Certain Properties of ω-Q-Fuzzy Subrings

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Abstract

In this paper, we define the ω-Q-fuzzy subring and discussed various fundamental aspects of ω-Q-fuzzy subrings. We introduce the concept of ω-Q-level subset of this new fuzzy set and prove that ω-Q-level subset of ω-Q-fuzzy subring form a ring. We define ω-Q-fuzzy ideal and show that set of all ω-Q-fuzzy cosets form a ring. Moreover, we investigate the properties of homomorphic image of ω-Q-fuzzy subring.

Keywords: Q-fuzzy set; Q-fuzzy subring (QFSR); ω-Q-fuzzy set; ω-Q-fuzzy subring (ω-QFSR); ω-Q-fuzzy ideal(ω-QFI).

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1. Introduction

The concept of fuzzy sets was introduced by Zadeh [17] in 1965. Rosenfeld [14] commenced the idea of fuzzy subgroups in 1971. The fuzzy subrings were initiated by Liu [8] Dixit et al. [5] described the notion of level subgroup in 1990. Gupta [6] defined many classical t-operators in 1991. Solairaju and Nagarajan [15] explored a new structure and construction of Q-fuzzy groups in 2009. Muthuraj et al. [10] proposed the study of lower level subsets of anti-QFS in 2010. The concept of Q-fuzzy normal subgroups and Q-fuzzy normalizer were established by Priya et al. [11] in 2013. Sither Selvam et al. [12] used the Biswas [4] work to modify the idea of anti-QFNS in 2014. Alsarahead and Ahmed $[1-3]$ commenced new concept of complex fuzzy subring, complex fuzzy subgroup and complex fuzzy soft subgroups in 2017. The Q-fuzzy subgroup in algebra was discussed in [7]. Rasuli [13] discussed Q-fuzzy subring with respect to t-norm in 2018. More development about fuzzy subgroup may be viewed in [9,16]. This paper is organized as the section 2 contains the elementary definition of Q-fuzzy subrings and related results which are thoroughly crucial to understand the novelty of this article. In section 3, we define the ω-Q-fuzzy subring and prove that the level subset of ω-Q-fuzzy subrings is a subring. We also define ω-Q-fuzzy ideal and discuss its properties. In section 5, we use the classical ring homomorphism to investigate the behavior of homomorphic image (inverse-image) of ω-Q-fuzzy subring.

2. Preliminaries

We recall first the elementary notion of fuzzy sets which play a key role for our further analysis.

Definition (2.1) [17]: A fuzzy set A of a nonempty set M is a function

$$A : P \to [0,1].$$

Definition (2.2) [12]: Let A be fuzzy subset of a ring R. Then A is said to a fuzzy subring if

i. $A(u-v) \geq \min\{A(u), A(v)\}$

ii. $A(uv) \geq \min\{A(u), A(v)\}$, for all $u, v \in R$.

Definition (2.3) [15]: Let M and Q be two nonempty sets. A Q-fuzzy subset A of set M is a function $A: X \times Q \to [0,1]$ for all $u, v \in M$ and $q \in Q$.

Definition (2.4) [13]: A function $A : R \times Q \longrightarrow [0,1]$ is a QFSRR of a ring R if

i. $A(u-v,q) \geq \min\{A(u,q), A(v,q)\}$

ii. $A(uv,q) \geq \min\{A(u,q), A(v,q)\}$, for all $u, v \in R$ and $q \in Q$.

Definition (2.5) [13]: Let the mapping $f: R_1 \to R_2$ be a homomorphism. Let A and B be ω-QFSRs of R_1 and R_2 respectively, then $f(A)$ and $f^{-1}(B)$ are image of A and the inverse image of B respectively, defined as

i. $f(A)(v,q) = \begin{cases} \sup\{A(u,q): u \in f^{-1}(v)\}, & \text{if } f^{-1}(v) \neq \emptyset \\ 0, & \text{if } f^{-1}(v) = \emptyset \end{cases}$, for every $v \in R_2$ and $q \in Q$

ii. $f^{-1}(B)(u,q) = B(f(u),q)$, for every $u \in R_1$ and $q \in Q$

Definition (2.6) [6]: Let $t_p: [0,1] \times [0,1] \to [0,1]$ be the algebraic product t-norm on $[0,1]$ and is described as $t_p\{a,b\} = ab$, $0 \leq a \leq 1$, $0 \leq b \leq 1$

3. Properties of $\boldsymbol{\omega}$-$\boldsymbol{Q}$-fuzzy subrings

Definition (3.1): Let M and Q be any two nonempty sets and A be a Q-fuzzy subset of a set P, any $\omega \in [0,1]$. Then fuzzy set A^ω of M is said to be ω-Q-fuzzy subset of M (w.r.t Q-fuzzy set A) and defined by

$$A^\omega(m,q) = t_p\{A(m,q), \omega\}, \qquad \text{for all } m \in M \text{ and } q \in Q$$

Remark (3.2): Clearly, $A^1(m,q) = A(m,q)$ and $A^0(m,q) = 0$.

Remark (3.3): If A and B be two Q-fuzzy sets of M. Then $(A \cap B)^{\omega} = A^{\omega} \cap B^{\omega}$.

Definition (3.4): A Q-fuzzy subset of a ring R is called ω-QFSR, and $\omega \in [0,1]$, if

i $A^{\omega}(m-n,q) \geq \min\{A^{\omega}(m,q), A^{\omega}(n,q)\}$, for all $m,n \in R$ and $q \in Q$.

ii $A^{\omega}(mn,q) \geq \min\{A^{\omega}(m,q), A^{\omega}(n,q)\}$, for all $m,n \in R$ and $q \in Q$

Theorem (3.5): If A is a ω-QFSR of a ring R, then

$A^{\omega}(m,q) \leq A^{\omega}(0,q)$, for all $m \in R$ and $q \in Q$ where 0 is the additive identity of R.

Proof: Consider$A^{\omega}(0,q) = A^{\omega}(m-m,q) \geq \min\{A^{\omega}(m,q), A^{\omega}(m^{-1},q)\}$

$= \min\{A^{\omega}(m,q), A^{\omega}(m,q)\} = A^{\omega}(m,q)$

Hence, $A^{\omega}(0,q) \geq A^{\omega}(m,q)$, for all $m \in R$

Theorem (3.6): If A is QFSR of a ring H, then A is an ω-QFSR of R.

Proof: Assume that A is a QFSR of a ring R and $\forall\, a,b \in R$ and $q \in Q$.

Assume that, $A^{\omega}(a-b,q) = t_p\{A(a-b,q),\mu\} \geq t_p\{\min\{A(a,q),A(b,q)\},\omega\}$

$= \min\left\{t_p\{A(a,q),\omega\}, t_p\{A(b,q),\omega\}\right\} = \min\{A^{\omega}(a,q), A^{\omega}(b,q)\}$

$A^{\omega}(a-b,q) \geq \min\{A^{\omega}(a,q), A^{\omega}(b,q)\}$

Further $A^{\omega}(ab,q) = t_p\{A(ab,q),\mu\} \geq t_p\{\min\{A(a,q),A(b,q)\},\omega\}$

$= \min\left\{t_p\{A(a,q),\omega\}, t_p\{A(b,q),\omega\}\right\} = \min\{A^{\omega}(a,q), A^{\omega}(b,q)\}$

$A^{\omega}(ab,q) \geq \min\{A^{\omega}(a,q), A^{\omega}(b,q)\}$

Consequently, A is ω-QFSR of R. In general, the converse may not be true.

Note 3.7: we take $Q = \{q\}$ in all the examples

Example (3.8): Let $R = \{0,1,2,3\}$, be a ring and $Q = \{q\}$. Let the Q-fuzzy set A of R described by

$$A(a,q) = \begin{cases} 0.3, & \text{if } a = 0 \\ 0.5, & \text{if } a = 1 \text{ or } 3 \\ 0.4, & \text{if } a = 2 \end{cases}$$

Take $\omega = 0$ then

$A^{\omega}(a,q) = t_p\{A(a,q),\omega\} = t_p\{A(a,q),0\} = 0$, for all $a \in R$

$\Longrightarrow\ A^{\omega}(a-b,q) \geq \min\{A^{\omega}(a,q), A^{\omega}(b,q)\}$

Further, we have $A^{\omega}(ab,q) \geq \min\{A^{\omega}(a,q), A^{\omega}(b,q)\}$

Consequently A is ω-QFSR of R and A is not QFSR of R.

Definition 3.9: Let A be ω-Q-fuzzy set of universe set M. For $t, \omega \in [0,1]$ the level subset A_t^{ω} of ω-Q-fuzzy set is given by

$$A_t^{\omega} = \{m \in M : A^{\omega}(m,q) \geq t\}.$$

Theorem (3.10): Let A is ω-Q-fuzzy subring of R then A_t^{ω} is subring of R for all $t \leq A(e,q)$.

Proof: It is quite obvious that A^{ω} is non-empty. Since A be ω-Q-fuzzy subring of a ring R, which implies that $A^{\omega}(m,q) \leq A^{\omega}(e,q)$, for all $m \in R$ and $q \in Q$. Let $m,n \in A_t^{\omega}$ then $A^{\omega}(m,q) \geq t\ a\ A^{\omega}(n,q) \geq t$.

Now

$A^{\omega}(m-n,q) \geq \min\{A^{\omega}(m,q), A^{\omega}(n,q)\} \geq \min\{t,t\} = t$,

$A^{\omega}(mn,q) \geq \min\{A^{\omega}(m,q), A^{\omega}(n,q)\} \geq \min\{t,t\} = t$

This implies that $m-n, mn \in A_t^{\omega}$. Hence, A_t^{ω} is subring of R.

Definition 3.11: Let A be a Q-fuzzy subset of a ring R and $\omega \in [0,1]$. Then A^{ω} is ω-Q-fuzzy left ideal (ω-QFLI) of R if

i $A^{\omega}(m-n,q) \geq \min\{A^{\omega}(m,q), A^{\omega}(n,q)\}$

ii $A^{\omega}(mn,q) \geq A^{\omega}(n,q)$, for all $m,n \in R$ *and* $q \in Q$

Definition 3.12: Let A be a Q-fuzzy subset of a ring R and $\omega \in [0,1]$. Then A^{ω} is ω-Q-fuzzy right ideal (ω-QFRI) of R if

i. $A^{\omega}(m-n,q) \geq \min\{A^{\omega}(m,q), A^{\omega}(n,q)\}$

ii. $A^{\omega}(mn,q) \geq A^{\omega}(m,q)$, for all $m,n \in R$ *and* $q \in Q$

Definition 3.13: Let A be a Q-fuzzy subset of a ring R and $\omega \in [0,1]$. Then A^{ω} is ω-QFI of R if

i. $A^{\omega}(m-n,q) \geq \min\{A^{\omega}(m,q), A^{\omega}(n,q)\}$

ii. $A^{\omega}(mn,q) \geq \max\{A^{\omega}(m,q), A^{\omega}(n,q)\}$, for all $m,n \in R$ *and* $q \in Q$

Definition 3.14: Let A be a ω-QFSR of a ring R and $\omega \in [0,1]$. For any $m \in R$ and $q \in Q$, The ω-Q-fuzzy coset of A in R is represented by $m + A^{\omega}$ as defined as

$(m + A^{\omega})(h,q) = t_p\{A(h-m,q), \omega\} = A^{\omega}(h-m)$, for all $m,h \in R$ and $q \in Q$

Theorem 3.15: Let A be ω-QFI of ring R. Then the set

$A_0^{\omega} = \{ m \in R : A^{\omega}(m,q) = A^{\omega}(0,q)\}$ is an ideal of ring R.

Proof: Obviously $A_0^{\omega} \neq \varphi$ because $0 \in R$. Let $m,n \in A_0^{\omega}$ be any elements.

Consider

$$A^{\omega}(m-n,q) \geq \min\{A^{\omega}(m,q), A^{\omega}(n,q)\} = \min\{A^{\omega}(0,q), A^{\omega}(0,q)\}$$

Implies that $A^{\omega}(m-n,q) \geq A^{\omega}(0,q)$. But $A^{\omega}(m-n,q) \leq A^{\omega}(0,q)$

Therefore, $A^{\omega}(m-n,q) = A^{\omega}(0)$

Implies that $m-n \in A_0^{\omega}$.

Further, let $m \in A_{(\alpha,\beta)}^{0}$ and $n \in R$. Consider

$A^{\omega}(mn,q) \geq \max\{A^{\omega}(m,q), A^{\omega}(n,q)\} = \max\{A^{\omega}(0,q), A^{\omega}(n,q)\}$, Implies that $A^{\omega}(mn, q) \geq A^{\omega}(0, q)$. But $A^{\omega}(mn, q) \leq A^{\omega}(0, q)$

Therefore, $A^{\omega}(mn, q) = A^{\omega}(0, q)$.

Similarly, $A^{\omega}(nm, q) = A^{\omega}(0, q)$

Implies that $mn, nm \in A_0^{\omega}$.

Implies that A_0^{ω} is an ideal.

Theorem 3.16: Let A_0^{ω} be an ω-QFI of ring R, $m, n \in R$ and $q \in Q$. Then

$$m + A^{\omega} = n + A^{\omega} \quad \text{if and if only } m - n \in A_0^{\omega}.$$

Proof: For any $m, n \in S$, we have $m + A^{\omega} = n + A^{\omega}$.

Consider,

$A^{\omega}(m - n, q) = (n + A^{\omega})(m, q) = (m + A^{\omega})(m, q) = A^{\omega}(0, q)$

Therefore, $m - n \in A_0^{\omega}$.

Conversely, let $m - n \in A_0^{\omega}$

Implies that $A^{\omega}(m - n, q) = A^{\omega}(0, q)$

Consider, $(m + A^{\omega})(h, q) = A^{\omega}(h - m, q) = A^{\omega}\big((h - n) - (m - n), q\big)$

$\geq \min\{A^{\omega}\big((h - n), q\big), A^{\omega}\big((m - n), q\big)\}$

$= \min\{A^{\omega}\big((h - n), q\big), A^{\omega}(0, q)\} = A^{\omega}\big((h - n), q\big) = (n + A^{\omega})(h, q)$

Interchange the role of p and q we get $(n + A^{\omega})(h, q) \geq (m + A^{\omega})(h, q)$

Therefore, $(m + A^{\omega})(h, q) = (n + A^{\omega})(h, q)$, for all $h \in R$

Definition (3.17): Let A be a ω-QFI of a ring R. The set of all ω-Q-fuzzy cosets of A denoted by $R/_{A^{\omega}}$ form a ring with respect to binary operation * defined by

$$(m + A^{\omega}) + (n + A^{\omega}) = (m + n) + A^{\omega}, where\ m + A^{\omega}, n + A^{\omega} \in$$

$R/_{A^{\omega}}, m, n \in R$.

$$(m + A^{\omega}) * (n + A^{\omega}) = (m * n) + A^{\omega}, where\ m + A^{\omega}, n + A^{\omega} \in$$

$R/_{A^{\omega}}, m, n \in R$. The ring $R/_{A^{\omega}}$ is called the factor ring of R with respect to ω-QFI A^{ω}.

Theorem (3.18): The set $R/_{A^{\omega}}$ forms a ring with respect to the above stated binary operation.

Proof: Let $m_1 + A^{\omega} = m_2 + A^{\omega}$ and $n_1 + A^{\omega} = n_2 + A^{\omega}$ for some $m_1, m_2, n_1, n_2 \in R$.

Let $g \in R$ be any element of R and $q \in Q$.

$$(m_2 + n_2 + A^{\omega})(g, q) = A^{\omega}(g - (m_2 + n_2), q)$$

$$= A^{\omega}(g - m_2 - n_2), q) = n_2 + A^{\omega}(g - m_2), q) = n_1 + A^{\omega}(g - m_2), q)$$

$$= A^{\omega}(g - m_2 - n_1), q) = m_2 + A^{\omega}(g - n_1), q) = m_1 + A^{\omega}(g - n_1), q)$$

$$= A^{\omega}(g - m_1 - n_1), q) = A^{\omega}(g - (m_1 + n_1), q) = (m_1 + n_1 + A^{\omega})(g, q)$$

Moreover,

$$(m_2n_2 + A^\omega)(g,q) = A^\omega(g - m_1n_1 - (m_2n_2 - m_1n_1), q)$$
$$\geq \min\{A^\omega(g - m_1n_1), A^\omega((m_2n_2 - m_1n_1), q)\}$$

But we have, $A^\omega\big((m_2n_2 - m_1n_1), q\big) = A^\omega((m_1n_1 - m_2n_1 + m_2n_1 - m_2n_2), q)$

$$= A^\omega\big((m_1 - m_2)n_1 + m_2(n_1 - n_2), q\big) \geq \min\{A^\omega(m_1 - m_2)n_1, q), A^\omega(m_2(n_1 - n_2), q)\}$$
$$= \min\{A^\omega((m_1 - m_2), q), A^\omega((n_1 - n_2), q)\}$$
$$= \min\{A^\omega(0,q), A^\omega(0,q)\}$$

, $A^\omega\big((m_2n_2 - m_1n_1), q\big) \geq A^\omega(0,q)$,

$$(m_2n_2 + A^\omega)(g,q) \geq A^\omega(g - m_1n_1), q)$$
$$= (m_1n_1 + A^\omega)(g,q)$$

Similarly, we can prove that $(m_2n_2 + A^\omega)(g,q) \leq (m_1n_1 + A^\omega)(g,q)$

Consequently, $(m_2n_2 + A^\omega)(g,q) = (m_1n_1 + A^\omega)(g,q)$.

Therefore $*$ is well defined. Now we prove that the following axioms of ring, for any $m, n \in R$.

1. $(m + A^\omega) + (n + A^\omega) = m + n + A^\omega$
2. $(m + A^\omega) + [(n + A^\omega) + (r + A^\omega)] = m + A^\omega + [n + r + A^\omega] = (m + n) + r + A^\omega = [m + n + A^\omega] + r + A^\omega = [(m + A^\omega) + (n + A^\omega)] + (r + A^\omega)$
3. $(m + A^\omega) + (n + A^\omega) = m + n + A^\omega = n + m + A^\omega = (n + A^\omega) + (m + A^\omega)$
4. $(0 + A^\omega) + (n + A^\omega) = (n + A^\omega)$
5. $(m + A^\omega) + (-m + A^\omega) = A^\omega$
6. $(m + A^\omega)(n + A^\omega) = mn + A^\omega$
7. $(m + A^\omega)[(n + A^\omega)(r + A^\omega)] = m + A^\omega + [nr + A^\omega] = mnr + A^\omega = [mn + A^\omega] + r + A^\omega = [(m + A^\omega)(n + A^\omega)](r + A^\omega)$
8. $(m + A^\omega)[(n + A^\omega) + (r + A^\omega)] = (m + A^\omega)[(n + r) + A^\omega] = m(n + r) + A^\omega = (mn + mr) + A^\omega = (mn + A^\omega) + (mr + A^\omega) = [(m + A^\omega)(n + A^\omega) + (m + A^\omega)(r + A^\omega)]$

Consequently, $\left({}^{R}\!/_{A^\omega}, +, *\right)$ is a ring.

4. Homomorphism of ω-Q-fuzzy subrings

In this section, we investigate the behavior of homomorphic image and inverse image of ω-QFSR.

Lemma 4.1: Let $f: M \to N$ be a mapping and A and B be two fuzzy subsets of M and N respectively, then

i. $f^{-1}(B^{\omega})(m,q) = \left(f^{-1}(B)\right)^{\omega}(m,q)$, for all $m \in M$ and $q \in Q$

ii. $f(A^{\omega})(n,q) = \left(f(A)\right)^{\omega}(n,q)$, for all $n \in N$ and $q \in Q$

Proof: (i) $f^{-1}(B^{\omega})(m) = B^{\omega}\left(f(m)\right) = t_p\{B\left(f(m)\right), \omega\} = t_p\{f^{-1}(B)(m), \omega\}$

$$f^{-1}(B^{\omega})(m) = \left(f^{-1}(B)\right)^{\omega}(m), \qquad \text{for all } m \in M$$

(ii) $f\left(A^{\omega}\right)(n,q) = \sup\{A^{\omega}(m,q): f(m) = y\} = \sup\{t_p\{A(m,q), \omega\}: f(m) = n\}$

$= t_p\left\{\sup\{\{A(m,q): f(m) = n\}, \omega\}\right\} = t_p\{f(A)(n,q), \omega\} = \left(f(A)\right)^{\omega}(n,q)$, for all $n \in N$

Hence, $$f(A^{\omega})(n,q) = \left(f(A)\right)^{\omega}(n,q)$$

Theorem 4.2: Let$f : R_1 \longrightarrow R_2$ be a homomorphism from a ring R_1 to a ring R_2 and A be a ω-QFSR of ring R_1 . Then $f(A)$ is a ω-QFSR of ring R_2.

Proof: Let A be a ω-QFSR of ring R_1. Let $n_1, n_2 \in R_2$ be any element. Then there exists unique elements $m_1, m_2 \in R_1$ such that $f(m_1) = n_1$ and $f(m_2) = n_2$ and for $q \in Q$.

Consider,

$\left(f(A)\right)^{\omega}(n_1 - n_2, q) = t_p\{f(A)(n_1 - n_2, q), \omega\} = t_p\{f(A)(f(m_1) - f(m_2), q), \omega\}$

$= t_p\{f(A)(f(m_1 - m_2), q), \omega\} = t_p\{A(m_1 - m_2, q), \omega\} = A^{\omega}(m_1 - m_2, q)$

$\geq \min\{A^{\omega}(m_1, q), A^{\omega}(m_2, q)\}$, for all $m_1, m_2 \in H_1$ such that $f(m_1) = n_1$ and $f(m_2) = n_2\}$

$\geq \min\{\sup\{A^{\omega}(m_1, q) : f(m_1) = n_1\}, \sup\{A^{\omega}(m_2, q) : f(m_2) = n_2\}\}$

$= \min\{f(A^{\omega})(n_1, q), f(A^{\omega})(n_2, q)\} = \min\{\left(f(A)\right)^{\omega}(n_1, q), \left(f(A)\right)^{\omega}(n_2, q)\}$

Thus, $\left(f(A)\right)^{\omega}(n_1 n_2, q) \geq \min\{\left(f(A)\right)^{\omega}(n_1, q)\left(f(A)\right)^{\omega}(n_2, q)\}$.

Further, $\left(f(A)\right)^{\omega}(n_1 n_2, q) = t_p\{f(A)(n_1 n_2, q), \omega\} = t_p\{f(A)(f(m_1)f(m_2), q), \omega\}$

$= t_p\{f(A)(f(m_1 m_2), q), \omega\} = t_p\{A(m_1 m_2, q), \omega\} = A^{\omega}(m_1 m_2, q)$

$\geq \min\{A^{\omega}(m_1, q), A^{\omega}(m_2, q)\}$, for all $m_1, m_2 \in H_1$ such that $f(m_1) = n_1$ and $f(m_2) = n_2\}$

$\geq \min\{\sup\{A^{\omega}(m_1, q) : f(m_1) = n_1\}, \sup\{A^{\omega}(m_2, q) : f(m_2) = n_2\}\}$

$= \min\{f(A^{\omega})(n_1, q), f(A^{\omega})(n_2, q)\} = \min\{\left(f(A)\right)^{\omega}(n_1, q), \left(f(A)\right)^{\omega}(n_2, q)\}$

Thus, $\left(f(A)\right)^{\omega}(n_1 n_2, q) \geq \min\{\left(f(A)\right)^{\omega}(n_1, q)\left(f(A)\right)^{\omega}(n_2, q)\}$.

Consequently, $f(A)$ is ω-QFSR of R_2.

Theorem 4.3: Let $f : R_1 \longrightarrow R_2$ be a homomorphism from ring R_1 into a ring R_2 and B be a ω-QFSR of ring R_2. Then $f^{-1}(B)$ is ω-QFSR of ring R_1.

Proof: Let B be ω-QFSR of ring R_2. Let $m_1, m_2 \in R_1$ be any elements, then

$$\left(f^{-1}(B)\right)^{\omega}(m_1 - m_2, q) = f^{-1}(B^{\omega})(m_1 - m_2, q) = B^{\omega}(f(m_1 - m_2), q)$$

$$= B^{\omega}(f(m_1) - f(m_2), q)$$

$$\geq \min\{B^{\omega}(f(m_1), q), B^{\omega}(f(m_2), q)\} = \min\{f^{-1}(B^{\omega})(m_1, q), f^{-1}(B^{\omega})(m_2, q)\}$$

$$= \min\left\{\left(f^{-1}(B)\right)^{\omega}(m_1, q), \left(f^{-1}(B)\right)^{\omega}(m_2, q)\right\}$$

Thus, $\left(f^{-1}(B)\right)^{\omega}(m_1 m_2, q) \geq \min\left\{\left(f^{-1}(B)\right)^{\omega}(m_1, q), \left(f^{-1}(B)\right)^{\omega}(m_2, q)\right\}$.

Further,

$$\left(f^{-1}(B)\right)^{\omega}(m_1 m_2, q) = f^{-1}(B^{\omega})(m_1 m_2, q) = B^{\omega}(f(m_1 m_2), q) = B^{\omega}(f(m_1) f(m_2), q)$$

$$\geq \min\{B^{\omega}(f(m_1), q), B^{\omega}(f(m_2), q)\} = \min\{f^{-1}(B^{\omega})(m_1, q), f^{-1}(B^{\omega})(m_2, q)\}$$

$$= \min\left\{\left(f^{-1}(B)\right)^{\omega}(m_1, q), \left(f^{-1}(B)\right)^{\omega}(m_2, q)\right\}$$

Thus, $\left(f^{-1}(B)\right)^{\omega}(m_1 m_2, q) \geq \min\left\{\left(f^{-1}(B)\right)^{\omega}(m_1, q), \left(f^{-1}(B)\right)^{\omega}(m_2, q)\right\}$.

Consequently, $f^{-1}(B)$ is ω-QFSR of a ring $R_{1.}$

5. Conclusion

In paper, we have proved the level subset of two ω-Q-fuzzy subrings is a subring. In addition, we have extended the study of this ideology to investigate the effect of image and inverse image of ω-QFSR under ring homomorphism.

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Conflict of interest

All authors declare no conflict of interest in this paper.

References

[1] M. O. Alsarahead, A. G. Ahmad, Complex fuzzy subgroups, Applied Mathematical Sciences, $11(2017), 2011 - 2021$.

[2] M. O. Alsarahead, A. G. Ahmad, Complex fuzzy subrings, International Journal of Pure and Applied Mathematics, $117(2017),563 - 577$.

[3] M. O. Alsarahead, A. G. Ahmad, Complex fuzzy soft subgroups, Journal of Quality Measurement and Analysis $13(2017), 17 - 28$.

[4] R. Biswas, Fuzzy subgroups and anti-fuzzy subgroups. Fuzzy Sets and Systems, $\mathbf{35}$ $(1990), 121-124$.

[5] V. N. Dixit, R. Kumar, N. Ajmal, Level subgroups and union of fuzzy subgroups. Fuzzy Sets and Systems, 37 $(1990), 359-371$.

[6] M. M. Gupta, J. Qi, Theory of T-norms and fuzzy inference methods, Fuzzy Sets and Systems, 40 (1991), 431-450.

[7] Dr. R. Jahir Hussain, A Review On Q-fuzzy subgroup in Algebra. International Journal of Applied Engineering Research 14 $(2019), 60-63$.

[8] W. J. Liu, Fuzzy invariant subgroups and fuzzy ideals. Fuzzy Sets Syst. 8 $(1982), 133-139$.

[9] B. B. Makamba, V. Murali, A class of fuzzy subgroups of finite reflection groups, Journal of Intelligent and Fuzzy Systems, 33 (2017) $979-983$.

[10] Dr. R. Muthuraj, P. M. Sitharselvam, M. S. Muthuraman, Anti Q-Fuzzy Group and Its Lower Level Subgroups. International Journal of Computer Application, $3(2010), 16-20$.

[11] Priya, T. Ramachandran, K. T. Nagalakshmi, On Q -fuzzy Normal Subgroups. International Journal of computer and Organization Trends, $3(2013), 39-42$.

[12] P. M. Sithar selvam, T. Priya, K. T. Nagalakshmi and T. Ramachandran On Some properties of anti - Q -fuzzy Normal Subgroups. General Mathematics Notes, $22(2014), 1-10$.

[13] R. Rasuli, Characterization of Q-fuzzy subrings (Anti Q-fuzzy subrings) with respect to a T-norm (T-conorm), Journal of Information and Optimization Sciences, 39(2018), $827-837$.

[14] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. $35(1971), 512-517$.

[15] A. Solairaju, and R. Nagarajan, A new structure and construction of Q- fuzzy groups. Advances in Fuzzy Mathematics, 4 $(2009), 23-29$.

[16] S. A. Trevijano, M. J. Chasco and J. Elorza, The annihilator of fuzzy subgroups, Fuzzy Sets and Systems 369, No $(2019), 122-131$.

[17] L.A. Zadeh, Fuzzy sets, Inform. and Control, 8 $(1965), 338 - 353$.