**Design and Implementation of Robust H∞ Loop Shaping Control for Ball Position Control of Ball and Beam System**

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**Abstract:**

Ball and Beam prototype is a system designed to simulate the controlling of aircraft flight and land in space application studies. This paper presents using MATLAB to design and implement of PID and robust H∞ Loop Shaping controllers to control the position of the rolling ball in a laboratory ball and beam system prototype. Applying constant input will cause the ball to be continuously rolling on the beam, which lead to an unstable open loop system. To achieve the desired position, a PID controller was used primarily to stabilize the system. To achieve performance requirement for system with uncertainties, robust H∞ Loop Shaping controller was used. Results for the step response shows that robust H∞ Loop Shaping controller response faster 80 times when compared to than response of PID controller, and there is no over shoot, and it's more effective and had better performance compared to other controllers in the control of Ball and Beam system

**Key words:** Robust control, Ball and Beam system, H∞ loop shaping.

**I. Introduction :**

Ball and Beam system (BBS) is one of the most important laboratory model for control, it is a highly nonlinear, unstable open loop system [1- 3]. BBS consist of beam connected from one end to a motor, and a ball rolling freely on it. By controlling the angular position of the motor, the ball position will be controlled. The system is a prototype to represent many of the complex dynamics associated with unstable system [4], such as horizontally stabilizing an airplane during landing and in turbulent airflow [2].

 BBS is a typical under actuated system with two degree of freedom (ball position and beam angle), and one degree of actuation. The under actuated systems are low energy consuming, light weight, but they are more complicated than fully actuated systems [5].

 Because of the system behavior, the ball and beam system considered as a complex system to be controlled. Many researchers consider it as an ideal prototype to learn different control strategies ( classical and modern). Different techniques and controllers had been used to control the BBS, ranging from conventional controllers to intelligent controllers.

 Mahmud and Rini [1] designed a two degree of freedom controller which is developed based on algebraic method. Abdulgani A. [2] in his paper presented an optimization technique ( PSO algorithm ) to optimize PID controller parameters to control linearized BBS, while Anjali T. and Shyju S. M. [4] used Genetic algorithm to optimize the PID controller parameters. A. I. I. et.al. [3] applied an intelligent hybrid fuzzy controller based PID controller for different testing conditions. Adaptive dynamic surface control based on T-S Fuzzy model was presented in [5] by Yeong-Hwa. N. S. et.al. [6] present a Fuzzy PID controller, while Reza [7] designed a PID controller, Mamdani Fuzzy Logic controller and Sugeno Fuzzy Logic Controller to control the non-linear model of the ball and beam system. M. K. and G. N. [8] used ESO based LQR controller, while R. S. and Dr. S. [9] used Sliding mode controller. Back propagation neural network was presented by G. L. and L. Y [10].

Since the ball roll on the beam, it has an acceleration, by adjusting the acceleration, the position of the ball could be controlled. The goal here is to stabilize the ball on a desired position for the longest possible time, and reach that position in shortest time by tuning the angle of the beam. PID controller will be used first to stabilize the system, then H∞ loop shaping controller will be used to achieve better requirement and to overcome the uncertainty in ball and beam system.

**II. System Description and Modeling:**

In this section, system description and mathematical model of Ball and Beam system will be present.

**A. System Description**

BBS consist of two main systems, as shown in figure (1), the electromechanical system (DC servomotor), and mechanical system (ball and beam). The beam is connected in one end to the motor by lever arm so it could move up and down, while the other end of the beam is pinned. When the motor rotate, the lever moves up and down so that the ball roll on the beam. The motor rotate when it receive an electrical signal from controller, that lead the beam to oscillate [2]. A linear potentiometric sensor is used to get the current position of the ball [8], and feed back this position to be compared with the desired position to examine if the ball reach the desired point or not. The beam will swing even if it is nearly horizontal, without active feedback, that lead to roll the ball till reach to the end of the beam [8].



Figure (1): Illustration of Ball and Beam System [7]

To control the position of the ball and stop it at that desired position, the motor rotation should be controlled by adjusting the angle of the gear [9] which effect directly the beam inclination angle ($α$).

**B. Mathematical Modeling**

Two different methods were used to derive the mathematical model of ball and beam system, Newton [2,3,9], and Lagrangian [4,6,7]. In this paper, a Newton's second low of motion will be considered to derive the system dynamic equation, It was assumed that the friction between ball and beam is negligible. The mathematical expression between inclination angle ($α$) and angular position of beam ($θ$) can be described as:

$α=\frac{d}{L} θ$ (1)

The movement of the ball on the beam can be expressed as:

$\left(\frac{J\_{b}}{R\_{b}^{2}}+M\right)\ddot{r}+ Mg\sin(α)=Mr(α)^{2}$ (2)

By simplification and taking Laplace to equation (2), the transfer function of system is:

$\frac{R(s)}{θ(s)}=-\frac{0.7}{s^{2}}$ (3)

Where, *L* is the length of the beam (cm), $θ$ is the angular position of the beam, *d* is the distance between contact point and the mid point (cm), *Jb* is the rotational inertia of the ball (kg.m2), *Rb* is the radius of the ball (m), *M* is the mass of the ball (kg), *r* is the ball position, and *g* is the gravity constant (m/s2).

The final state space representation of the ball and beam system after linearizing the non-liner model around equilibrium point ( $r=0, \dot{r}=0, θ=0, \dot{θ}=0$ ) is as follows [9] :

$\dot{X}=AX+BU$ , $Y=CX+DU$ (4)

$\left[\begin{matrix}\dot{x}\_{1}\\\begin{matrix}\dot{x}\_{2}\\\dot{x}\_{3}\\\dot{x}\_{4}\end{matrix}\end{matrix}\right]=\left[\begin{matrix}\begin{matrix}0&1&\begin{matrix}0 & 0\end{matrix}\end{matrix}\\\begin{matrix}\begin{matrix}0&0&\begin{matrix}-7.0071 &0\end{matrix}\end{matrix}\\\begin{matrix}0& 0&\begin{matrix}0 &1\end{matrix}\end{matrix}\\\begin{matrix}-24.5225&0&\begin{matrix}0&0\end{matrix}\end{matrix}\end{matrix}\end{matrix}\right]\left[\begin{matrix}x\_{1}\\\begin{matrix}x\_{2}\\x\_{3}\\x\_{4}\end{matrix}\end{matrix}\right]+\left[\begin{matrix}0\\\begin{matrix}0\\0\\49.9950\end{matrix}\end{matrix}\right]u$ (5)

$y=\left[\begin{matrix}1&0&\begin{matrix}0&0\end{matrix}\end{matrix}\right]\left[\begin{matrix}x\_{1}\\\begin{matrix}x\_{2}\\x\_{3}\\x\_{4}\end{matrix}\end{matrix}\right]$ (6)

Where matrix A is the dynamic behavior of the system, matrix B defines the behavior of system actuator, matrix C is the relation between states and output of system [1].

**III. System Behavior and Controllers Design**

 BBS is unstable, controllable, observable, and nonlinear. The open loop behavior of the system is completely unstable as shown in figure (2) for time response and figure (3) for frequency response. It is clear that the system needs to be stabilized first, and meet a reasonable performance specifications for the controlled Ball & Beam system. For this reason, two different kind of controllers have been designed.



Figure(2): Open loop time domain response of the Ball and Beam system model.



Figure(3): Open loop frequency domain response of the Ball and Beam system model.

**PID control design :**

A conventional simple PID controller is designed first to stabilize the system, it's block diagram is shown in figure (4). We assume it's parameters as: proportional *P=-0.716*, derivative *D=-2.164*, and integral *I=-0.05.* Figure (5) shows represent the step response which indicates the position of ball controlled by PID controller. Although there is no steady state error, it can be noted that there is an overshoot (14.3%) which is not preferable in such systems, and the response is somewhat slow (ts = 8 seconds ).



Figure (4): PID control system Block



Figure (5): Step response of PID control system

***H∞* Loop shaping control design:**

This controller combines classical loop-shaping ideas with an effective method for robustly stabilizing the feedback loop [11]. The process of designing this controller required two stages, proposed by McFarlane and Glover [11]: First, determining the pre and post compensators (*W1* and *W2*) to give the desired shape to the singular value of the system frequency response, and second: robustly stabilize the resulting shaped plant with respect to coprime factore uncertainty. Figure (6) shows the block diagram of the controller designed, where *W1* and *W2* are the weights, while *G* is the transfer function of the system.



Figure (6): The shaped plant and controller [11]

To robustly stabilize the shaped plant, the normalized coprime factorization method of robust controller design should be applied [13, 14], it is important to find K(s) and $γ\_{min}$ such that:

$\left‖\left(\begin{matrix}I\\K\end{matrix}\right)(1-GK)^{-1} (IW\_{2}GW\_{1})\right‖\_{\infty }=γ\_{min}=ε\_{max}^{-1}$ (7)

Where the minimal value is given by :

$γ\_{min}=ε\_{max}^{-1}=\sqrt{1+λ\_{sup}(XY)}$ (8)

The procedure of designing pre- and post-filters (W1 and W2) involves [13]:

1. Scaling the plant inputs and outputs to improve the design problem condition.

2. Selecting proper pre- and post- ﬁlters in such a way that the shaped plant has high gain at low frequencies.

MATLAB was used to determine the weighting functions *W1*and *W2* , their values were:

$W\_{1}=\frac{10(s+0.5)}{s(s+0.4)}$ , $W\_{2}=\frac{20\left(s+0.03\right)(s+0.5)}{s(s+1)}$

Maximum stability margin is estimated as performance criterion and is defined as $e\_{max}={1}/{γ\_{min}}$ , where $γ\_{min}$ has been described as *H∞* optimal cost [12]. The maximum stability margin could be found by using the (ncfsyn) command in MATLAB, $e\_{max}$ is obtained as $0.7071$. The closed loop step response for ball position of *H∞*loop shaping controlled system is shown in figure (7).



Figure (7): Step response of *H∞*loop shaping robustly controlled system.

It is clear from figure (7), there is no over shoot and the response is very fast (settling time less than 0.1 sec.). The resulted magnitude and phase for controlled system is shown in figure (8), it is clear that the right choice of shaping functions guaranteed the stability of the system.



Figure (8): Bode diagram of *H∞*loop shaping robustly controlled system

To test the strength of the controller, 20% uncertainty was added to the plant using the same values of *W1*and *W2*. The step response for ball position after adding the uncertainty is shown in figure (9).



Figure (9): Step response of *H∞*loop shaping robustly controlled system with uncertainty 20%

The results of the two controllers is shown in table 1. It can be concluded that the settling time for the *H∞*loop shaping controller was improved 80 times than in PID controller, at the same time the overshoot was eliminated. A comparison of *H∞*loop shaping controller with other controllers is shown in table 2.

Table 1: Step response results for the designed controllers.

|  |  |  |
| --- | --- | --- |
| Performance Index | PID Controller | *H∞*loop shaping controller |
| Settling Time (sec.) | 8 | Less than 0.1 |
| Overshoot | 14.3% | 0 |

Table 2: Step response results for different controllers.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Performance Index** | **2 DOF Controller [1]** | **PSO [2]** | **Hybrid Fuzzy Controller [3]** | **Fuzzy PID [6]** | ***H∞* loop shaping controller** |
| **Settling Time (sec.)** | 0.19 | 3.7 | 2.2664 | 5.34 | Less than 0.1 |
| **Overshoot** | 1.8% | 3.67% | 23.11% | 0 | 0 |

**Conclusions:**

1. Although there is no error steady state, the designed PID controller did not meet the requirement of settling time and overshot.
2. The designed *H∞*loop shaping controller achieve all requirements ( no over shoot and very fast response).
3. Using *H∞*loop shaping controller robustly overcome the uncertainty added to the system.
4. Robust *H∞*loop shaping controller gives better performance than different types of controllers.

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