Nash equilibrium learning in non-cooperative reputation game in social networks

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ABSTRACT
Recently, people became more dependent on online social networks with the increasing use and the rapid development of information technology. Those environments constitute an important area where users interact and create communication ties to maintain their relationships. Furthermore, the time life of these relationships is depending on reputations of the users. Every source (information provider) has a reputation which depends on his frequency of publishing, but also the opinions given by the observers (others users) has an important impact on the determination of this reputation. Since, everyone is trying selfishly to keep a good reputation; a competition is met within these networks. This paper aims to solve this kind of competition through a game theoretic approach; we formulate the said competition as a non-cooperative game, demonstrate the uniqueness of the existent Nash Equilibrium which seems to be the convent solution in this case, then present results clarifying and illustrating the proposed modeling method.

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1. INTRODUCTION
Nowadays, social networks became a very important tool in our daily life, they allow to group and to connect people. In fact, they are environments where subscribers share some kind of interactions and relationships. In order to maintain these relationships, a main tool and an important concept called reputation took place. Therefore, the evaluation of a reputation helps consumers of information and content to differentiate between trustworthy and untrustworthy sources, and then decide who to interact with and who to keep in touch with. As a result, good reputation management ensures anticipation in order to better control the message chosen to be broadcasted on one’s own page or on the walls of groups in social networks, while focusing on positive information and contents that users are interested in. Initially, Akerlof discussed the concept of reputation when he introduced the lemon market’s problem in [1]. The author highlighted a major problem in these markets, which is the asymmetry of information between buyers and sellers. On the one hand, buyers are aware of their own behavior and their products’ quality. On the other hand, sellers guess information that buyers gathered about them, especially their reputation in the market. Moreover, in order to reduce this asymmetry in terms of information gathered about both of the actors in the considered market, trading partners employ reputation and then facilitate trusting business relationships. Years later, reputation has become a research topic
treated in many fields. Within this context, several researchers have established various researches and papers in different domains such as computer science, economics, scientometrics, sociology, and evolutionary biology [2], [3]. In addition, reputation has recently received considerable attention within social networks [4]-[11]. In these works, the authors have given us many studies regarding the reputation of members in these social environments. Raj and Babu [7], focused on indicating the characteristics determining a user’s trust within social networks employing Benchmark databases. They proposed a probabilistic trust prediction model. This model was tested and validated on three databases, namely: the Wikipedia database, the Epinions database, and the Slashdot database.

Trying to increase their reputation in social networks and acquire an important and relevant level, users behave selfishly and then cause competition over reputation. To control this behavior and model this kind of competition, many technics are adopted; among whom those provided by game theory. This theory is a mathematical discipline launched in 1944 by John Von Neumann (mathematician) and Morgenstern (economist) who worked to justify the idea of maximizing the expected payoff in game theory. Many years later, the researcher John Nash introduced his contributions on 1950 [12], then many researchers adopted this theory to improve their approaches seeking to reach a conventual equilibrium situation in different studies [13]-[16]. Therefore, the main objective of the said theory is balancing the situation concerning the interactions among players and developing strategies helping to reach an optimal result. There are main keys of the game theory, among whom, we can mention: the dependence of a player’s actions on the choices of others, rather than on their own strategies. This dependence is the most essential criterion in the system under consideration.

Consequently; in an online reputation system, the behavior of an information provider (IP) is derived from the interactions of various users. Therefore, The successful approaches established for modeling reputation employ game theory and particularly Bayesian or evolutionary games [17], [18]. An exhaustive survey of proposed models for reputation is presented in [19]. Thus, a Bayesian game is characterized by representing a sort of incomplete information in terms of the type of each player (Honest/Dishonest); a player is not able to know about the type of other players with whom he is interacting. Henceforth, Bayesian games represent models of situations, with interactive decisions. Then, the decision-makers don’t have complete information about the data concerning the game and other players. In this case, Nash equilibrium is a list of beliefs and behavior such that each player is looking to boost his outcome, following his beliefs about other players’ behavior. Thereby, the Bayesian game doesn’t seem to be a game that is regular, but we can employ the concept of Nash equilibrium, based on the best response concept to find the solution concept of Nash–Bayes equilibrium.

In contrast to these approaches, we model the situation of competition over reputation between information providers as a non-cooperative game with complete information (the players are aware, at the time of decision making, of the set of players, the set of strategies of the other players and the objective functions of all of them), then analyze it by exploring concave game theory.

In order to achieve the aim of our approach, we organize this paper as follows: we present a formulation of the non-cooperative game between selfish and competing users and we generate a detailed description of the game, in section 2; this analysis is based on algorithmic game theory and in particular on the Best Response algorithm, which converges to a unique point that we call the Nash equilibrium. In section 3, we present detailed numerical results in order to illustrate our analysis and validate our model. Finally, we conclude this work, devoted to the learning of the Nash equilibrium in a reputation competition in social networks, with a general conclusion in section 4.

2. GAME FORMULATION

This section aims to formulate the competition over reputation in a social network. A reputation is calculated when an interaction is set between two users of this structure. Actually, a source of information can be an observer, an observed or both at the same time as it’s presented in Figure 1. After an interaction, each observer evaluates it to calculate the reputation of the observed agent. Since interactions take place all the time within social networks, the propagation of reputations from one side to the other one is guaranteed identically to what it is happening in reality. So, each information provider, using specific strategies, looks to improve and promote his reputation. This situation leads to a competition over reputation which needs to be formulated and analyzed using the game theory because it is the main way to analyze this sort of competition problematics.
2.1. Game modeling

Let $G = [\mathcal{X}, \Lambda_i, U_i(\lambda)]$ be the non-cooperative frequency game, where $\mathcal{X}$ is the set of indices that identifies the number of information sources, $\Lambda_i = [0, \lambda_i^{max}]$ is the frequency strategy set appropriate for $IP_i$, $\lambda_i^{max}$ is the maximum publication frequency that information provider $i$ can achieve. $U_i(.)$ is the utility function that $IP_i$ must maximize by adopting a publication frequency $\lambda_i$. Officially, the objective function of $IP_i$ is described, by the (1), as follows:

\[
U_i(\lambda_i, \lambda_{-i}) = \lambda_i \gamma_i + A_i
\]

where, $\lambda_{-i} = (\lambda_1, ..., \lambda_{i-1}, \lambda_{i+1}, ..., \lambda_K)$ represents the strategy vector of the other information providers. And $A_i$ is the observers’ view of the information provider $i$.

In this context, the observers’ opinions can be a value among real ones then it is impossible to confirm that the predicted values will be in the right range. In order to overcome these range problems, we transform the opinions’ function and provide a model for the transformation as a linear function using the Logit model presented in [20] as bellows: let $A_i = \lambda_i \sum_{\lambda_{-i}} K$ be the linear probability model of users’ opinions. To make it as we need, we across two steps. The first one permits to move from the probability $A_i$ to the odds which is defined as the quotient of the probability and its complement, it’s presented by the following (2).

\[
\text{odds}_i = \frac{A_i}{1 - A_i}
\]

The second step is about calculating logit($A_i$) or log(odds). Supposing that the logit of the underlying probability $A_i$ is a linear function of the predictors, it’s given as (3).

\[
\text{logit}(A_i) = \lambda_i \frac{K}{\sum_{\lambda_{-i}}}
\]

Exponentiating the (3), we find that the odds for the $i^{th}$ unit is given by (4).

\[
\text{odds}_i = \frac{A_i}{1 - A_i} = \exp(\lambda_i \frac{K}{\sum_{\lambda_{-i}}})
\]

Solving for $A_i$ in (4), we obtain (5):

\[
A_i = \frac{\exp(\lambda_i \frac{K}{\sum_{\lambda_{-i}}})}{1 + \exp(\lambda_i \frac{K}{\sum_{\lambda_{-i}}})}
\]

As a result, the Utility function, given in (1), of the player $i$ is defined as (6).

\[
U_i(\lambda_i, \lambda_{-i}) = \lambda_i \gamma_i + \frac{\exp(\lambda_i \frac{K}{\sum_{\lambda_{-i}}})}{1 + \exp(\lambda_i \frac{K}{\sum_{\lambda_{-i}}})}
\]
2.2. Nash equilibrium

Since the information providers in a social network are characterized by selfishness; each one tries, individually, his own strategies to improve his revenue. Thereby, they bring out a non-cooperative game that accepts as a solution a strategy profile such that no IP has a chance to take advantage by changing his strategy unilaterally. This profile, named the Nash equilibrium [21], pushes us to employ the concave games’ theory to demonstrate and prove its existence and uniqueness [22]. Therefore, a non-cooperative game is concave if the utility functions of all players are strictly concave with respect to their corresponding strategies [22]. Following the analysis and the process, Rosen presented in [22], the existence of a Nash equilibrium is guaranteed in a concave game if the considered strategy set is compact and convex, and the utility functions of all players are concave and continuous at each point belonging to the said set.

Definition 1: a frequency vector \( \lambda^* = (\lambda_1^*, ..., \lambda_K^*) \) is a Nash equilibrium if:

\[
\forall i \in \mathcal{K}, \ U_i(\lambda_i^*, \lambda_{-i}^*) = \max_{\lambda_i \in \Lambda_i} U_i(\lambda_i, \lambda_{-i}^*)
\]

furthermore, attending the equilibrium state, no information provider could increase his revenue by switching his strategy in an individual way. Theorem 1: for the game G which is concave the Nash equilibrium exists. Proof 1: to demonstrate the existence of the Nash equilibrium, the second derivative of \( U_i \) is calculated as follows:

\[
\frac{\partial^2 U_i(\lambda_i, \lambda_{-i})}{\partial \lambda_i^2} = \left( \frac{K}{\sum_{i \neq -i} K} \right)^2 \frac{\exp(\lambda_i \sum_{i \neq -i} K) - (\exp(\lambda_i \sum_{i \neq -i} K))^2}{(1 + \exp(\lambda_i \sum_{i \neq -i} K))^3}
\]

we have \( \exp(\lambda_i \sum_{i \neq -i} K) - (\exp(\lambda_i \sum_{i \neq -i} K))^2 < 0 \) and \( (\exp(\lambda_i \sum_{i \neq -i} K))^2 > \exp(\lambda_i \sum_{i \neq -i} K) \), then

\[
\frac{\partial^2 U_i(\lambda_i, \lambda_{-i})}{\partial \lambda_i^2} < 0
\]

as a conclusion, the formulated game is concave then the existence of the Nash Equilibrium is guaranteed. We use the best algorithm defined in section 2.3 to verify the uniqueness of the Nash equilibrium.

2.3. Learning the Nash equilibrium

Seeing that the Nash equilibrium exists (proved in section 2.2), this section will be a window to study the learning of the Nash equilibrium employing the best response’s algorithm; an algorithm belonging to the set of algorithms used in strategic decision-making. Actually, we use it in our analysis to converge to the equilibrium’s publishing frequency. In the named algorithm, a best response sequence is determined as a conclusion, the formulated game is concave then the existence of the Nash Equilibrium is guaranteed. We use the best algorithm defined in section 2.3 to verify the uniqueness of the Nash equilibrium.

Algorithm 1 Best response Algorithm

1. Initializing the strategies’ vector \( \lambda \);
2. For each IP \( i \in K \) at the iteration \( t + 1 \): \( \lambda_i(t + 1) = \arg\max_{\lambda_i \in \Lambda_i} (U_i(\lambda_i, \lambda_{-i})) \).
3. IF \( \forall i \in K, |\lambda_i(t + 1) - \lambda_i(t)| < \epsilon \), STOP.
4. ELSE, \( t \leftarrow t + 1 \) and go back to step (2).

2.4. Price of anarchy

Koutsoupias and Papadimitriou [23], explained the concept of “price of anarchy” is mentioned for the first time. Then various studies and researches were established dealing with this term [24]. In addition, the popularity of this critical measure of the loss of equilibrium’s efficiency has increased due to Roughgarden and Tardos, who wrote [25]. In this work, the authors opened the window to studying the PoA in atomic and non-atomic congestion games. The concept was, also, studied for creating networks in [26] and to facilitate location in [27]. Following [24], the egoism of players produces an inefficiency that we define as the ratio of...
the social welfare acquired at the Nash equilibrium (according to Maille and Tuffin, it’s the sum of the utilities of all the actors) in [28] and the maximum value of the social welfare. This price is described in (10):

\[ P_{oA} = \frac{\min W_{NE}(\lambda)}{\max W(\lambda)} \] (10)

where: \( \max W(\lambda) = \max_{\lambda} \sum_{i=1}^{K} U_i(\lambda) \) is the social welfare function, and \( W_{NE}(\lambda) = \sum_{i=1}^{K} U_i(\lambda^*) \) the sum of the utilities of all the sources at Nash equilibrium.

3. RESULTS AND DISCUSSIONS

In established game modeling, game theory is the main tool to study the competitive interaction produced over reputation in social networks. In order to validate the theoretical approach, we reserve this section for the generation of figures presenting numerical results for the equilibrium publishing frequency \( \lambda^* \).

To do so, we suggest studying a network with two selfish information providers \((K = 2)\). The strategies’ space is \( \Lambda_i = [1,1000] \) and both of the IPs have to pay the same fixed price \( \gamma_i \). In Figure 2, we start by plotting the two utility functions as a function of the publishing frequency \( \lambda \). The curves, in Figure 2, represent the utility functions of the two IPs as a function of the publishing frequencies. We notice that the \( U \) are concave functions on all the considered strategies. This concavity allows us to validate the theoretical results. Then, we turn to the learning of this equilibrium by applying the best response (BR) algorithm. Figure 3 shows the convergence curves to the Nash equilibrium of the publishing frequency. According to Figure 3, we observe that the best response algorithm (algorithm 1) converges to the unique Nash equilibrium in very few iterations (about 6 iterations are sufficient to learn the equilibrium point). In other words, the information providers’ reactions tend towards the Nash equilibrium very quickly. Thus, the learning of the equilibrium state (using the BR algorithm) accomplished with a relatively high speed reinforces what we proved in the theoretical study presented in the previous section.

Subsequently, we focus on revealing the impact of the publishing price on the evolution of the utility function, and also on the publication frequencies reached in the Nash equilibrium. The curves, in Figure 4 and Figure 5, describe the evolution of the Nash equilibrium utility functions and publishing frequencies as a function of the publishing price of the content on the walls of the set system. In Figure 4, we notice that the utility function at Nash equilibrium evolves in an increasing way with respect to the publishing price \( \gamma \). Similarly, in Figure 5, the increase of the publishing price \( \gamma \) influences positively the evolution of the publishing frequencies found at the equilibrium states. The income from the publication of the contents increases, which involves the increase of the income of the IPs in terms of reputation within the social network they are subscribed to. Therefore, they think to publish more contents to maintain their reputation and subsequently attract more people to have contact and interact with. And so, they can make their community grows up.
In what follows, we discuss the impact of this price on the system’s efficiency, under consideration, using the PoA. Figure 6 shows the curve of variation of the PoA as a function of publishing price \( \gamma \). In Figure 6, we remark that the PoA increases with the publication price. When the publication price is low, the PoA is low. Then, the Nash equilibrium is not socially efficient, Information providers are considered to be selfish, and each one does his best to maximize his profit individually. However, when the publication price increases, the Nash equilibrium becomes more and more socially efficient. This increase finds the simple intuition that when the publication price is high, IPs are not selfish anymore, so each one takes into account the strategies of his opponents while adopting his own strategies to reach an optimal equilibrium state (he chooses his decisions as a result of the decisions made by his competitors).

4. CONCLUSION

This paper has dealt with the case of a system where many participants interact; a social network where users compete and do their best to improve their reputation within this environment of sharing information and thoughts. As game theory seems to be the natural tool to model situations with many actors, we employed it to study the interactions of information providers considered rational and selfish and then resolve this competition produced among them as a non-cooperative game. We consider the fact that the utility function of an information provider is depending on his own publishing frequency and on the publishing frequency and the opinions of other users, and that the sending of messages has a price that is proportional to his publishing frequency. Then, we set the game formulation for the considered situation employing technics of concave games accepting the Nash equilibrium as the reliable and adequate solution. Thus, a social network with two IPs seeking selfishly to have a good reputation is studied, we employed the BR algorithm to reach the equilibrium state very quickly (a convergence with high speed) and we presented the obtained numerical investigations. Furthermore, the evolution of the price of anarchy showed the optimality and the efficiency of the Nash equilibrium found in this case. In this context, by achieving a progressive e-reputation, these IPs attract more content consumers who make them convey a dynamic and quite original image. Since all the strategies adopted by the IPs always
converge to the already mentioned Nash equilibrium, then the stability in the social network is ensured. Given that, the good management of one’s reputation is always an essential element in the race between competitors, it allows the user to stand out from the others while enhancing his prestige and qualifying his image in the social network. As a result, classical users employ their reputation to establish more relationships and maintain these links for a long time. In fact, those users look for interaction with their community, and professional ones (companies and organizations) use it to ameliorate their commercial and financial image marque within the social network they use to attend some kind of lucrative purposes.

REFERENCES

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