(2): 212-223.
- [11] Valenzuela MA, Lorenz RD. Startup and commissioning procedures for electronically line-shafted paper machine drives. *IEEE Trans. Industry Applications*. 2002; (4): 966-973.
- [12] Perez-Pinal FJ. Improvement of the electronic line-shafting. *35<sup>th</sup> Annual IEEE Power Electronics Specilists Conference*, Aachen Germany: IEEE Power Electronics Specilists Conference. 2004: 3260-3265.
- [13] Lee SW, Kim JH. Robust adaptive stick-slip friction compensation. *IEEE Trans. on Industrial Electrons*. 1995; 42(5): 474-479.