Cyclic redundancy check-aided successive cancellation-based polar decoders

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ABSTRACT
Research on channel coding for network transmission using polar codes has produced excellent results. By removing error redundancy from the decoding process, cyclic redundancy check (CRC) is frequently used by researchers to increase a system's performance. In prior research, the application of decoder algorithms for polar codes was examined but not thoroughly compared. For the general capabilities of the previously proposed algorithms to be ascertained, it is crucial to analyze the employment of polar decoders especially successive cancellation (SC)-based polar decoders and the use of CRC in additive white gaussian noise (AWGN). Hence, this paper analyzes the performance of CRC with SC-based polar decoders in AWGN. In the simulation setup, (256,128) polar codes and CRCs with three-bit sizes (6, 8, and 11) were utilized. SC-based polar decoders, such as SC, soft-output cancellation, SC list (SCL) and simplified SC, were applied at the decoder part. The outcomes show that CRC-aided SC-based polar decoders reduced redundancy error. Among all the decoders, the SCL decoder with 11-bits CRC performed well when the normalized signal-to-noise ratio was high. Based on the analysis, removing errors at the highest level is possible using a suitable CRC size for SC-based polar decoders.

Keywords:
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Simplified successive cancellation
Soft-output cancellation
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Successive cancellation list

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1. INTRODUCTION
Polar codes with a low block error rate (BLER), low complexity, and low latency performances can be achieved depending on the design of decoders [1]. Polar codes are the first to strictly achieve channel capacity in binary-input discrete memoryless channels [2]. Over the last decade, there has been a steady increase in academic and industrial interest in polar codes [3]. Such codes have been adopted as channel coding for uplink and downlink control information for enhanced mobile broadband communication services as part of the third-generation partnership project’s ongoing 5G wireless systems standardization process. Thus, much effort has gone into developing polar codes that are simple to implement, have low description complexity, and provide good error correction performance across a wide range of code and channel parameters. The successive cancellation (SC) decoder was the first decoder to be integrated into polar codes that could provide low computational complexity [1]. This decoder is essentially a kind of greedy algorithm; thus, its output is based on optimal local solutions and results in a high BLER. With the aid of cyclic redundancy check (CRC)
and list decoding, known as a CRC-aided SC list [4]. This decoder with polar codes was claimed to have superior BLER performance compared to other codes. After its introduction in 1962, it was first used as a decoder by Arikan [5], Sun et al. [6].

Some improvements, such as the implementation of post-processing techniques [1], [5], bit-flip [1], [7] and a critical set [7], were made by researchers to enhance the BLER performance of this decoder. In other research, soft-output cancellation (SCAN) was claimed to be the best decoder. It was allegedly better than the SC decoder and could provide soft output for parity check-polar codes [8]. Later, a simplified SC (SSC) decoder was introduced by [9] to reduce the decoding latency and complexity of traditional SC and SCL while maintaining the bit error rate (BER) and BLER. The enhancement of the SSC decoder was subsequently proposed. For example, the fast simplified SCL was introduced, which uses the fastest approach to decoding rate-1 nodes in polar codes [10]. Further, an SSC with medium-dimensional binary kernels [11] and an adaptive path strategy based on path metric (PM) were proposed [12].

In each of the above-mentioned works, the researchers strongly emphasized complexity, efficiency, latency, and error correction performance for system evaluation. Overall, each decoder method and its improvement schemes improved the error detection of the original schemes but at the cost of increasing decoding complexity, latency, and high error correction performances [13]. However, previous research has not analyzed and compared all SC, SCAN, SCL, and SSC decoders. Additionally, some methods were not explicitly focused on the usage of CRC with the SC-based polar decoders in additive white gaussian noise (AWGN) [14]. Various decoding enhancements were evaluated for only specific decoders with CRC, such as an early termination for the SCL decoder [15] and a bit-flip criterion for the SCL decoder [16]. Therefore, this study contributes to the literature by analyzing and comparing the performance of these decoding algorithms in AWGN while focusing on a polar code with CRC.

The remainder of this paper is organized as follows: In sections 2 and 3 describe the polar encoder and decoder algorithms integrated with the CRC. In section 4 describes the parameters used for the simulation work. The simulation results are presented and briefly discussed in section 5. Finally, the conclusions drawn from the analysis of the experimental simulation are presented in section 6.

2. CRC-AIDED POLAR CODES

Figure 1 is a block diagram of the transmitter system integrated with a CRC generator and polar encoder schemes. The message bits, \( k \) bits, are encoded with CRC bits and produce CRC-message bits, \( K \). At the CRC generator, there are data word bits, \( k \) bits, and codeword bits, \( n \) bits. The generator receives the \( k \) bits result, and the data word is then expanded by adding \( 0s \) (e.g. 3 bits) to the word’s right side [17]. Then, \( K \) bits are encoded using a polar encoder, and the encoded bits, \( N \) bits, are produced. After that, all the encoded bits are modulated in the modulator scheme and, finally, transmitted through the independent transmission channel, \( W \).

![Figure 1. Transmitter system with CRC generator and polar encoder [18], [19]](image)

Figure 2 presents the CRC schemes with a polar decoder for the general receiver of the network transmission system. The received bits are fed into the demodulator scheme, where the original information-bearing signal will be extracted from a carrier wave. Then, the demodulated bits are decoded as original messages through the polar decoder schemes. In a CRC, a sequence of redundant bits from the polar decoder are added to the checker and perform division. At the receiver, the received codeword could be corrupted. All \( n \) bits of the codeword are fed into the checker and produce the remainder.

The generated CRC bits are then compared against the decoded CRC bits to perform a CRC. The remainder is a syndrome of 3-bits (\( n-k \) bits), which is fed into the decision logic analyzer. A remainder of 0 means that no errors were presented during transmission and the data were received in their original, error-free state. Otherwise, the data are rejected at the receiver, and a retransmission request is made since the codeword was corrupted during transmission [20]. A successful CRC is recorded if the data are equivalent, in which case no errors are detected. Otherwise, the decoded information sequence and CRC bits are fed into the CRC generator to attain the bit syndrome [19].
3. **POLAR DECODER ALGORITHMS**

The polar decoder is essential for the receiver to decode and correct the received data from noise. This section discusses the different SC types of decoders: SC, SCAN, SCL, and SSC. All algorithms are explained in the following subsection.

3.1. Successive cancellation

Polar codes are decoded by SC decoding based on [2] and [4] as (1). The estimated bits are denoted by ūiN, and the received codeword is denoted by y1N. When the source bit, ūi, is frozen, it is assigned to the fixed value; otherwise, it is calculated using (1). Any N is represented as the i-th number. The metric value h(yiN, ūi−1) is calculated as shown in (2).

\[
\hat{u}_i = \begin{cases} 
0 & h(y_i^N, \hat{u}_i^N) > 0 \\
1 & \text{otherwise} 
\end{cases} 
\]  
(1)

\[
h(y_i^N, \hat{u}_i^{-1}) = M_i^{(1)}(u_i = 0, \hat{u}_i^{-1}|y_i^N) - M_i^{(1)}(u_i = 1, \hat{u}_i^{-1}|y_i^N) 
\]  
(2)

In (2), M_i^{(1)} is the logarithmic posteriori probability, which can be determined recursively from the logarithmic posteriori probabilities of the channel denoted as W. If N = 1, then M_1^{(1)}(x|y) is calculated using (3). Here, (y|x) represents the channel transition probabilities, x ∈ X, y ∈ Y, where x denotes the input alphabet and y denotes the output alphabet based on communication over a generic symmetric binary-input, discrete, memoryless channels, W [11], [18].

\[
M_1^{(1)}(x|y) = \log \frac{W(y|x)}{\sum_y W(y|x)} 
\]  
(3)

3.2. Soft-output cancellation (SCAN)

The SCAN decoder calculates the log-likelihood ratio with channel observations, y0y1 (u1 is assumed to be equally likely to be 0 or 1), using (4) [22]. L_j(φ, ω) is the log-likelihood value corresponding to the node, which is defined by the trio (λ, φ, ω). λ is represented as columns indexed by λ ∈ {0, ..., n}. Each column consists of 2λ groups indexed by φ ∈ {0, ..., 2λ − 1}, and each group consists of 2n − λ nodes, represented by ω ∈ {0, ..., 2n − λ − 1} [23]. The message bits for all ω ∈ {0, ..., N − 1} are estimated using (5). \( \tilde{m}_i \) is the estimated message bits, \( \tilde{c} \) is the complement as the set of ‘frozen indices,’ and L_0(i, 0) denotes the LLRs received from the channel.

\[
L_2(φ, ω) = L_{λ−1}(ψ, 2ω) \oplus L_{λ−1}(ψ, 2ω + 1) 
\]  
(4)

\[
\tilde{m}_i = \begin{cases} 
0 & \text{if } i \notin \tilde{c} \text{ or } L_0(i, 0) \geq 0 \\
1 & \text{otherwise} 
\end{cases} 
\]  
(5)

3.3. Successive cancellation list

Assuming that the transmitted codeword is W^N0−1 and the obtained code word is y^N0−1, the log-
likelihood ratio, L_N, of the approximated ūi of the information bits, ui, can be expressed as in (6). In this equation, the transfer likelihood of the i-th subchannel is expressed by W_i^{(1)}(y_i^N−1, u_i^N−1|u_i) [24]. Thus, ū = \( \delta(L_N^{(1)}) \) in the formula \( \delta(x) = \frac{1}{2}(1 - \text{sign}(x)) \).

\[
L_N^i = \left(y_i^N−1, u_i^N−1\right) \ln \frac{W_i^{(1)}(y_i^N−1, u_i^N−1)}{W_i^{(1)}(y_i^N−1, u_i^N−1)} 
\]  
(6)

As shown in (7)-(10), the PM is needed to determine the final decoded output (when the path exceeds L). It is also to indicate which part needs to be deleted. According to Wang et al. [12], Sarkis and Gross [25], PM is well-approximated, as shown in (10).
\[ PM_i^l = -\ln \left( P \left[ U_i^l \mid Y = y_0^{N-1} \right] \right) \]  
\[ = PM_i^{l-1} + \ln \left( 1 + e^{-|g_i^l|} \right) \]  
\[ = PM_i^{l-1} + \ln \left( 1 + e^{-(|g_i^l|+|h_i^l|)} \right) \]  
\[ PM_i^l \approx \begin{cases} 
PM_{i-1} & \text{if } \bar{u}_i[1] = \delta(L_N^{(i)}) \\
PM_{i-1} + \lceil L_N^{(i)} \rceil & \text{otherwise}
\end{cases} \]  

### 3.4. Simplified successive cancellation

The conventional SC decoding process is modified by changing the local decoding algorithm at a rate of one node, \( v \). Each node, \( v \), acts as a decoder for its constituent code. When it is activated, \( \beta_v \) is calculated as in (11). \( \beta_v \) represents a summary of all codewords computed in the subtree rooted at \( v \), while \( h \) represents the binary quantizer that takes on a value of either 0 or 1.

\[ \beta_v = h(\alpha_v) \]

This vector is replaced by a soft information vector, \( \alpha_v \), when the decoder at node \( v \) receives \( \alpha_v \) from its parent and produces a codeword, \( \beta_v \). All bits with indices, \( J_v \), are immediately decoded using (12) and (13). \( J_v \) indicates the indices of all leaf nodes and \( G_{n-d_v} \) is the generator matrix for polar codes of a block length of \( 2^n \). At the nodes of \( V_v \), the improved method effectively replaces soft information (which is laboriously obtained) with hard information (which is easily computed).

\[ J_v = \{ \ell(u): u \in V_v \text{ and } u \text{ is a leaf node} \} \]

\[ \hat{u}[\min J_v], \ldots, \hat{u}[\max J_v] = \beta_v G_{n-d_v} \]

### 4. SIMULATION PARAMETERS

A simulation for (256, 128) polar codes was conducted using MATLAB. The polar codes were tested with SC, SCAN, SCL, and SCL decoder, a list size of 32 was selected. Binary phase-shift keying modulation was applied to the encoded bits and then transmitted through the AWGN, \( W \sim N(0, 0.9) \).

### 5. RESULTS AND DISCUSSION

For this analysis, the presence of CRC bits and sizes was used to evaluate each decoder’s BER and frame error rate (FER) performance. The BER and FER performances are compared based on the 6, 8, and 11 CRC sizes. The results are presented in the following subsections.

#### 5.1. CRC-aided SC decoder

Figure 3 shows the performance of SC decoding with three CRC values. The decoding process with a CRC size of 6-bits provided the lowest error rate for both BER and FER in terms of signal-to-noise per bit (Eb/No), with an Eb/No of less than 4 dB. SC decoding with a CRC size of 8-bits showed surprising outcomes, as it achieved the lowest error rate for BER among all CRC sizes when Eb/No approached 5 dB. This finding indicates that the CRC with a size of 8-bits effectively eliminates the error rate as Eb/No increases. Meanwhile, FER for SC decoding with a CRC size of 11-bits decreased FER at 4.5 dB, which is lower than when the CRC size was 8-bits. However, all CRC sizes achieved similar FER performance, as they all reached 5 dB of Eb/No.
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5.2. CRC-aided SCAN decoder

The simulation results for the CRC-aided SCAN decoding scheme are shown in Figure 4. This decoder was performed with 10 iterations. SCAN decoding with an 11-bit CRC had the highest rates of BER and FER, followed by the 8-bit and 6-bit CRCs, with Eb/No ranging from 1 to 3.5 dB. The 6-bit CRC gained less BER and FER at 4.5 dB compared to the 8- and 11-bit CRCs. However, the 6-bit CRC could not maintain its superior results as Eb/No reached 5 dB; in this case, the 8-bits and 11-bit CRCs achieved lower BER and FER readings.

5.3. CRC-aided SCL decoder

Figure 5 depicts the BER and FER performances of CRC-aided SCL decoders. Figure 5(a) shows the results of CRC-aided SCL decoding for N=256-bits. SCL-CRC with 6 bits could not continue achieving the lowest error when compared with SCL-CRCs with 8- and 11-bits. For BER, both CRC sizes continued reducing error readings as Eb/No increased. At 3 dB for Eb/No, the CRCs with sizes of 8-bits and 11-bits stopped reading the BER and FER errors on the received message. Meanwhile, the SCL-CRC results for N=512-bits are depicted in Figure 5(b). CRCs with 8-bits and 11-bits reduced the FER reading efficiently, while the CRC with 6-bits exhibited a much higher FER reading after 2.0 dB. The CRC with 8 bits achieved a lower FER reading than the CRC with 11-bits. Meanwhile for BER, CRCs with 8-bits and 11-bits achieved the lowest error readings (they provided equal readings). Thus, a CRC with a large size can reduce the redundancy errors found in the messages received by the network receiver for the SCL decoder. When Eb/No increased, the CRCs with 8 and 11-bits achieved zero FER and BER readings at 2 dB, while the CRC with 6-bits did so at 2.5 dB. Overall, using an SCL decoder with the help of CRC can reduce the redundancy error in received messages, and increasing the size of the CRC on the decoder can increase the reduction of redundancy error. In addition, increasing the values of N and K for the data sent and received reduced errors with the help of CRC.
5.4. CRC-aided SSC decoder

Figure 6 shows the SSC decoding results. The figure shows significant changes in BER and FER readings as Eb/No approached 4 dB. Initially, the CRC with 6 bits achieved lower BER and FER than other CRC bit sizes. However, the lowest BER and FER readings were provided by the 11-bit CRC when Eb/No exceeded 4 dB. When network transmission experienced this level of Eb/No, the SSC aided by the 11-bit CRC reduced redundancy error more effectively and efficiently than when aided by either of the other two CRCs. This shows that SSC decoding with a high CRC value efficiently reduces the redundancy errors in received messages at high Eb/No values. According to the results, utilizing CRCs with more bits can improve error correction for data received at a high Eb/No value. The largest CRC (with 11-bits) produced the lowest decoding error rate for almost all decoder systems, especially the SCAN, SCL and SSC decoders. These three decoders reduced the redundancy error on the received data, especially as Eb/No increased. Overall, the SCL decoder with a large CRC achieved the best reductions in BER and FER readings when Eb/No was high.

6. CONCLUSION

The effectiveness of each SC-based polar decoder with the aid of CRC was analyzed. Using CRC can increase the number of redundancy errors found in the received data. Specifically, it can reduce redundancy errors before they are decoded back into their original form (i.e., before they are transmitted across noisy channels). Using a suitable size of CRC can eliminate error at the highest level and can significantly reduce space, thus lowering the system’s computational complexity. In comparison to the other decoders employed in this study, the SCL decoder with a large CRC achieved the lowest error detection rate for both BER and FER. Thus, a list decoder for SC with the aid of CRC can enhance the elimination of errors found in the received
data and further reduce the error rate in the transmitted data. In conclusion, SCL decoding was the most suitable decoding method to be applied to CRC in the decoding process; it eliminated redundancy errors effectively and provided the lowest BER and FER readings.

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