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Performance Study of Load Flow Algorithms in Well and Ill-Conditioned Systems

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Abstract

Existing transmission systems are classified as either ill systems or healthy systems. Most of the load flow algorithms works proficiently under well-conditioned systems. However, some of those algorithms fail to produce the accurate results for ill-conditioned systems. This paper investigates the performance of eight load flow algorithms based on the conventional Newton-Raphson, Fast-decoupled and Second-order Load Flow methods for a wide range of electrical bus system sizes. Tests are carried out for each load flow algorithm on six different standard bus systems, each with five different ill-conditioning levels. The results show that improved load flow model with constant Jacobian has advantages over the conventional load flow approach in both well and ill-conditioned system, especially for large-scale system.

Keywords*: Newton-Raphson, Fast-decoupled Load Flow, Second-order Load Flow, Constant Jacobian, ill-conditioned system*

1. Introduction

Electrical energy plays an important role in the social and economic growth of a country. The power flow analysis predicts all flows and voltages in the network when given the status of generators and loads [1]. Load flow analysis enables the planning of the future growth of power systems in the most economical way and determining the best operation of existing systems. The power flow analysis works as an essential tool for power system research such as power system security, optimum power flow, state estimation, continuation power flow [2-3]. The studies for the power flow analysis started with the Ward & Hale method in 1956 [4]. Newton-Raphson (NR) method is one of the most popular and widely used method currently [5]. In 1977, Sachdev and Medicherla presented Second Order Load Flow (SOLF) technique which was a formulation based on the Taylor series expansion of a multivariable function and an algorithm for obtaining digital solutions by the proposed approach is described [6]. Besides, fast decupled load flow (FDLF) is also widely used for their good convergence, fast calculation speed and modest computer storage request [7-8]. FDLF method is much faster comparing with the Newton method.

Power system becomes more important as the load demand increases worldwide [9]. Real life power transmission system involves many buses and there are over 150,000 major components in a power flow model. Computation for such wide-area systems require huge amount of computation time. Therefore, two prominent features that load flow tool should exhibit are efficiency and robustness. A study of the available literature indicates that practically adopted load flow algorithms are based on either Newton-Raphson technique or Fast-Decoupled technique. These algorithms are reported to be effective. However, they seem to have some limitations. NR technique requires large computing time and encounters convergence problems on the ill-conditioned systems where the iterative process may diverge or oscillate. On the other hand, the FDLF technique does not perform well for the systems where decoupling assumption is not completed with.

Economic, environmental, and technical problems such as the difficulty in construction of new transmission lines, new generation plants, and the raising load demand have let power system to operate near to its limit capacity. Transmission lines with high Resistance/Inductance (R/X) ratio and bus connections with very high resistance and very low impedance cause the

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power system to become ill-conditioned [10]. Much effort has been made to find efficient methods for solving the power flows of ill-conditioned power systems.

In this research work, the performances of robust load flow methods are compared in both well and ill-conditioned systems. The performance and robustness of eight load flow analysis algorithms based on conventional methods namely Newton-Raphson (NR) method, Second-order load flow method, and Fast-Decoupled method were investigated and simulation results for some IEEE test systems are shown to validate the analysis.

2. Research Method

The real and reactive power at bus i is:

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad (1)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \quad (2)$$

where $Y_{ij} = |Y_{ij}|\angle\theta_{ij} =$ shunt admittance between bus i and bus j and $V_i = |V_i|\angle\delta_i$ =voltage at bus i

Equations (1) - (2) are a set of nonlinear algebraic equations comprising of the independent variables, per unit voltage magnitude, and phase angle. There are two equations for each PQ bus, given by Equations (1) - (2), and one equation for each PV bus, given by Equation (1).

The terms $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are known as the power residuals and can be calculated by

$$\Delta P_i^{(k)} = P_i^{sch} - P_i^{(k)} \quad (3)$$

$$\Delta Q_i^{(k)} = Q_i^{sch} - Q_i^{(k)} \quad (4)$$

where P_i^{sch} is the scheduled real power and Q_i^{sch} is the scheduled reactive power. Expansion of Equations (1) - (2) in Taylor's series about the first-order estimate and neglecting all the higher order terms gives the following set of linear equations.

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \hline \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \left[\begin{array}{ccc|ccc} \frac{\partial P_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_2}{\partial \delta_n}^{(k)} & \frac{\partial P_2}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial P_2}{\partial |V_n|}^{(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_n}{\partial \delta_n}^{(k)} & \frac{\partial P_n}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial P_n}{\partial |V_n|}^{(k)} \\ \hline \frac{\partial Q_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_2}{\partial \delta_n}^{(k)} & \frac{\partial Q_2}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial Q_2}{\partial |V_n|}^{(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_n}{\partial \delta_n}^{(k)} & \frac{\partial Q_n}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial Q_n}{\partial |V_n|}^{(k)} \end{array}\right] \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_n^{(k)} \\ \hline |\Delta V_2^{(k)}| \\ \vdots \\ |\Delta V_n^{(k)}| \end{bmatrix} \quad (5)$$

Bus 1 is assumed to be the slack bus. The Jacobian matrix describes the linear relationship between small changes in the voltage angle $\Delta\delta_i^{(k)}$ and the voltage magnitude $|\Delta V_i^{(k)}|$ with small changes in real power, $\Delta P_i^{(k)}$and reactive power, $\Delta Q_i^{(k)}$. Elements of Jacobian matrix are partial derivatives of Eqs. (1) - (2), evaluated at $\Delta\delta_i^{(k)}$ and $|\Delta V_i^{(k)}|$.

The equation is solved iteratively in Newton-Raphson method and voltage increments at each bus is updated every iteration. The new estimates for bus voltages can be evaluated by using Equatoions (6) - (8).

$$\begin{bmatrix} \Delta\boldsymbol{\delta} \\ \Delta|\boldsymbol{V}| \end{bmatrix} = \begin{bmatrix} \boldsymbol{J_1} & \boldsymbol{J_2} \\ \boldsymbol{J_3} & \boldsymbol{J_4} \end{bmatrix}^{-1} \begin{bmatrix} \Delta\boldsymbol{P} \\ \Delta\boldsymbol{Q} \end{bmatrix} \quad (6)$$

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)} \quad (7)$$

$$|V_i^{(k+1)}| = |V_i^{(k)}| + \Delta|V_i^{(k)}| \quad (8)$$

The iteration process is continued until the voltage increment components $\Delta|V_i^{(k)}|$ and $\Delta\delta_i^{(k)}$ are less than the specified accuracy ϵ.

Simulation carried out by other reaserchers mainly focused on conventional algorithms [11-12]. In this research work, the performance and robustness of eight forms of load flow analysis algorithms are investigated. These algorithms are either in the orginal form or enhanced versions of conventional methods namely Newton-Raphson (NR) method, Second-order load flow method, and Fast-Decoupled method. These algorithms include four forms of NR, two forms of SOLF and two methods on FDLF.

The four forms of NR are NR in polar coordinate (NRpm), NR in rectangular coordinate (NRcm), improved version of NRpmJ by fixing the Jacobian matrix constant in both polar coordinate (NRpmJ) and rectangular coordinate (NRcmJ) [13-14]. The two forms of SOLF are conventional SOLF and SOLF with constant Jacobian matrix (SOLFJ) [15]. The two methods of FD are Fast Decoupled in polar form (PQpm) and rectangular form (PQcm). The constant Jacobian matrix load flow method was used on the NR method and SOLF method by keeping the Jacobian matrix constant, thus eliminating the need of calculating the matrix in every iteration. These proposed methods exhibit better computation speed.

Constant Jacobian implementation is a solution technique which avoids the time-consuming lower and upper triangular (LU) factorization. This technique is implemented to reduce the computation time taken for the load flow process. Figure 1 shows the Flowchart of Newton-Raphson method with constant Jacobian.

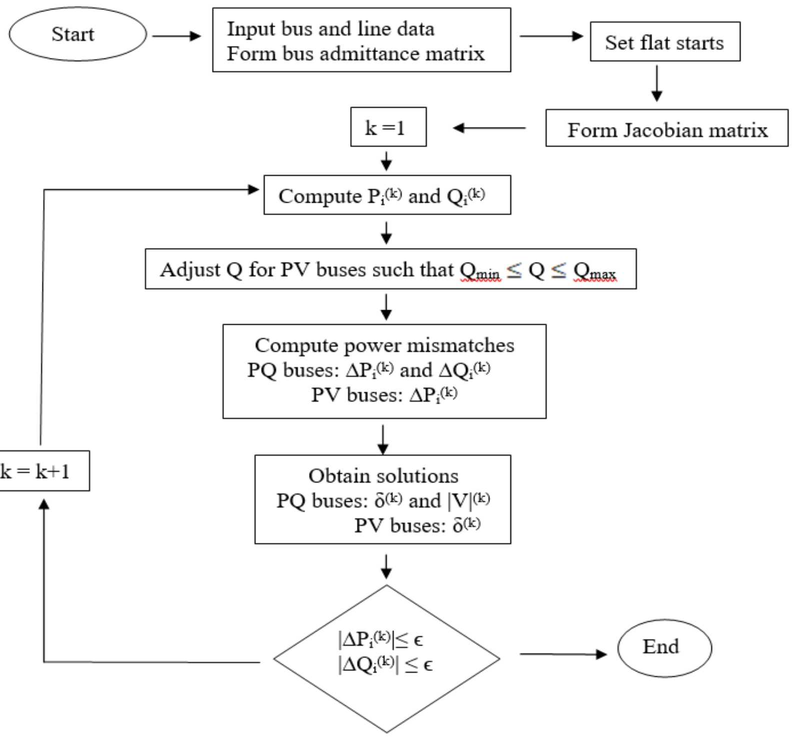

Figure 1. Flowchart for Newton-Raphson method with constant Jacobian

Analysis was carried out to evaluate the convergence rate and computation speed of the improved and conventional algorithms on six different standard bus systems with each bus system set at five different ill-conditioning levels. The approach was tested by computing load flows of the six standard bus systems: IEEE 14 bus test case, IEEE 24 bus RTS, IEEE 30 bus test case, 39 bus New England system, IEEE 57 bus test case, IEEE 118 bus test case. The ill-conditioned systems are simulated by stepping up the R/X ratio from 1 to 3 using a 0.5 step on all the six standard bus systems. Simulations were carried out on Matrix Laboratory

(MATLAB) platform with standard Institute of Electrical and Electronics Engineers (IEEE) bus systems, IEEE Reliability Test System (RTS), and New England system. The simulations are conducted on a personal computer running on operating system Microsoft Windows 8 Version 6.2 (Build 9200) 64-Bit, with Intel® Core™ i5-3317U CPU @ 1.70GHz and total installed memory (RAM): 8.00 GB (7.89 GB usable).

3. Results and Analysis

The tests were carried out and average of 5 sample computation times are tabulated. The maximum standard variation for the computation times are within 0.005 seconds. The accuracy of the simulation output is verified to be within 0.001 per unit voltage in comparison with the standard test bus data. The simulation time and number of iterations are recorded for the 8 algorithms implemented on 6 standard bus systems. The results on simulation time and number of iterations are shown in Table 1 and Table 2 respectively.

Table 1. Simulation time for the six test systems

Algorithm	Simulation time (seconds)					
(ill-conditioning level)	14	24	30	39	57	118
NRpm	0.022743	0.025645	0.029779	0.045105	0.058728	0.197917
NRpm(1.5)	0.022787	0.026079	0.030642	0.042182	0.069437	0.187589
NRpm(2)	0.022872	0.026551	0.033626	0.042096	0.066353	0.185811
NRpm(2.5)	0.021821	0.026422	0.033621	0.042891	0.069593	0.187133
NRpm(3)	0.022454	0.02666	0.033091	0.042032	0.069177	0.182669
NRpmJ	0.021162	0.024807	0.0267	0.035331	0.049973	0.117431
NRpmJ(1.5)	0.018759	0.02201	0.024989	0.032813	0.045606	0.103678
NRpmJ(2)	0.018925	0.02202	0.026497	0.032305	0.045374	0.10262
NRpmJ(2.5)	0.01884	0.021552	0.027461	0.034268	0.049272	0.11436
NRpmJ(3)	0.018939	0.021289	0.027761	0.035511	0.055402	0.125063
NRcm	0.023475	0.027269	0.030856	0.046845	0.06389	0.165171
NRcm(1.5)	0.021593	0.024287	0.029416	0.035483	0.054056	0.147375
NRcm(2)	0.021725	0.024369	0.029304	0.035534	0.053863	0.150003
NRcm(2.5)	0.021514	0.024609	0.029243	0.035448	0.053527	0.14918
NRcm(3)	0.021601	0.024544	0.029613	0.035407	0.05412	0.184775
NRcmJ	0.022064	0.025333	0.027399	0.037997	0.04505	0.121578
NRcmJ(1.5)	0.020552	0.022774	0.025814	0.031397	0.042611	0.121269
NRcmJ(2)	0.020235	0.02273	0.025805	0.031457	0.04256	0.132277
NRcmJ(2.5)	0.020121	0.022843	0.025681	0.031857	0.042672	0.145958
NRcmJ(3)	0.020262	0.022896	0.026835	0.033611	0.046656	0.173311
SOLF	0.035443	0.040024	0.044394	0.048829	0.074263	0.296851
SOLF(1.5)	0.032215	0.03704	0.042985	0.045744	0.067076	0.504641
SOLF(2)	0.033077	0.037513	0.046743	0.045465	0.066447	
SOLF(2.5)	0.03383	0.03715	0.057483	0.05182	0.081642	
SOLF(3)	0.04796	0.039262		0.051452	0.094859	
SOLFJ	0.033946	0.038646	0.042647	0.047427	0.069135	0.294543
SOLFJ(1.5)	0.032222	0.033702	0.040294	0.043716	0.064566	
SOLFJ(2)	0.031966	0.035283	0.042553	0.047626	0.064056	
SOLFJ(2.5)	0.032077	0.036824	0.042509	0.047801	0.07389	
SOLFJ(3)	0.032222	0.038448	0.04757	0.047667	0.081962	
PQpm	0.01929	0.021347	0.024308	0.112522	0.05014	0.152435
PQpm(1.5)	0.016945	0.01815	0.024813	0.103556	0.062876	0.112718
PQpm(2)	0.017879	0.018255	0.029526	0.097015	0.094048	0.108568
PQpm(2.5)	0.01918	0.019849	0.037502	0.094859	0.157625	0.144024
PQpm(3)	0.020563	0.020959	0.049075	0.087782	0.248907	0.175536
PQcm	0.02564	0.025189	0.043499	0.181684		0.225068
PQcm(1.5)		0.02376				0.487285
PQcm(2)		0.027827				
PQcm(2.5)		0.034865				
PQcm(3)		0.062471				

Generally, NR methods prove to have high robustness, yielding accurate result for all test cases and all levels of ill-conditioned systems. The applications of constant Jacobian on conventional Newton-Raphson power flow algorithm in both polar and rectangular coordinates perform well in most bus system with different levels of ill-conditioned system with shorter computation time as compared to corresponding conventional method.

On the other hand, SOLF in its pure form has good robustness but failed to provide a solution for a few of the test cases, mostly in ill-conditioned system. SOLFJ with constant Jacobian implementation has improved robustness that is achieved at shorter simulation time generally. The computation time for algorithms with Constant Jacobian modification is shorter due to the time saved by avoiding factorization in every iteration.

Despite of the slight increase in the number of iterations for constant jacobian algorithms in both NR and SOLF, the computation time per iteration is much shorter, especially in large-scale networks. Therefore, the constant Jacobian version of NR and SOLF become more efficient compared to the basic methods for both well and ill-conditioned system for the larger test bus system.

Fast-decoupled method has the lowest robustness and for the two Fast-decoupled algorithms tested, only PQpm yield accurate result for all test cases. Fast-Decoupled Load Flow with rectangular coordinate fails to converge to load flow solution for ill-conditioned system of 14, 30, 39, 57 and 118 bus system.

Table 2. Number of Iterations for the six test systems

Algorithm	Number of iterations					
(ill-conditioning level)	14	24	30	39	57	118
NRpm	3	3	3	4	3	4
NRpm(1.5)	3	4	3	4	4	4
NRpm(2)	3	4	4	4	4	4
NRpm(2.5)	3	4	4	4	4	4
NRpm(3)	4	4	4	4	4	4
NRpmJ	4	7	5	7	6	5
NRpmJ(1.5)	4	6	5	7	6	5
NRpmJ(2)	4	6	5	7	6	5
NRpmJ(2.5)	4	5	6	7	7	6
NRpmJ(3)	5	5	7	8	9	7
NRcm	4	4	4	4	4	4
NRcm(1.5)	4	4	4	4	4	4
NRcm(2)	4	4	4	4	4	4
NRcm(2.5)	4	4	4	4	4	4
NRcm(3)	4	4	4	4	4	5
NRcmJ	5	6	5	5	5	5
NRcmJ(1.5)	5	6	5	6	5	6
NRcmJ(2)	5	6	5	6	5	7
NRcmJ(2.5)	5	6	5	6	5	8
NRcmJ(3)	5	6	6	6	6	10
SOLF	4	4	3	3	3	5
SOLF(1.5)	4	4	4	3	3	10
SOLF(2)	5	4	5	3	3	
SOLF(2.5)	6	4	8	4	4	
SOLF(3)	21	5		4	5	
SOLFJ	6	5	5	4	4	8
SOLFJ(1.5)	6	4	5	4	4	
SOLFJ(2)	7	5	6	5	4	
SOLFJ(2.5)	7	6	6	5	5	
SOLFJ(3)	8	7	8	5	6	
PQpm	5	6	7	52	8	10
PQpm(1.5)	8	6	11	53	14	8
PQpm(2)	11	6	16	51	23	8
PQpm(2.5)	15	9	25	49	42	11
PQpm(3)	20	11	38	46	67	14
PQcm	14	7	21	81		14
PQcm(1.5)		9				34
PQcm(2)		15				
PQcm(2.5)		25				
PQcm(3)		64				

4. Conclusion

Eight different variations of algorithms based on NR, SOLF and FDLF methods have been implemented on the MATLAB platform and tested on six standard test bus systems with each bus system set at five different ill-conditioning levels. Improved load flow model with constant Jacobian presented in this paper has advantage of shorter computation time over the conventional load flow approach in both well and ill-conditioned system, especially for large-scale system. NR method works well in all the ill-conditioning levels as compared to SOLF and FDLF techniques which fail to converge in some ill-conditioned systems.

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