

# A Novel Decoding Algorithm for BICM-ID Embedded Turbo Codes

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## Abstract

*Bit-interleaved coded modulation iterative decoding (BICM-ID) embedded turbo code is widely used in wireless communication because of its excellent performance. This paper proposes an improved logarithmic maximum a posteriori (Log-MAP) algorithm for BICM-ID embedded turbo codes. It can yield excellent bit error rate (BER) performance with much lower complexity. The proposed algorithm exploits a linear interpolation and Least Squares approximation function to replace the logarithmic correction in the Jacobian logarithmic function, which avoids complicated logarithm look-up table operations in Log-MAP algorithm. Simulation results show that the novel algorithm can offer almost equivalent performance to the optimal algorithm with much less computation. Compared with the improved MAX-Log-MAP algorithm proposed by Talakoub, the proposed algorithm can reduce about 36% of computational complexity, meanwhile it achieves 0.1db-0.16db performance gains. In addition, it obtains 0.35-0.4db gains than MAX-Log-MAP algorithm.*

**Keywords:** BICM-ID, Log-MAP, interpolation function, least squares, turbo decoding

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## 1. Introduction

Bit-interleaved coded modulation iterative decoding (BICM-ID) [1] is an effective transmission scheme without bandwidth expansion, which combines bit-interleaved coded modulation (BICM) [2] with iterative decoding. Turbo code [3], which was first proposed by C.Berrou in 1993, makes good use of the Shannon channel coding theory and is close to the Shannon capacity limit. Naturally, people combine BICM-ID with turbo code so as to simultaneously achieve large coding gain and high bandwidth efficiency, which leads to BICM-ID embedded turbo codes. It is a very promising technology compared to the classical turbo codes. As an improved structure, BICM-ID embedded turbo code has been utilized in many wireless communication systems to achieve better performance [4, 5]. The structure of the BICM-ID embedded turbo code is shown in Figure 1.

In the decoding algorithms, BCJR algorithm [6] i.e. the maximum a posteriori (MAP) algorithm has the best performance but it can not decode until the decoder receives the entire bit sequence. This leads to large decoding delay. The Log-MAP [7] algorithm is one approximation of MAP algorithm, and its performance is nearby MAP. However, reading data from a big logarithm table is a time consuming process for the Log-MAP. The MAX-Log-MAP [8] is another desirable candidate because of its simplicity, while it loses lots of performance. The improved MAX-Log-MAP algorithm [9] achieves the balance between complexity and performance, but it involves lots of redundant computations without performance gains. Additionally, its performance is equal to the MAX-Log-MAP under some circumstances. So it is essential to propose a novel algorithm that owns simple complexity and superior performance for BICM-ID embedded turbo codes.

In this paper, we propose a novel decoding algorithm based on Log-MAP and the improved MAX-Log-MAP algorithm. Our algorithm proposes a novel approximation function of logarithmic correction using mathematical method. It is more superior than conventional decoding algorithms in terms of performance and complexity.

The rest of this paper is organized as follows. In Section 2, we briefly introduce the traditional turbo decoding algorithms. The proposed algorithm is presented in Section 3. We analyze complexity and compare the performance of the proposed algorithm with the other decoding algorithms in Section 4. Finally, the conclusion is given in Section 5.

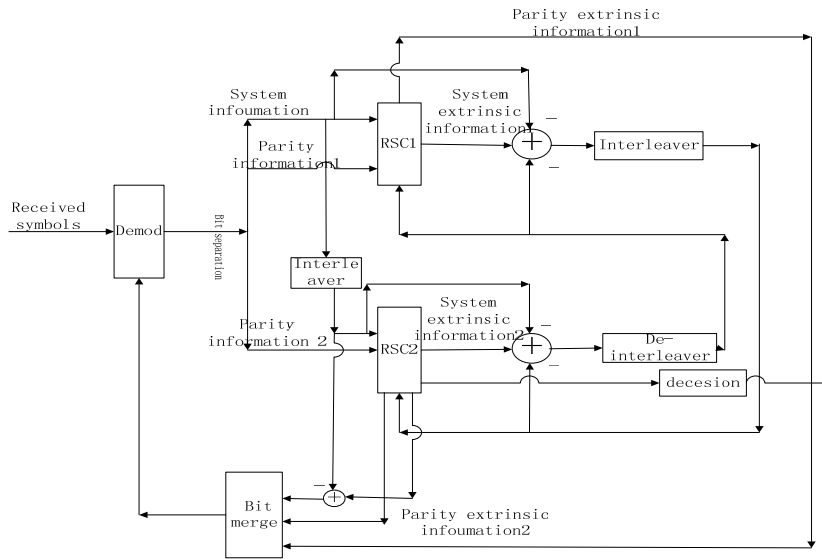


Figure 1. The Structure of the BICM-ID Embedded Turbo Codes

## 2. The Conventional Decoding Algorithms

At present, the classical turbo decoding algorithms mainly include Log-MAP algorithm, MAX-Log-MAP algorithm, and the improved MAX-Log-MAP algorithm.

The goal of the Log-MAP algorithm is to compute log-likelihood ratio (LLR) [9, 10]:

$$L(u_l) = \ln \left[ \sum_{(s_{l-1}, s_l), u_l=+1} e^{\alpha_l^*(s_{l-1}) + \beta_{l+1}^*(s_l) + \gamma_l^*(s_{l-1}, s_l)} \right] - \ln \left[ \sum_{(s_{l-1}, s_l), u_l=-1} e^{\alpha_l^*(s_{l-1}) + \beta_{l+1}^*(s_l) + \gamma_l^*(s_{l-1}, s_l)} \right] \quad (1)$$

Where  $u_l$  is the information bits,  $S_l$  and  $S_{l-1}$  denote the state at  $l$ th and  $l-1$ th time instant. To compute Equation (1), we need to recursively calculate forward and backward metrics, denoted as  $\alpha_l(s_l)$  and  $\beta_l(s_l)$ .

Define the following function:

$$\max^*(x, y) = \ln(e^x + e^y) = \max(x, y) + \ln(1 + e^{-|x-y|}) \quad (2)$$

Where  $\ln(1 + e^{-|x-y|})$  is a correction, which makes Log-MAP be an optimal algorithm.

According to Equation (2), the forward and backward metrics can be computed as:

$$\begin{cases} \alpha_l^*(S_l) = \ln(\alpha_l(S_l)) = \max_{S_{l-1} \in \sigma_{l-1}}^* (\gamma(S_{l-1}, S_l) + \alpha_{l-1}^*(S_{l-1})) \\ \beta_l^*(S_l) = \ln \beta_l(S_l) = \max_{S_{l+1} \in \sigma_{l+1}}^* (\gamma(S_l, S_{l+1}) + \beta_{l+1}^*(S_{l+1})) \end{cases} \quad (3)$$

Where  $\sigma_{l-1}$  and  $\sigma_l$  are collection of all states at the moment  $l-1$  and  $l$  respectively, and  $\gamma$  is the branch metrics.

Finally, we rewrite Equation (1) as:

$$L(u_l) = \max_{(s_{l-1}, s_l), u_l=+1}^* [\beta_l^*(S_l) + \gamma(S_{l-1}, S_l) + \alpha_{l-1}^*(S_{l-1})] - \max_{(s_{l-1}, s_l), u_l=-1}^* [\beta_l^*(S_l) + \gamma(S_l, S_{l-1}) + \alpha_{l-1}^*(S_{l-1})] \quad (4)$$

The MAX-Log-MAP algorithm is obtained by omitting the logarithmic part of Equation (2), so it is a suboptimal algorithm.

The improved MAX-Log-MAP is an algorithm which modifies Equation (2) into Equation (5) using the MacLaurin Series [9].

$$\max^*(x, y) \approx \max(x, y) + \max\left(0, \ln 2 - \frac{1}{2}|x - y|\right) \quad (5)$$

### 3. The Proposed Algorithm

For convenience, we let  $|x - y| = t$ , and then, Equation (5) is expressed as:

$$\max^*(x, y) \approx \begin{cases} \max(x, y) + \ln 2 - |x - y|/2, & t < 2 \ln 2 \\ \max(x, y), & t > 2 \ln 2 \approx 1.3862 \end{cases} \quad (6)$$

When  $t < 1.3862$ , after removing the absolute value, we get the following equation:

$$\max(x, y) + \ln 2 - |x - y|/2 \equiv (x + y)/2 + \ln 2 \quad (7)$$

When  $t > 1.3862$ , the improved MAX-Log-MAP algorithm is completely equal to the MAX-Log-MAP algorithm, so it is suboptimal too.

We add the correction to solve this problem, then Equation (6) becomes:

$$\max^*(x, y) \approx \begin{cases} (x + y)/2 + \ln 2, & t < 1.3862 \\ \max(x, y) + \ln(1 + e^{-|x-y|}), & t > 1.3862 \end{cases} \quad (8)$$

However, other problems will follow. Firstly, lookup tables are required for a wide range of operating signal-to-noise ratios (SNRs), which increases the hardware cost. Secondly, saving the results of  $\ln(1 + e^{-|x-y|})$  in a lookup table would introduce a quantization error caused by truncation [9]. Thirdly, reading data from logarithm tables is a time consuming process. So it is necessary to exploit another function instead of  $\ln(1 + e^{-|x-y|})$ . We deduce as follows:

$$t < 2 \ln 2 \Rightarrow 0 < e^{-t} < 1/4 \quad (9)$$

Let  $e^{-t} = m$ , then we can get:

$$\ln(1 + e^{-|x-y|}) = \ln(1 + m), 0 < m < 1/4 \quad (10)$$

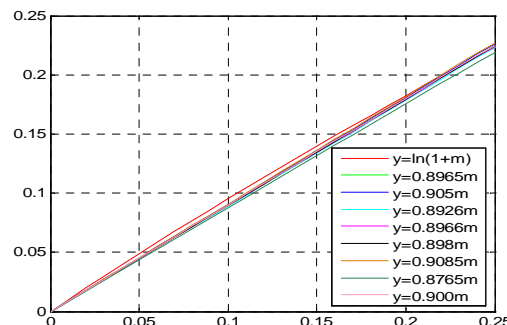


Figure 4. The Comparison of the Approximation Straight Lines

In  $(0, 1/4)$  such a small range,  $\ln(1+m)$  will probably be accurately approximated by a linear function. Selecting eight points around  $1/4$ , we can obtain eight straight lines by the method of interpolation function approximation [11]. The lines are shown in the Figure 4.

Observing carefully, we find the optimal approximation straight line is:

$$\ln(1+m) \approx 0.8966m. \quad (11)$$

By Equation (11) we can get:

$$\ln(1+e^{-t}) \approx 0.8966e^{-t}. \quad (12)$$

Moreover, we can find an optimal linear least squares approximation function replace  $e^{-t}$  of equation (12) in  $(1.3862, 2.3862)$  range. Derivation process is as follows.

Let  $S$  be the subspace of all linear functions in  $C[1.3862, 2.3862]$ . Although the functions 1 and  $t$  span  $S$ , they are not orthogonal. We seek a function of the form  $t - a$  that is orthogonal to 1.

$$\langle 1, t - a \rangle = \int_{1.3862}^{2.3862} (t - a) dt = 1.8862 - a \quad (13)$$

Thus  $a=1.8862$ . Since:

$$\|t - 1.8862\| \approx \sqrt{0.0833}, \quad (14)$$

It follows that:

$$\begin{cases} f_1(t) = 1 \\ f_2(t) = \frac{1}{\sqrt{0.0833}}(t - 1.8862) \end{cases} \quad (15)$$

Form an orthonormal basis for  $S$ .

Let,

$$\begin{aligned} c_1 &= \int_{1.3862}^{2.3862} f_1(t)e^{-t} dt = e^{-1.3862} - e^{-2.3862} \\ c_2 &= \int_{1.3862}^{2.3862} f_2(t)e^{-t} dt = \frac{1}{\sqrt{0.0833}}(0.5e^{-1.3862} - 1.5e^{-2.3862}) \end{aligned} \quad (16)$$

The projection:

$$\begin{aligned} e^{-t} &= c_1 f_1(t) + c_2 f_2(t) \\ &= 32.9652 e^{-2.3862} - 10.3217 e^{-1.3862} + (18.0072 e^{-2.3862} - 6.0024 e^{-1.3862})t \\ &\approx -0.1555 t + 0.4514, 1.3862 < t < 2.3862 \end{aligned} \quad (17)$$

Is the best liner least squares approximation to  $e^{-t}$  in  $(1.3862, 2.3862)$ .

Similarly, in the range  $(2.3862, 3.3862)$  and  $(3.3862, 4.3862)$ ,  $e^{-t}$  can be expressed as follows:

$$e^{-t} \approx \begin{cases} -0.0572 t + 0.2233, 2.3862 < t < 3.3862 \\ -0.0210 t + 0.1032, 3.3862 < t < 4.3862 \end{cases} \quad (18)$$

According to p. Robertson's study, just selecting the  $t$  between 0 and 5 could obtain the ideal approximation [12], so we obtain the following equation:

$$e^{-t} \approx 0, t > 4.3862 \tag{19}$$

Combine the expressions (8)(12)(17)(18) and (19), we deduce the approximately expression of  $\max^*(x, y)$  as follows:

$$\max^*(x, y) \approx \begin{cases} (x+y)/2 + \ln 2, t < 1.3862 \\ \max(x, y) + kt + b, 1.3862 < t < 4.3862 \\ \max(x, y), t > 4.3862 \end{cases} \tag{20}$$

Here, the values of  $k, b$  are shown in Table 1.

Range of t	k	b
(1.3862,2.3862)	-0.1394	0.4047
(2.3862,3.3862)	-0.0515	0.2002
(3.3862,4.3862)	-0.0188	0.0925

With (20) calculating the  $\alpha_i(s_i), \beta_i(s_i)$  and the LLR, we can accomplish better performance with much lower complexity than the conventional decoding algorithms.

#### 4. Simulation Results

In the simulation, two systematic recursive convolutional (RSC) encoders are employed. We consider two circumstances which the generator polynomials respectively are  $g_1 = [7,5]$  and  $g_2=[11,13]$ . Simulations were respectively executed for code rate(denote R) of 1/2 and 3/4 respectively. The interleaving length is set to 1024 and the maximum number of iterations is 6. We only consider BPSK, additive white gaussian noise (AWGN) channels in the transmission environment.

##### 4.1. Performance Comparison

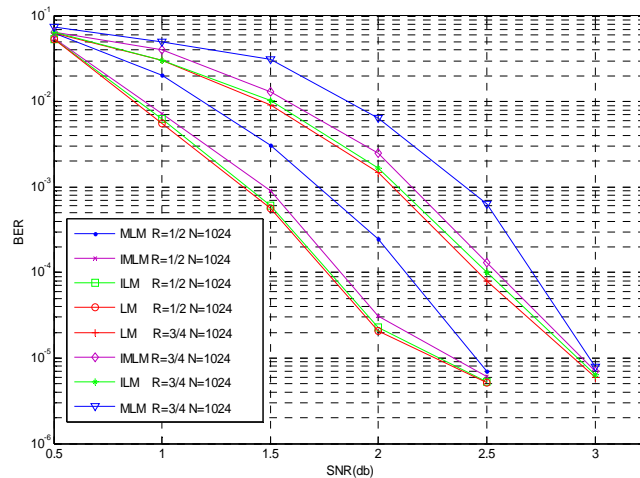


Figure 3. BER Performance Comparison, N=1024, Six Iterations, AWGN Channels,  $g_1=[7,5]$

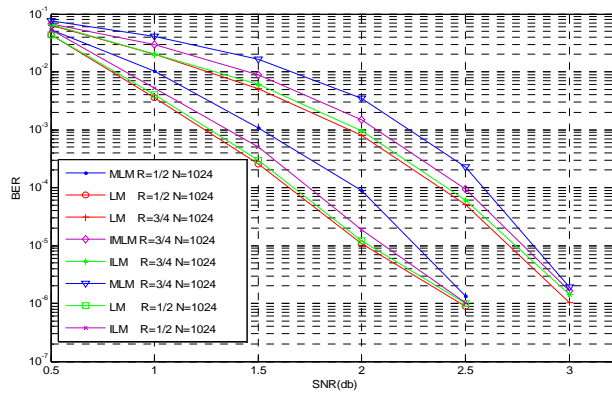


Figure 4. BER Performance Comparison, N=1024, Six Iterations, AWGN Channels,  $g_2 = [11, 13]$

The performance results with different schemes are shown in Figure 3 and Figure 4. We employ the following abbreviations to replace the four algorithms.

IMLM: the improved MAX-Log-MAP algorithm

ILM: the improved Log-MAP algorithm

LM: the Log-MAP algorithm

MLM: the MAX-Log-MAP algorithm

Figure 3 shows BER performance for the generator polynomial  $g_1 = [7, 5]$ . The curves show the BER for ILM compared with LM, MLM and IMLM for  $R = 1/2$  and  $R = 3/4$  respectively. As can be seen from Figure 3, ILM, IMLM and LM have obvious performance advantage in comparison to MLM. ILM can offer almost equivalent performance compared with LM and it obtains about 0.35db performance gains than MLM and 0.1db gains than IMLM.

Figure 4 presents the same results while its generator polynomial is  $g_2 = [11, 13]$ . From Figure 4, we can see that ILM achieves about 0.4 db and 0.16db gains than MLM and IMLM respectively. With the increasing of SNR, the performance of the four algorithms converges gradually.

In conclusion, the proposed algorithm is superior than MLM and the IMLM, and its performance is close to the optimal algorithm.

#### 4.2. Complexity Analysis

We obtain the probabilities of calculating  $\alpha, \beta$  and APP L-value when  $t$  locates in different range by statistics to compute the complexity. Selecting 5 SNRs to make large numbers of statistics, and then, we respectively calculate the average in each range and write as  $p_1, p_2, p_3, p_4, p_5$

$$\begin{cases} p_1 = 0.0562 \\ p_2 = 0.0390 \\ p_3 = 0.0394 \\ p_4 = 0.0365 \\ p_5 = 0.8285 \end{cases} \quad (33)$$

Using these data, we can compute the computation calling the formula (32) once.

The number of addition is:

$$2 \times (1 - p_5) \approx 0.3430 \quad (34)$$

The number of multiplication is:

$$1 \times (1 - p_5) \approx 0.1715 \quad (35)$$

The number of comparison is:

$$(2 - p_1) + (1 - p_5) \left[ 1 + \sum_{i=2}^4 p_i (1 + \sum_{i=3}^4 p_i) \right] \approx 2.1365 \quad (36)$$

Through complex calculating, we obtain the Table 2 by referring to [10, 12], where  $M$  is the constraint length of encoder.

Table 2. The Complexity of Turbo Decoding Algorithms

Algorithms	LM	MLM	IMLM	ILM
Comparisons	$5 \times 2^M - 2$	$5 \times 2^M - 2$	$10 \times 2^M - 4$	$10.6825 \times 2^M - 4.273$
Additions	$15 \times 2^M + 9$	$10 \times 2^M + 11$	$20 \times 2^M + 15$	$11.143 \times 2^M + 2.6002$
Multiplications	8	8	$4 \times 2^M + 6$	$0.686 \times 2^M + 7.657$
Look-ups	$5 \times 2^M - 2$	0	0	0

From Table 2 we can see that ILM avoids the logarithm calculation for LM. Additionally, the complexity of ILM is reduced about 36% compared with IMLM.

## 5. Conclusion

We propose an improved Log-MAP algorithm for BICM-ID embedded turbo code in this paper. The novel method utilizes a new function to expand the correction function of the Log-MAP algorithm by the method of interpolation function and Least Squares approximation. As are shown in the complexity analysis and simulation results, the proposed algorithm can accomplish almost equivalent performance to the optimal algorithm with a much lower complexity. In addition, it offers about 0.1-0.16db performance gains than the improved MAX-Log-MAP algorithm with only 74% computations. To sum up, the novel algorithm has obvious advantages in contrast to the traditional algorithms and it can be easily implemented in BICM-ID embedded turbo codes.

## References

- [1] X Li, A Chindapol, JA Ritce. Bit-interleaved coded modulation with iterative decoding and 8PSK signaling. *IEEE Trans. on Commun.* 2002; (50): 1250-1257.
- [2] E Zehavi. 8-PSK trellis codes for a Rayleigh channel. *IEEE Trans. Commun.* 1992; (40): 873-884.
- [3] Ji-Hoon Kim. In-CheolPark, ExpressBriefs. Bit-Level extrinsic information exchange method for double-binary turbo codes. *IEEE Transactions.* 2009; (56): 81-85.
- [4] Shah C. P Tsimenidis CC, Sharif BS, Neasham JA. Low-Complexity iterative receiver structure for time-varying frequency-selective shallow underwater acoustic channels using BICM-ID: design and experimental results. *IEEE Oceanic Engineering.* 2011; (36): 406-421.
- [5] Liyao Wang, Jianping Li. *A new iterative scheduling for BICM-ID embedded turbo codes*, IEEE International Conference. 2012; 39: 953-956.
- [6] L Bahl, J Cocke, J Jelinek, J Raviv, F Raviv. Optimal decoding of linear codes for minimizing symbol error rate. *IEEE Trans. Inform. Theory.* 1974; (20): 284-287.
- [7] Xiaoyi Li, ZhaoFang. Research on improved turbo-codes decoding algorithm. *Computer.* 2009; (25): 230-232.
- [8] Robertson P, Hoeher P. Optimal and sub-optimal maximum a posteriori algorithm suitable for turbo dccoding. *Eur. Transactions. Telecommun.* 1997; (8): 119-125.
- [9] S Talakoub, L Sabeti, B Shahrava, M Ahmadi. An improved Max-Log-MAP algorithm for turbo decoding and turbo equalization. *IEEE Trans. on Instrumentation and measurement.* 2007; (56): 1058-1063.
- [10] S Lin, Daniel J Costello Jr. 2004. Error Control Coding. Jian Yan, Yuanzhi He, Yahan Pan. China Machine Press. 2007.
- [11] Istvan Haller, Sergiu Nedeveschi. Design of interpolation functions for subpixel-accuracy stereovision systems. *IEEE Transactions on image processing.* 2012; (21): 889- 898.
- [12] Robertson P, Villebrun E, Hoeher P. *A comparison of optimal and sub-Optimal MAP decoding algorithms operating in the Log domain.* IEEE International Conference on 1822. 1995: 1009-1013.