Electromagnetism-like mechanism algorithm for hybrid flow-shop scheduling problems

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ABSTRACT
Given the interest and complexity, of the hybrid flow shops (HFS) problem in industry, he has been extensively considered, the HFS, is a complex combinatorial problem supported in many original world applications. We consider a hybrid flow shop FH4(P3, P2)||Cmax to applied in this paper. In this papers we attempt to optimize the makespan which refers to the last task completion time by an adequate meta-heuristic algorithm based on electromagnetism mechanism (EM). We also present analysis on the performance of the EM-algorithm adapted to HFS scheduling problems. The electromagnetism-like mechanism method gave us efficient and fair results comparing to particle swarm and genetic algorithm.

Keywords:
Electromagnetism-like mechanism
Hybrid flow shop
Makespan
Metaheuristics
Optimization

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1. INTRODUCTION
Lee and Vairaktarakis [1] define a hybrid flow shops (HFS) model as a multi-stage flowshop with parallel machines at each stage. He is a general case of the simple FS problem, as shown by Gourgand et al. [2]. We study the scheduling of a batch of identical jobs in an homogeneous parallel machines at each stage, to minimize makespan Cmax, ie. (completion time of the last job in the batch). Resolution of HFS problem through the use of metaheuristic techniques represents a common practice in literature, according to [3]–[10]. Recently, a complete review on research carried out in the field of HFS problem has been presented by Mirabi [11], and Ruiz and Vázquez-Rodríguez [12].

In this article, we use the electromagnetism mechanism (EM) algorithm to arrangement with the HFS problem presented by Figure 1. In section 2, we propose the problem statement and the proposed EM algorithm. The empirical results and their interpretation are described in section 3. Finally, section 4 summarizes our article and indicates future research.

2. PROBLEM STATEMENTS AND ELECTROMAGNETISM-LIKE ALGORITHM
2.1. HFS problem
Xu and Wang [13] assume that the HFS problem as following: for all machines the release time is negligible; one job is treated on only one machine at a time; and one machine can process only one transaction. The buffers between stages are unlimited in FHS problem and all n jobs are convenient to be refined at the initial time and are independent. For all of the n jobs, in each machine the treatment times are known and
The formulated of HFS problem is:

\[
\begin{align*}
\text{min} & \quad C_{\max} \\
\text{s.t.} & \\
C_{\max} &= \max(C_{ik}), \quad k = 1, ..., s; \quad i = 1, ..., n \quad (1) \\
C_{ik} &= S_{ik} + P_{ik} \quad (2) \\
\sum_{i=1}^{m(k)} X_{ijk} &= 1, \quad k = 2, ..., s \quad (3) \\
S_{ik} &\geq C_{i,k-1} \quad k = 2, ..., s \quad (4) \\
S_{ik} &\geq C_{jk} - MY_{ijk}, \quad \forall (i, j) \quad (5) \\
S_{ij} &\geq C_{ik} - (1 - M)Y_{ijk}, \quad \forall (i, j) \quad (6) \\
S_{i1} &\geq R_i \quad \forall i = 1, ..., n \quad (7) 
\end{align*}
\]

where \(C_{ik}\) represent the completion time of job \(i\); \(m(k)\) is the number of the machines at stage \(k\); \(s\) is the total number of stages; \(n\) is the total number of jobs. \(X_{ijk} = 1\) if at stage \(k\), job \(j\) is defined to machine \(i\); in another way, \(X_{ijk} = 0\) if at stage \(k\), job \(i\) predate job \(j\); in another way, \(Y_{ijk} = 0\), \(S_{ik}\) represent the starting time of job \(i\) at stage \(k\), \(R_i\) represent the release time of job \(i\), \(P_{ik}\) represent the process time of job \(i\) at stage \(k\), \(M\) is a tall constant.

The formulations that are mentioned before described as: (1) designate the objective function to minimize \(C_{\max}\); (2) denotes the maximum realization time of all jobs; (3) represent the computation of \(C_{ik}\); (4) guarantee that at every stage, one job is treated explicitly on one machine; (5) and (6) confirm that only one job can process on one machine at one time; (7) indicate that one job cannot be treated until its prior job is accomplished ; (8) determine the starting time of a job. In Figure 1, we illustrate the HFS problem, for scheduling problems and its development in [14], [15], and adopting the recognised three-field notation \(\alpha|\beta|\gamma\), the problem considered here can be viewed as \(FH4(P_3, P_2)||C_{\max}\) i.e 2 stages with three machines in the first stage and two machines in the second stages, and a stock at the access of the first stage and between the stages. Gourgand et al. [5] definet that the total number of feasible solutions is \(n! \left( \prod_{i=1}^{m_1} m_i \right)^n\). The flexible flowshop problem (FFS) with only two stages with a single machine is non-deterministic polynomial-time hardness (NP-hard) [16], so the FHS is also NP-hard since it is a general case of the FFS.

2.2. Electromagnetism-like algorithm

Birbil et al. [17], [18] present an adequate meta-heuristic algorithm based on EM for unconstrained global optimization of non-linear functions. The charge is related with each solution to the objective function value in a multi-dimensional space. A set of solutions points (population), is create, in whatever solution point will apply repulsion or attraction on more points, the weightiness of which is inversely proportional to the square of the distance and proportional to the product of the charges between the solutions points (Coulomb's Law).
The foundation of the EM-algorithm is that the repulsive solutions will forbid a move in their direction by repelling more points in the population and that attractive point’s will facilitate moves in their direction. It is a set up of LS in solution space. The fundamental dissimilarity with exist methods is that the moves are controlled by forces that conformed the method of electromagnetism [4].

Recently, EM has been applied to deal with combinatorial optimization problems, let us quote the following cases: Debels et al. [4] for an hybrid scatter search/electromagnetism for project scheduling problem, and has also been used by Chang et al. [19] to single machine scheduling problem, and by Maenhout and Vanhoucke [20] for the nurse scheduling problem. EM method is converged rapidly with the generated sound solutions [21]–[23]. Other approaches have been based on EM-algorithm, such as [22], [24]–[31]. The processus of EM-Algorithm on the optimization have the following form:

$$\min f(x), \text{ subject to } x \in S$$

where $f(x)$ is a nonlinear function,

$$S = \{ x \in R^n | -\infty < l_m \leq x_m \leq u_m < \infty, m = 1, ..., n \}$$

is a bordred appropriate zones of problem. $n$ denote the size of the situation, $u_m$, $l_m$ are successively the upper and lower bounds of the $m^{th}$ size.

EM is a simulation based on Coulomb’s law that describes the repulsion-attraction tool of electromagnetism approach. In Algorithm 1 EM, the solution is represented by a charged particle such that the best solution aspect of the particle represents the highest charge.

The fixed charge of solution $i$ is calculated by:

$$q_i = \exp \left( -n \frac{\sum_{m=1}^{m} (f(x^m) - f(x^{best}))}{f(x^i) - f(x^{best})} \right)$$

where, $m$ is the population size, $f(x^{best})$, $f(x^i)$ and $f(x^k)$ represent respectively, the first-rate solution and the fair values of points $k$ and $i$.

The amplitude of the effect of repulsion and attraction in the set of solutions is represented by the quality of the solution of each particle. The unacceptable solution dismay more particles to change against other region, while the more excellent solution reassure them to meet to interesting valleys. The total force $F_i$ given by (11) represents the resultant of the force of the solution $i$ compared to the solution $j$; $F_i$ make use of each solution, indicates the order of the displacement of the particles towards other diversified solutions.

$$F_i = \sum_{j=1, j \neq i}^{p} F_j^i$$

$F^i$ decide the direction of evolution for the equivalent point at the successive iterations.

Algorithm 1. EM (M_IT, LS_IT, i, α)

M_IT: max iterations

LS_IT: max iterations of L.S.

$\alpha \in [0, 1]$: L.S. parameter,

Initialize

While (has not met stop criterion) do

L.S. (Local Search)

C.T.F. F() (Calculate Total force)

Move the point by F()

Evaluate the New points

End while

As shown in the fundamental EM algorithm 1, it consists of five steps described as follows:
a) Initialization: initialization procedure is applied to choose randomly k points, pretented to be evenly scattered between the corresponding upper bound \(u_m\) and lower bound \(l_m\) from the feasible domain of the conjoint variables can be determined as follows:

\[ x_i^m = l_m + \alpha \cdot (u_m - l_m) \text{ for } m = 1, \ldots, n \text{, where } \alpha \in U(0, 1). \]

The initialization ends with the choice of the best point \(x^{best}\) which has the best function value in the midlle of the k identified points, where \(x^{best} = \arg \min f(x^i)\).

b) Local search (L.S.): L.S. procedure is applied only to the best point \(x^{best}\) in order to research the region of this solution in the population. It’s a random line search algorithm [24]. Firstly, to ensure that the L.S. only achieve suitable points, the procedure L.S. calculates the highest suitable step length \(\alpha(\max_t(u_m-l_m))\), secondly it uses a provisional solution \(y\) to assign it the best solution \(x^{best}\) this for each coordinate \(m\). Then point \(y\) is moved forward that direction and a arbitrary number is preffered as the step length. If an improvement is observed, with in the LS_IT iterations, the perfect solution is updated by \(y\) and the search forward this dimension \(m\) ends. Finally, some points are updated. The reader is referred to [13], [14], [25] for details.

c) Calculation of charge: for each iteration, the charge of each point changes as it is coincident to their objective function values, thus points that have better objective values have higher charges. The charge \(q_i\) of each point \(i\) is evaluated by (9), it determines the power of attraction or repulsion of point \(i\).

d) Calculation of total force vector C.T.F.: the T.F.P expend on point \(i\) is evaluated by the (11). In (10) shows that the solution which has a superior objective function rate acts as an absolute attracting point and invite all more solutions in the population to a improved region. On the contrary, the point with the worst rate of the objective function repels the others.

e) Move correspondent to the T.F.: using a arbitrary step of length \(\alpha\) consistently distributed bounded by 0 and 1, and a vector RNG whose components designate the allowable appropriate move towards the upper destined or the lower destined of the joint variables, in (12) evaluates the new coordinate \(x^t\) of the displacement of point \(i\) in the order of the T.F. \(F^t\)

\[ x^t = x^t + \alpha \frac{p^t}{||p^t||}\text{(RNG)} \text{ for } i = 1, 2, \ldots, k \]

The calculation of one iteration of the EM algorithm is completed when the execution of the above five procedures is finished, thus the positions of the points are updated.

3. SIMULATION RESULTS AND ANALYSIS

The EM was tested on HFS containing 2 stages, with 3 machines and a stock on the entry of the first stage and 2 machines in the second stage, and stock among the first stage and the second stage. The number of jobs varies between five and twenty. Figure 1 represents the simplified model of the FSH using Vignier's notation [15], ie FH4(P3,P2)||C\(_{\text{max}}\). Simulations under MATLAB of the EM algorithm was performed on the Intel® core™ 2 Duo 3.06 GHz computer. Table 1 presents a comparison between our final results - presented in bold - in terms of average \(C_{\text{max}}\) and the results recently published in the literature [3]. The Average CPUs for EM in (FH4, \(C_{\text{max}}\)) of N jobs as presented in Table 2.

<table>
<thead>
<tr>
<th>N jobs</th>
<th>Iterations 100</th>
<th>Iterations 500</th>
<th>Iterations 1000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GA</td>
<td>PSO</td>
<td>ME</td>
</tr>
<tr>
<td>5</td>
<td>130</td>
<td>148</td>
<td>131</td>
</tr>
<tr>
<td>10</td>
<td>214</td>
<td>271</td>
<td>206</td>
</tr>
<tr>
<td>15</td>
<td>302</td>
<td>367</td>
<td>298</td>
</tr>
<tr>
<td>20</td>
<td>421</td>
<td>556</td>
<td>391</td>
</tr>
</tbody>
</table>

**Table 2. Average CPU, for EM in (FH4, \(C_{\text{max}}\))**

<table>
<thead>
<tr>
<th>N jobs</th>
<th>Average CPUs (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.56</td>
</tr>
<tr>
<td>10</td>
<td>1.21</td>
</tr>
<tr>
<td>15</td>
<td>4.15</td>
</tr>
<tr>
<td>20</td>
<td>7.18</td>
</tr>
</tbody>
</table>
In general, we compare the results given by the application of (EM) to the FSH studied with other approaches in the literature; we note that these results are competitive. Figures 2 and 3 (value of cost average $C_{max}$) shows the comparison between the EM, PSO and GA for 5, 10, 15, and 20 jobs at different number of iterations (100, 500, and 1,000).

In Figure 2, the coordinate x represents the number of jobs as the coordinate y represents the average $C_{max}$. In order to show that the EM is qualified to produce fairness results given the additional processing time, we increased the number of iterations up to 1,000 iterations. The average $C_{max}$ is speedily diminished at the starting point of the search where the $N_{Jobs}=10$ for $N_{Iter}=500$ and $N_{Iter}=1,000$ Figure 2, hence the expectation of having more acceptable solutions in these experiments overdue to the competence of the EM to research various regions of the solution domain. The Figure 3 as well shows that by extending $N_{Iter}=1,000$, EM is qualified to find a good values compared to PSO and GA. According to the comparison between EM, GA and PSO to HFS scheduling problems, the proposed approach may be better than PSO and GA.

![Figure 2. Comparison between the EM, PSO and GA for 5, 10, 15 and 20 jobs (100, 500 and 1,000 iterations)](image)

![Figure 3. Comparison between the EM, PSO and GA for N=5 jobs](image)
4. CONCLUSION AND PROSPECTIVE ENDEAVOR
The present paper reports the results of a population-based metaheuristic approach that was originally advanced to optimize unconstrained continuous functions located on an similarity with electromagnetism concept applied to problems of FSH scheduling. We compare the simulation results of the EM algorithm applied to FSH problems with the meta-heuristics of the literature; we notice that they are very competitive. Therefore, we believe that the hybridization between EM and Tabu search could give better results by replacing the local search used by Tabu search. In the proceptive, we seek to develop the accomplishment of the EM algorithm to solve combinatorial problems using parallelism techniques. Moreover, the ME algorithm can be useful for multi-objective functions.

REFERENCES


BIOGRAPHIES OF AUTHORS

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