Variable length error correcting code for image in OFDM and PAPR reduction

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ABSTRACT

Data transmission in orthogonal frequency division multiplexing (OFDM) system needs source and channel coding, the transmitted data suffers from the bad effect of large peak to average power ratio (PAPR). Source code and channel codes can be joined using different joined codes. Variable length error correcting code (VLEC) is one of these joined codes. VLEC is used in mat lab simulation for image transmission in OFDM system, different VLEC code length is used and compared to find that the PAPR decreased with increasing the code length. Several techniques are used and compared for PAPR reduction. The PAPR of OFDM signal is measured for image coding with VLEC and compared with image coded by Huffman source coding and Bose-Chaudhuri-Hocquenghem (BCH) channel coding, the VLEC code decreases the data transmitted size and keep the same level of PAPR reduction with respect to data coded by Huffman and BCH code when using PAPR reduction methods.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is widely used for wireless multimedia communication and has the solution for high bit rate communication, where digital data including image is transmitted as parallel with multicarrier technique. One of the challenges in OFDM system is the high peak to average power ratio (PAPR), this challenge worries the manufacturers because the high PAPR makes the transmitter power amplifier works in the nonlinear region, the peaks when pass in the amplifier at the nonlinear region will cause signal distortion [1], [2], increasing the dynamic range of the amplifier to solve the problem will increase the cost, power consumption and complexity of the amplifier. Many methods are used to decrease the PAPR and decreasing the problem such as clipping technique, companding technique, coding technique, tone reservation (TR), selective mapping (SLM), partial transmission sequence (PTS) and so on [3]-[11].

Data transmission needs to be coded by source code to remove redundancy and coded by channel code for error detection and correction to decrease the channel effect, the channel code increases the data redundancy to recover the channel errors, therefore it’s better to combine the source and channel codes in single code to reduce redundancy and correct errors at the same time [12]. Variable length code (VLC) is widely used for audio and video compression, variable length error correcting code (VLEC) is a class of VLC with free distance between the code words like the channel code, and therefore VLEC offers source coding and channel coding by the single code [13].
In this work we use VLEC code in the OFDM system to decrease data transmitted size and at the same time to detect and correct errors, PAPR is a common problem in OFDM system, which will decrease the transmission efficiency, therefore; it’s important to know the relationship between the transmission of image coded with VLEC code and PAPR values, several methods are tested for PAPR reduction. The PAPR is measured for transmitted image in AWGN OFDM channel and coded by VLEC. The PAPR is high for data coded with VLEC code, companding and DFT methods are tested to decrease the PAPR. Huffman and Bose-Chaudhuri-Hochquenghem (BCH) codes are used as a source and channel code respectively to encode the image and compare it with the one coded with VLEC code. The comparing is related to size of data and PAPR values. Many papers take care of studying joining source code with channel code and more than that studying PAPR reduction and the effect of coding type on it [14], suggests using Goppa coding for PAPR reduction and for error detection and correction [15]. Suggests fragmentation transmit sequence (FTS) technique to decrease PAPR in OFDM system by combining the SPIHT source code for image with LDPC channel code in a joint source channel code (JSCC) [16], joint source code with channel code by using VLEC and studying the effect on image transmitted in LTE OFDM [12]. Suggests an irregular mapping method for small data size coded with VLC and modulated with 8PSK for bit-interleaved coded modulation with iterative decoding (BICM-ID) system, this method decreases the iteration count for decoding small size [17]. Suggests time varying VLEC (TV-VLEC) to obtain code length variable according to the symbol position, this code will keep good synchronization over the standard VLEC [18]. Transmit image in OFDM system has multiple input multiple output (MIMO), using the residue number system (RNS) as a technique for coding this image to decrease PAPR.

This paper has organized as follows: section 2 explains the definitions of an OFDM system and PAPR. Section 3 explains the companding and DFT methods which are used in this paper, section 4 explains VLEC encoding and decoding. The measurements for image quality is explained in section 5, proposed work in section 6, section 7 displays results and discussion, conclusion in section 8.

2. METHOD
2.1. OFDM and PAPR
Parallel transmission of N data symbols with N orthogonal subcarriers are used in OFDM system, the discrete time domain \( x(n) \) signal for N subcarriers, which is the output of the IFFT stage in the transmitter side is expressed as:

\[
    x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N}, \quad 0 \leq n \leq N - 1
\]  

(1)

\( X[k] \) is the modulated signal by one of the modulation types expressed in frequency domain. The PAPR is the ratio between the maximum instantaneous power and the average power of the signal transmitted in the OFDM system:

\[
    PAPR(x) = \frac{\max|x(n)|^2}{\bar{|x(n)|^2}}
\]

(2)

Cumulative distribution function (CDF) is:

\[
    F(z) = 1 - e^{-z}
\]

(3)

where \( z \) is probability of PAPR. CCDF is the probability distribution function of PAPR which is more than a certain threshold \( z \) and expressed as [3], [4], [19]-[22]:

\[
    Pr(PAPR > Z) = 1 - F(Z) = 1 - (1 - e^{-z})^N
\]

(4)

2.2. PAPR reduction techniques
The high PAPR problem causes nonlinear distortion [1], [2]. Several techniques are usually used for PAPR reduction to overcome this distortion, the techniques differ in system complexity and ability to reduce PAPR without distortion [6], companding and precoding techniques are simple techniques which are used for PAPR reduction, these techniques don’t need for side information transmission, but they need inverse operation at the receiver. They are used in this paper and explained.

2.2.1. Companding technique
The term companding is taken from compression and expansion processes, this technique compresses the large signals and expands the small signals amplitudes to increase the average signal power,
as a result PAPR will reduce, see (2) [23]-[25]. Several types of companding such as $\mu$-law, exponential companding, hyperbolic tangent companding, logarithmic function companding and the logarithmic companding and de-companding formulas are given by:

$$\hat{x} = k_1 \log(1 + k_2 x)$$  \hspace{1cm} (5)

$$\hat{y} = (e^{y/k_1} - 1)/k_2$$  \hspace{1cm} (6)

$\hat{x}$ is the signal $x$ after companding process and $\hat{y}$ is the signal $y$ after de-companding, $k_1$ and $k_2$ are positive constants determine the compression level [24], [26].

### 2.2.2. Precoding technique

The precoding technique is used to decrease the PAPR, the PAPR reduction must decrease the effect of the amplifier nonlinearities to reduce the BER. Each OFDM modulated data block is multiplied by $P$, which is the precoding matrix with size $N \times N$ at the transmitter, where $N$ represents the subcarriers number in the OFDM system. The received data is multiplied by $P^{-1}$, $P^{-1}$ represents the inverse of the precoding matrix, different transforms are used as a precoding matrix $P$ and need inverse transformation at receiver. DFT, DST, DCT and are used for precoding PAPR reduction technique [27], [28].

$$P = \begin{bmatrix}
    p_{11} & \cdots & p_{1N} \\
    \vdots & \ddots & \vdots \\
    p_{N1} & \cdots & p_{NN}
\end{bmatrix}$$  \hspace{1cm} (7)

### 2.3. Variable length error correcting code

VLEC represents good data compression and error correcting code, it is used instead of the classical separate source and channel codes as shown in the classical OFDM system in Figure 1, practically the system complexity can be reduced when using VLEC, but certainly with keeping a good performance [29]. The basic concepts of VLEC that, $X = x_1 x_2 x_3 \ldots \ x_n$, $X$ has $n$ binary code source symbols with fixed length $L$ for each symbol, $C = c_1 c_2 c_3 \ldots c_n$. $C$ has $n$ code words with $\sigma$ different length $L_1, L_2, L_3 \ldots L_\sigma$ where $L_1 \leq L_2 \leq L_3$ and so on, the length of each code word depending on the probability of occurrence, the less probability with the greatest length, where:

$$P(x_i) = \sum_{i=1}^{n} p(x_i) = 1$$  \hspace{1cm} (8)

$$L_{av} = \sum_{i=1}^{n} l_i p(x_i)$$  \hspace{1cm} (9)

$p(x_i)$ is the probability of occurrence of each source symbol. The binary code source symbols $X$ are mapping to code words $C$, the code words have number of different lengths is called $\sigma$ as shown in Table 1, in this table an example of three symbols with different probabilities are coding with $\sigma = 2$. Hamming distance between two vectors is defined to be the difference number of places between them, minimum block distance $b_m$ represents the minimum hamming distance among all code words of the same length and $b_{min}$ is the overall minimum block distance where:

$$b_{min} = \min b_m, m = 1, 2, \ldots, \sigma$$  \hspace{1cm} (10)

<table>
<thead>
<tr>
<th>Source symbols</th>
<th>Probabilities</th>
<th>Code words</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.25</td>
<td>000</td>
</tr>
<tr>
<td>B</td>
<td>0.2</td>
<td>0101</td>
</tr>
<tr>
<td>D</td>
<td>0.15</td>
<td>0011</td>
</tr>
</tbody>
</table>

Table 1. Example for source symbols and code words

Each different code word length has minimum block distance, the diverge distance $d$ and the converge distance $c$ are defined as the unequal lengths code words Hamming distance. The minimum diverge and converge distances of VLEC are $d_{min}$ and $c_{min}$, where $d_{min}$ is the minimum diverge distance from all possible diverge distances of unequal length code words and $c_{min}$ is the minimum converge distance from all possible converge distances of unequal length code words. The minimum Hamming distance between all paths of the same length in bits diverging in a state and converging in the same or another state is called the free distance $d_{free}$, it is defined by [17], [30], [31]:

$$d_{free} \geq \min (b_{min}, d_{min} + c_{min})$$  \hspace{1cm} (11)
2.3.1. VLEC encoder

Multiple algorithms are used for VLEC coding but the heuristic algorithm is the best, this algorithm offers minimum block, converge and diverge distances, then minimum $d_{\text{free}}$ is obtained. The heuristic VLEC algorithm steps is as follows [17], [31]. At first, VLEC code word builds a set $C$ with a length code $L_1$ with a minimum distance $b_{\text{min}}$ by using fixed length algorithms such as majority voting algorithm (MVA) and greedy algorithm (GA). Increase the code length one bit if the words in set $C$ is not enough for the needed code words number, the new code words with length $L_2$ will be stored in set $W$, the number of code words will be twice after the addition of this bit. All words in set $W$ that do not have at least a distance $d_{\text{min}}$ and $c_{\text{min}}$ from all code words of $C$ are deleted, then, only the words with minimum distance $c_{\text{min}}$ and $d_{\text{min}}$ are in the set $W$. Choose words from $W$ that has at least distance $b_{\text{min}}$ from each other and added them to $C$ set. Repeat this procedure until obtaining the enough number of words.

2.3.2. VLEC decoder

At the receiver the VLEC is decoding to retrieve the original source words, maximum likelihood algorithm is used for this purpose because the VLEC code is similar to the convolutional code, that the new generated code word of the message depends on the code word generated before it. Maximum likelihood decoder finds the closest code word to the received data [30]. The trellis diagram and modified Viterbi algorithm are used for good decoding process, where Viterbi is a dynamic programming procedure to implement maximum likelihood algorithm [32].

2.4. Image quality measurement

The image is retrieved by decoding the received data, several measurements are used to measure the image quality such as mean squared error (MSE) and peak signal to noise ratio which is used in this paper.

MSE measures the squared of the difference between the original image intensities and the decoded image:

$$MSE = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (I_T(i,j) - I_R(i,j))^2$$  \hspace{1cm} (12)

where $M$ and $N$ are the dimensions of the image, $I_T$ and $I_R$ are the transmitted and received image pixels respectively, PSNR measures the accuracy of the decoded image, it can be measured depending on the MSE value [19], [33]:

$$PSNR = 10 \log_{10} \left( \frac{255^2}{MSE} \right)$$  \hspace{1cm} (13)

3. PROPOSED WORK

Transmitting image in the OFDM system demands encoding the image with a source and channel codes, the transmitted data will have big data size, another problem in the OFDM system is the large value of PAPR. In the proposed work VLEC coding is suggested to join the source and channel codes into single code, Heuristic algorithm is used to generate VLEC code, the maximum likelihood decoding algorithm is used for decoding. The PAPR problem will appear, which has high value in OFDM and its value affected by the coding type. It is necessary to use PAPR reduction techniques in the proposed work, DFT precoding technique and logarithmic companding technique are used in this work. The proposed work block diagram can be shown in Figure 1.

![Figure 1. Proposed work block diagram](image-url)
4. RESULTS AND DISCUSSION

Colored Lena image with size 128x128 has sent in the OFDM system, Mat-lab 2018a is used in the simulation, the OFDM system is used with 256 subcarriers, 20 MHz bandwidth and 16QAM modulation technique. Additive white gaussian noise channel (AWGN) is used in the simulation. CCDF is measured for Lena image coded with VLEC code without using any PAPR reduction at first, then it is measured with using companding and DFT PAPR reduction, $k_1=10$ and $k_2=1$ are chosen for the logarithmic companding as in (5), Figure 2 explains the relationship between CCDF and PAPR for different values of $d_{free}$ for the VLEC codes, from this figure it’s clear that PAPR is reduced with increasing $d_{free}$ and the PAPR values decreased to close values when PAPR reduction techniques are used independent on $d_{free}$ length, using DFT method gives results better than using companding method.

Figure 2. Comparison of CCDF for image coded by VLEC code with different $d_{free}$ value with and without PAPR reduction

Transmitted lena image is encoded using VLEC code with $d_{free}=3$, then it is encoded using Huffman and BCH codes, the two types of coding are compared, Figure 3 shows the relationship between CCDF and PAPR for the two types of coding, Figure 3 explains that the PAPR of data coded with Huffman and BCH codes is less than the PAPR of data coded with VLEC without using PAPR reduction techniques, but with using PAPR reduction techniques the PAPR values of the two different coded methods are close to each other, especially when DFT method is used. The transmitted data size for the two types of coding is compared, where the transmitted data size for lena image coded by using Huffman and BCH codes is 666,435 bytes, while the size of Lena image coded by using VLEC code is 534,629 bytes, it is clear that the coding with VLEC code decreases the size of transmitted data. The BER of the received images coded with the previous two types of coding is measured, the relationship between BER and signal to noise ratio (SNR) are shown in Figure 4, the figure shows that BER of the data with small $d_{free}$ is more than BER of the data with larger $d_{free}$, BER for the Huffman and BCH separated codes is being very small at SNR=8dB, VLEC codes with $d_{free}=3$ and $d_{free}=4$ are also being very small at SNR=8dB.

The transmission of images are tested for several values of $d_{free}$ and SNR=8dB, the decoded images are shown in Figure 5. Figure 5(a) $d_{free}=2$ is used and the decoded image has PSNR=14.063, (b) $d_{free}=4$ is used and the decoded image has PSNR=46.2695, from Figure 5, we found that peak signal to noise ratio (PSNR) increases for the decoded images as long as $d_{free}$ increases for constant value of SNR. The transmission are tested for several values of SNR and $d_{free}=2$ as shown in Figure 6. Figure 6(a) SNR=8dB is used and the decoded image has PSNR=14.063, (b) SNR=10 dB is used and the decoded image has PSNR=23.7368, (c) SNR=12 dB is used and the decoded image has PSNR=71.5244, from Figure 6 we show that the PSNR increases for the decoded images when SNR increases with constant $d_{free}$.
Figure 3. Comparison of CCDF for image coded with VLEC once and with huffman and BCH another time with and without PAPR reduction.

Figure 4. Comparison of BER for coded Lena image with respect to SNR change.

Figure 5. Lena decoded images with different d_free. (a) d_free=2 is used and the decoded image has PSNR=14.063 and (b) d_free=4 is used and the decoded image has PSNR=46.2695.

Figure 6. Lena decoded images with different SNR. (a) SNR=8dB is used and the decoded image has PSNR=14.063, (b) SNR=10 dB is used and the decoded image has PSNR=23.7368, and (c) SNR=12 dB is used and the decoded image has PSNR=71.5244.
5. CONCLUSION

Image transmission is needed source and channel coding, which demand large data size for separated coding, so VLEC joining code is used, VLEC will decrease the transmitted data size and recover the channel errors at the same time, the effect of VLEC code on the PAPR is tested with different lengths of \(d_{\text{free}}\), the CCDF and BER are compared for the transmitted data coded by using VLEC joining code and using Huffman and BCH separated codes, two PAPR reduction methods are tested DFT and logarithmic companding methods. The results indicate that using VLEC in the OFDM minimize the large data size problem and the channel errors problem of the received data, larger \(d_{\text{free}}\) is better for smaller PAPR and BER, the PAPR of the VLEC code is more than that of the Huffman and BCH codes, but when the PAPR reduction techniques are used, the PAPR will reduce to the same value for the two methods of coding, using precoding (DFT) for PAPR reduction is better than using companding method, so the proposed work can be used to overcome the image transmission problems in the OFDM system. It gives performance close to separated source and channel codes but it has less data size than separated coded.

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