Fuzzy based back stepping controller for glucose level regulation under meal disturbance

Yousra Abd Mohammed¹, Rokaia Shalal Habeeb²
¹Communication Engineering Department, Technology University, Baghdad, Iraq
²Computer Engineering Department, College of Engineering, Mustansiriya University, Baghdad, Iraq

ABSTRACT
Diabetes is one of the most common and critical diseases around the world, which need insulin injections to control the body’s glucose rate. A robust back stepping (BS) controller design based on fuzzy system is introduced in this paper to control the glucose level with the presence of meal disturbance. The controller’s design is based on Bergman’s mathematical model. Different fuzzy controller structures are implemented (fuzzy PI, fuzzy PD, and fuzzy PID) controllers along with BS controller named as (BS-fuzzy PI, BS-fuzzy PD, BS-fuzzy PID) controllers. Simulation results using MATLAB/Simulink show efficiency and robustness of the proposed design in terms of controlling the insulin concentration level in blood under meal disturbance and retaining the glucose level to its Basal value.

Keywords:
Back stepping controller
Bergman mathematical model
Blood glucose level
Diabetes
Fuzzy logic controller

1. INTRODUCTION
Diabetes is an illness caused by an insulin deficiency of blood. This causes an abnormal increase in blood sugar levels for a long time while the factors causing the disease are either hereditary or environmental [1]. Blood glucose is normally regulated by the insulin hormone, which is produced by beta cells in the pancreas. In type 1 diabetes mellitus (T1DM), the beta cells are incapable of providing enough insulin due to the body’s attack (autoimmune destruction) and the glucose level becomes uncontrollable. This type needs to inject insulin into the human body to maintain a normal glucose concentration [2]. Hyperglycemia (Glucose concentration in bloodstream is higher than the normal level) leads to life-threatening of a patient. Unless the patient with diabetes mellitus is given sufficient insulin, his life will be at risk and makes the body vulnerable to many serious problems such as renal failure, nerve damage and limb amputation [3], [4]. A normal human's blood glucose concentration is in a narrow range (70-110) mg/dL. Diabetes is diagnosed when the human body is unable to control the normal interaction of glucose and insulin [5]. Therefore, insulin must be injected to regulate the level of glucose in the blood [6]. The closed-loop glucose regulation system is generally formed up of three main elements, which are: control method, insulin pump, and glucose sensor to estimate the insulin dose depending on glucose monitoring [7]. Figure 1 illustrates the block diagram of the insulin regulation system. Controllers are needed to set insulin treatment according to the measurements of glucose concentrations as shown in Figure 1.

Various kinds of control units were introduced by many researchers in the literature. To name few, a second-order sliding mode control design is introduced to control the glucose concentration level [8]. Two robust control methods are presented in [9], to regulate the glucose-insulin system for Type I, LQ and H∞
control are proved to be effective for providing acceptable control performance (By refusing to disturbances food meal). A new back-stepping based sliding mode control (B-SMC) approach is presented as a way to ensure practical glucose concentration tracking [10]. A back stepping controller design is proposed, based on Lyapunov stability theorem for the glucose-insulin model of type I diabetes in [11]. A comparison between back stepping sliding mode controller and the conventional state feedback controller is presented in [12]. A single model predictive controller to regulate insulin, and glucagon, is presented in [13]. An integrated of adaptive backstepping is proposed in [14], based non-linear controller with artificial pancreas, to control type I diabetes. Three nonlinear controllers are proposed in [15]. Integral backstepping controller, backstepping controller and fuzzy logic controller, for the automatic regulation of the blood glucose level in diabetic patients. Under parametric uncertainties. A nonlinear explicit model predictive control, to regulate blood glucose in type-1 diabetic patient, is presented in [16]. A simulation model of glucose metabolism process as well as a backstepping linear quadratic Gaussian controller design, is proposed in [17], to regulate the blood glucose level in type-1 diabetic patients.

\[ g(t) = x_1 \rightarrow \dot{g}(t) = \dot{x}_1(t) \]
\[ x(t) = x_2 \rightarrow \dot{x}(t) = \dot{x}_2(t) \]
\[ y(t) = x_3 \rightarrow \dot{y}(t) = \dot{x}_3(t) \]
\[ i(t) = u \]  

(2)

Figure 1. Closed loop insulin regulation system block diagram [8]

In this paper, a fuzzy-PID, fuzzy-PI and fuzzy-PD controllers are combined with a back stepping (BS) controller and implemented to control the glucose level in the human body under meal disturbance. The paper is organized as follows into five sections. In section 2, the model is presented; BS design is explained and fuzzy logic system (FLS) is presented in section 3. Section 4 shows the proposed controller's efficiency with simulation results. Finally, a conclusion is presented in section 5.

2. MATHEMATICAL MODEL OF BERGMAN'S SYSTEM

Many different mathematical models of diabetic systems have been studied in previous works. The mathematical model presented in [18] is adopted in many researches considering the dynamics of glucose and insulin interaction in the blood system [18–20]. In this work, the mathematical model in [18] is used. The nonlinear differential equation is given by [20]. Physical meanings of the variables in (1) are shown in Table 1.

\[
\begin{align*}
\dot{G}(t) &= -p_1 G(t) - (G(t) + G_b) X(t) + h(t) \\
\dot{X}(t) &= -p_2 X(t) + p_3 Y(t) \\
\dot{Y}(t) &= -p_4 (Y(t) + Y_b) + i(t)/V_L
\end{align*}
\]

(1)

Table 1. Physical meanings of the variables in Bergman’s system equations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G(t)$</td>
<td>Plasma glucose deviation</td>
<td>mg/dL</td>
</tr>
<tr>
<td>$X(t)$</td>
<td>Remote compartment insulin utilization</td>
<td>1/min</td>
</tr>
<tr>
<td>$u(t)$</td>
<td>Exogenous glucose infusion (control variable)</td>
<td>mU/min</td>
</tr>
<tr>
<td>$Y(t)$</td>
<td>Plasma insulin deviation</td>
<td>mU/dL</td>
</tr>
<tr>
<td>$h(t)$</td>
<td>The disturbance</td>
<td>mg/dL/min</td>
</tr>
<tr>
<td>$G_b$</td>
<td>Basal glucose level</td>
<td>110 mg/dL</td>
</tr>
<tr>
<td>$V_b$</td>
<td>Basal insulin level</td>
<td>1.5 mg/dL</td>
</tr>
<tr>
<td>$V_L$</td>
<td>The insulin distribution volume</td>
<td>120 dL</td>
</tr>
<tr>
<td>$p_1, p_2, p_3, p_4$</td>
<td>The model parameters</td>
<td>1/min, 1/min, dL/mU/min, 1/min</td>
</tr>
</tbody>
</table>

In (1), can be written as:

\[
\begin{align*}
\dot{x}_1(t) &= -p_1 x_1 - (x_1 + G_b) x_2 + h(t) \\
\dot{x}_2(t) &= -p_2 x_2 + p_3 x_3 \\
\dot{x}_3(t) &= -p_4(x_3 + y_B) + 1/V_L u
\end{align*}
\]  

(3)

the linearity form in the state space of the model was taken at the points of equilibrium with following specified values:

\[
G_0 = x_0 = y_0 = 0 \quad h = 0
\]  

(4)

and for \(i_0 = p_4 y_B V_L = 16.667\).

It is assumed that the state variable \(X(t)\) is also called slow variable, that is \(\dot{X}(t)\):

\[
\begin{align*}
\dot{G}(t) &= -p_1 G(t) - (G(t) + G_B) X(t) + h(t) \\
0 &= -p_2 \dot{X}(t) + p_2 Y(t) \\
\dot{Y}(t) &= -p_4 (Y(t) + Y_B) + i(t)/V_L \\
X(t) &= \frac{p_1}{p_2} Y(t)
\end{align*}
\]  

(5)

(6)

after applying in (6), the (1) becomes:

\[
\begin{align*}
\dot{G}(t) &= -p_1 G(t) - (G(t) + G_B) \frac{p_1}{p_2} y(t) + h(t) \\
\dot{Y}(t) &= -p_4 (Y(t) + Y_B) + i(t)/V_L
\end{align*}
\]  

(7)

This is due to the fact that the non-measurable \(X(t)\) variable (the use of isolated insulin) is a slow variable, which can be considered zero, thus eliminating the second equation of model (1) [21]. As a result, the linear low model is [22].

\[
\begin{bmatrix}
\dot{G}(t) \\
\dot{y}(t)
\end{bmatrix} = \begin{bmatrix}
-p_1 - \frac{p_2}{p_1} y_0 \\
0
\end{bmatrix} G(t) + \begin{bmatrix}
0 \\
1/V_L
\end{bmatrix} i(t) + \begin{bmatrix}
1 \\
0
\end{bmatrix} h(t)
\]  

(8)

Since and,

\[
y_0 = g_0 = 0
\]  

(9)

\[
G(t) = x_1 Y(t) = x_2 \\
\dot{x}_1 = \begin{bmatrix}
-p_1 & -p_2 \\
0 & -p_4
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
1/V_L
\end{bmatrix} u(t) + \begin{bmatrix}
1 \\
0
\end{bmatrix} h(t)
\]  

(10)

the system can be written as:

\[
\begin{align*}
\dot{x}_1 &= -p_1 x_1 - \frac{p_2}{p_2} G_b x_2 + h \\
\dot{x}_2 &= -p_4 x_2 + 1/V_L u
\end{align*}
\]  

(11)

(12)

3. BACKSTEPPING CONTROLLER (BSC) DESIGN FOR BLOOD GLUCOSE

The Backstepping approach is an effective tool for high-order system control problems [23], [24]. The goal of the proposed BS is to stabilize the system when starting from any initial state and regulate the state to the origin (until reaching the origin). Let us consider the (11) as the first subsystem and the (12) as the second subsystem. Accordingly, consider in (11) with \(x_1\) as the state variable which is required to stabilize it via \(x_2\). \(x_2\) is considered here as a virtual controller, namely, we rewrite the (11) as follows:

\[
\begin{align*}
x_2 &= v \\
\dot{x}_1 &= -p_1 x_1 - \frac{p_2}{p_2} G_b v + h
\end{align*}
\]  

(13)

(14)
in (14) is a first-order system with $v$ as a control input. Let $v$ be chosen as:
\[
v = \frac{1}{(\frac{p_2}{p_2^2})G_B} \left[ -p_1x_1 + \lambda x_1 \right] \rightarrow v = \frac{1}{(\frac{p_2}{p_2^2})G_B} \left[ -p_1 + \lambda \right] x_1
\] (15)
where $\lambda > 0$. Hence in (14) became:
\[
\dot{x}_1 = -\lambda x_1
\] (16)
the second state is:
\[
y = x_2 - v
\] (17)
\[
\dot{y} = \dot{x}_2 - \dot{v}
\]
\[
y = -p_4x_2 + 1/V_2u - \dot{v}
\] (18)
where:
\[
\dot{v} = \frac{1}{(\frac{p_2}{p_2^2})G_B} (-p_1 + \lambda) \dot{x}_1
\]
according to the previous equations:
\[
\lambda_1 = \frac{1}{(\frac{p_2}{p_2^2})G_B} (-p_1 + \lambda)
\]
where the derivative $v$ function is:
\[
\dot{v} = \lambda_1 \dot{x}_1
\]
\[
v = \lambda_1(-p_1x_1 - (\frac{p_2}{p_2^2})G_Bx_2 + h)
\]
Let $u = V_L[\dot{v} - ay + p_4x_2]$
then in (18) becomes:
\[
\dot{y} = -ay, \ a > 0
\]
the output $y$ goes exponentially asymptotically to zero as $t \to \infty$. As $y$ approach zero value $x_2 \to v$. Finally, the control law is given by:
\[
u = \dot{v} - ay + p_4x_2
\]
\[
u = V_L \left[ \lambda_1(-p_1x_1 - (\frac{p_2}{p_2^2})G_Bx_2 + h) - a(x_2 - v) + p_4x_2 \right]
\]
\[
u = V_L \left[ \lambda_1(-p_1x_1 - (\frac{p_2}{p_2^2})G_Bx_2 + h) - ax_2 - a\lambda_1x_1 + p_4x_2 \right]
\] (19)
where:
\[
k_1 = V_L(-\lambda_1p_1 - a\lambda_1)
k_2 = V_L(-\lambda_1(\frac{p_2}{p_2^2})G_B - a + p_4)
\]
by suitable selection for the parameters of the BS controller, these parameters are:
\[
\lambda = 0.114
\lambda_1 = 0.15, a = 0.02
k_1 = -0.864
k_2 = -1.536
\]

### 3.1. BS Controller based fuzzy system design

BS controller based on fuzzy logic system (FLS) is widely used to solve the real-world problems due to its ability to compensate for structured and unstructured uncertainty. In this paper, a fuzzy PI, PD and PID are combined with BS controller to improve its performance. The proposed BS controller control law based on the fuzzy system is given by:
Fuzzy based back stepping controller for glucose level regulation (Yousra Abd Mohammed)

\[ u(t) = -k_1x_1 - k_2x_2 + u_{\text{fuzzy}} \]  
(20)

where \( u_{\text{fuzzy}} \) is fuzzy PI, PD, PID controller with:

\[ u_{PI} = k_p x_1 + k_i \int x_2(t) \]  
(21)

\[ u_{PD} = k_p x_1 + k_d \frac{dx_2(t)}{dt} \]  
(22)

\[ u_{PID} = k_p x_1(t) + k_i \int x_2(t) + (k_p x_1(t) + k_d \frac{dx_2(t)}{dt}) \]  
(23)

where \( k_p, k_d, \) and \( k_i \) are the proportional gain, the derivative gain, and the integral gain respectively. In this paper the standard procedure of fuzzy controller's design is followed. Generally, the structure of a fuzzy logic controller consists of three basic steps fuzzification, control rule base establishment, and defuzzification.

In the fuzzification step, the physical values of the input and output are transformed into fuzzy linguistic sets. The fuzzy sets are normalized and arranged in subsets (intervals) according to the range of the input or output associated with membership function. These membership functions describe the degrees of the confidence of the input or output related to this range. This step is done to make the input physical values suitable with the fuzzy control rule base [25]. The membership functions considered in the design of the fuzzy PD, PI and PID controllers, are the same and are chosen as follows: i) fuzzy set 'plasma glucose' has two memberships P (positive) and N (negative), ii) fuzzy set 'plasma insulin' has two memberships P (positive and N (negative), and iii) the output controller is represented by three membership functions. And iv: Fuzzy set 'out' of PI, PD has three memberships P (out_positive), N (out_negative), and Z (out_zero) as shown in Figure 2(c). Fuzzy membership functions as shown in Figure 2. The Fuzzy memberships of Input for Plasma Glucose (g), Input for plasma insulin (y), and output (out) are illsetrated in Figure 2(a), Figure 2(b), and Figure 2(c) respectively.

\[ \text{IF (set conditions satisfied) Then (deductive result set).} \]

Fuzzy inference rules (FIR) are set of IF-THEN rules that control the relation between input and output of the fuzzy system and it is stated as follows: IF (set conditions satisfied) Then (deductive result set). These rules consist of a linguistic group associated with the input, output and control variables. It may also include fuzzy operators such as OR and AND [25]. In this work, the fuzzy rule systems for PI, PD and PID controllers are the same and it is represented by four rules as follows:

- If \( g = \text{N} \) AND \( y = \text{N} \) THEN OUT=\( \text{N} \).
- If \( g = \text{P} \) AND \( y = \text{P} \) THEN OUT=\( \text{P} \).
- If \( g = \text{P} \) AND \( y = \text{N} \) THEN OUT=\( \text{Z} \).
- If \( g = \text{N} \) AND \( y = \text{P} \) THEN OUT=\( \text{Z} \).
In the defuzzification step the fuzzy controller outputs are converted into crisp real signal \( f_{\text{out}} \) and then send it as a control action to the physical plant [20]. There are several methods for defuzzification, the center of mass (COM) formula is used in this paper.

\[
f_{\text{out}} = \frac{\mu u(1) \cdot \text{output}(1) + \mu u(2) \cdot \text{output}(2) + \ldots + \mu u(n) \cdot \text{output}(m)}{\mu u(1) + \mu u(2) + \ldots + \mu u(m)}
\]

(24)

Where:
- The control output as a crisp value is \( f_{\text{out}} \).
- The number of rules is \( m = 4 \).
- The membership value of the output \( n \) is \( \mu u(m) \)

4. SIMULATION RESULTS

The proposed controller has been tested using MATLAB-Simulink (2016) software and its performance is assessed. The Simulink module is shown in Figure 3. The experimental results are carried out in a personal Computer where Windows 10 Pro is installed with 2.6 GHz processor and 8 GB of installed memory (RAM).

![Simulink model of the proposed controller](image)

The controller has been applied to the diabetic model which is represented in (7) and the response of the system is tested with disturbance meal existence. Typical values considered for the parameters used in Bergman’s system equations are shown in Table 2. The parameters considered in the experimental results for the BS fuzzy controller are shown in Table 3. The behavior function for meal disturbance representing the exogenous glucose infusion is shown in Figure 4.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_B )</td>
<td>Basal glucose level</td>
<td>110 mg/dL</td>
</tr>
<tr>
<td>( Y_B )</td>
<td>Basal insulin level</td>
<td>1.5 mg/dL</td>
</tr>
<tr>
<td>( V_L )</td>
<td>The insulin distribution volume</td>
<td>120 dL</td>
</tr>
<tr>
<td>( p_1 ), ( p_2 )</td>
<td>The model parameters</td>
<td>( p_1 = 0.028 ) 1/min, ( p_2 = 0.0251 ) 1/min</td>
</tr>
<tr>
<td>( p_3 ), ( p_4 )</td>
<td>The model parameters</td>
<td>( p_3 = 0.00013 ) dL/ml/min, ( p_4 = 5/54 ) 1/min</td>
</tr>
</tbody>
</table>

Table 2. Parameters’ values considered in Bergman’s system equations

![image](image)
Fuzzy based back stepping controller for glucose level regulation ... (Yousra Abd Mohammed)

Table 3. The parameters of proposed BS controller and PID fuzzy

<table>
<thead>
<tr>
<th>Parameter of BS controller</th>
<th>Parameter of BS fuzzy (PI, PD, and PID)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda = 1.5, \alpha = 0.02 )</td>
<td>( k_p = 1.4 )</td>
</tr>
<tr>
<td>( k_i = 0.864 )</td>
<td>( k_p = 0.5 )</td>
</tr>
<tr>
<td>( k_d = 1.536 )</td>
<td>( k_i = 1.2 )</td>
</tr>
</tbody>
</table>

Figure 4. Disturbance meal function (exogenous glucose infusion) [9]

Figure 5 shows two plasma glucose deviations. Figure 5(a) illustrates the simulation results for the plasma glucose deviation \( G(t) \) when BS-fuzzy controllers are used. The figures show the glucose deviation \( G(t) \) values do not exceed (120.45 mg/dl) when applying the BS-fuzzy PD and BS-fuzzy PID controllers which means a (9.5\%) change over the basal value (110 mg/dl) while for a BS-fuzzy PI controller the maximum glucose level is around (123 mg/dl) i.e. (11.8 \%) change over the basal value.

Figure 5(b) shows a comparison between the simulation results for the plasma glucose deviation \( G(t) \) when BS controller is used and when BS-fuzzy PD is used. It’s obvious that when using the BS controller, the maximum glucose level reaches higher values of glucose deviation and hence permits higher percentage change over the basal value. Figure 6 shows the insulin deviation \( Y(t) \) when applying BS-fuzzy (PD, PI, PID) controllers. It is obvious from the figure that the insulin deviation does not rise over the Basal value for the insulin level (1.5 mU/dl). When applying the controllers this level is regulated to zero after less than 400 minutes.

Figure 7 shows two insulin infusion controls. Figures 7(a) and (b) illustrates the insulin infusion control \( I(t) \) when applying BS, BS-fuzzy (PI, PD, PID) controllers. The insulin infusion is done to maintain the blood glucose level in the patient’s blood after having a meal. It is clear that the convergence time is less than 400 minutes. Higher infusion level can be seen for the BS-fuzzy PD and BS-fuzzy PID controller.

Figure 5. Plasma glucose deviations: (a) when applying BS-fuzzy (PI, PD, PID) and (b) when applying BS and BS-fuzzy (PD) controllers
5. CONCLUSION

In this work, an improved BS controller based on different fuzzy controller structure was suggested to the nonlinear three-state minimal blood glucose model of Bergman, to control the blood glucose level. Simulation results are demonstrated through the Matlab tool, the efficiency of the proposed (BS and BS with fuzzy controller) under meal disturbance has obtained. BS and BS-fuzzy PI controllers show low performance as the glucose level exceeded the value of 123 mg/dL with lower insulin infusion as compared to the BS-fuzzy PD and BS-fuzzy PID. Both BS-fuzzy PD and BS-fuzzy PID show better performance as the maximum glucose level is less than 120 mg/dL and higher insulin infusion.

REFERENCES


Yousra Abd Mohammed (✉) is a lecturer at the Communication Engineering Department, Technology University, Baghdad, Iraq since 2005. She received her B.Sc. in Electronic and Communication Engineering from Technology University/Baghdad, Iraq in 1992 and her M.Sc. degree in Computer Engineering from Technology University/Baghdad in 2004. Her research interests include Control Systems, Encryption and Decryption Algorithms. She can be contacted at email: 30021@uotechnology.edu.iq.

Rokaia Shalal Habeeb (✉) is a lecturer at the Computer Engineering Department, College of Engineering, Mustansiriyah University, Baghdad, Iraq since 1997. She received her B.Sc. in Electrical Engineering from Mustansiriyah University/Baghdad, Iraq in 1995 and her M.Sc. degree in Electronic and Communication Engineering from Mustansiriyah University/Baghdad in 2002. Her research interests include but not limited to, Intelligient Control Systems, Optimization Algorithms, and Image Processing. She can be contacted at email: rokaia.shalal@uomustansiriyah.edu.iq.