

Parameter Estimation of DC Motor using Adaptive Transfer Function based on Nelder-Mead Optimisation

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Article Info

Article history:

Received Sep 21, 2017

Revised Dec 30, 2017

Accepted Jan 17, 2018

Keywords:

Adaptive response

DC motor

ITAE and ITSE based objective function

Matlab environment

Nelder-Mead optimisation

Method

ABSTRACT

This paper explains an adaptive method for estimation of unknown parameters of transfer function model of any system for finding the parameters. The transfer function of the model with unknown model parameters is considered as the adaptive model whose values are adapted with the experimental data. The minimization of error between the experimental data and the output of the adaptive model have been realised by choosing objective function based on different error criterions. Nelder-Mead optimisation Method is used for adaption algorithm. To prove the method robustness and for students learning, the simple system of separately excited dc motor is considered in this paper. The experimental data of speed response and corresponding current response are taken and transfer function parameters of dc motors are adapted based on Nelder-Mead optimisation to match with the experimental data. The effectiveness of estimated parameters with different objective functions are compared and validated with machine specification parameters.

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1. INTRODUCTION

The mathematical model such as state-space form or transfer function of any system is required to know the dynamic behaviour and design a proper controller to get the specific requirement responses without affecting the stability of the system. This demands the exact value of model parameters. Some parameters of the transfer function model of the system can be determined through conducting different experiments, but some other parameters are difficult to be determined. In addition to that, the accuracy of determined parameters is mainly depends on the accuracy of measuring instruments which is connected in the experimental system. In order to overcome the above difficulties, the replica of the actual system can be designed in the form of transfer function and the values are optimised, so that the responses of replica model named as adaptive model will exactly match with the corresponding response of actual (reference) system model.

In this paper, effort has been made to find the five parameters of separately excited dc motor named as: armature resistance, armature inductance, Back emf constant, Moment of Inertia and Viscous friction coefficient. The experimental data of armature current and speed response with respect to time are collected by running the separately excited motor at full rated voltage with load torque of 5 N-m using current sensor and voltage sensor respectively. The transfer functions of dc motors are built in MATLAB environment with randomly chosen initial parameters. The error between response of experimental data and that of randomly chosen initial parameters transfer function is used to formulate the objective function. The objective function is based on different integral criterion well defined in control engineering [1]. The Simpson's one-third rule is used for integration of objective function. The minimisation of the objective function in order to adapt the response of adaptive model to the experimental response can be realised by Nelder-Mead simplex direct

search method. The function `fminsearch` in optimisation toolbox of MATLAB uses Nelder-Mead algorithm is used for searching of the unknown coefficients of adaptive transfer function model [2]. A Neuro-Genetic controller is used to control the dc motor speed by obtaining the controller parameter for each load torque [3]. An adaptive parameter adjustment mechanism is combined with fuzzy controller solve various problems associated with brushless dc motor drive [4]. A fractional order PID controller for dc motor is designed using Genetic Algorithm and Simulated Annealing and the result is compared with the PID [5].

1.1. Motivation, Objectives and Procedures

The main motivation factor has to find out the unknown parameters of system through adaption method. This can be achieved by simulating the unknown mathematical model repeatedly applying search method for fulfilment of the objective function. The objective is to design the adaptive model of the actual unknown model, which can be used to develop the control algorithms for controlling the actual system.

- a. The procedures set up by the authors can be summed up as follows
- b. Built the transfer function of DC motor model in simpower system in MATLAB environment with unknown variables and it will act as adaptive model. The experimental data of speed response and armature current response with respect to time are collected and placed in one dimensional lookup table in same MATLAB simulink file and it will be regarded as reference model response .The model should be stored with name. The model is shown in Figure 1.

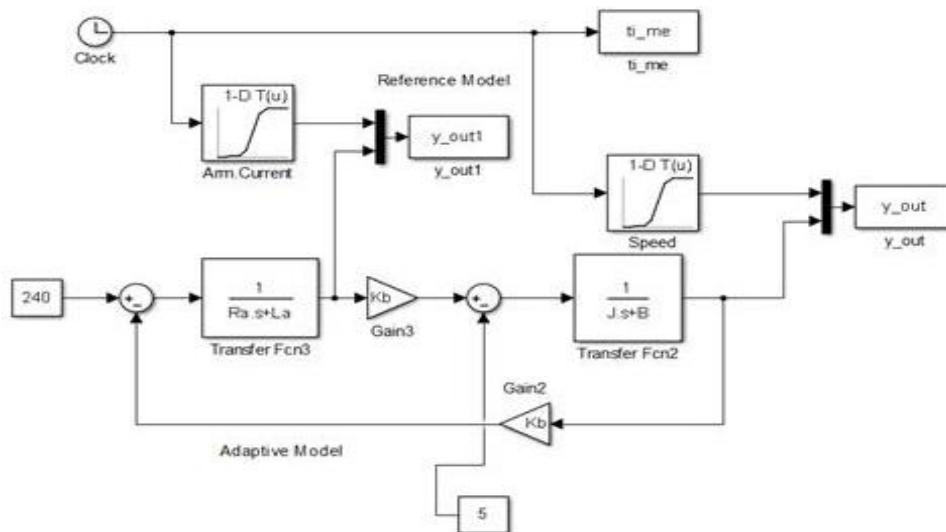


Figure 1. Reference and adaptive model

- c. The objective function should be carefully chosen. Here, integral criterion of ITAE and ITSE are considered for comparison as an objective functions. A function is written in MATLAB and Simpson's one-third rule is used to integrate the objective function. Inside the MATLAB objective function the simulink model of DC motor is called. The MATLAB program of objective function is given in Appendix.

Calling model command:

```
[t,y]=sim('bkn_ac6_pidest',tt);
```

- Nelder-Mead simplex direct search method is used here to adapt the response of adaptive model with the experimental response by minimising the objective function. The built-in Nelder-Mead simplex direct search is stored in the name of `fminsearch` in optimisation toolbox of the MATLAB [6-8]. A main program consist of `fminsearch` function has been written in MATLAB and it should be called for running of the system. The MATLAB program of main function is given in Appendix.

Calling objective function:

```
[X,FVAL,EXITFLAG,OUTPUT]=fminsearch(@byam_obj_ac6,x0,[],t_end,h)
```

- Running of the main program, the `fminsearch` function calls the objective function. Since objective function contains running command of MATLAB, the model will run. The difference between the experimental data and adaptive model data are passed into objective function. The output of objective function will processed through the iterative searching method, defined in `fminsearch`. The unknown five

parameters are optimised till the matching occurs between the adaptive response and experimental data. The iterative optimised searching method is discussed briefly in subsequent sections.

2. MODEL OF SEPARATELY EXCITED DC MACHINE

The two equations are used to develop the transfer function of separately excited DC motor. The equations are as follows:

$$R_a i_a + L_a \frac{di_a}{dt} + K_b \omega_m = V \quad (1)$$

$$K_b i_a - T_L = J \frac{d\omega_m}{dt} + B \omega_m \quad (2)$$

The above equation can be written in state space form as:

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega_m}{dt} \end{bmatrix} = \begin{bmatrix} -R_a & -K_b \\ L_a & L_a \\ K_b & -B \\ J & J \end{bmatrix} + \begin{bmatrix} V \\ L_a \\ -T_L \\ J \end{bmatrix} \quad (3)$$

where i_a , ω_m , and V are the armature current (ampere) and mechanical speed (rad/sec) and armature voltage (Volt) respectively. The five unknown parameters R_a , L_a , K_b , J and B are armature resistance (Ω) and armature inductance (H), back emf constant (Volt-sec/rad.), moment of inertia (kg-m^2) and viscous friction (Volt-sec/rad.) respectively. In this paper efforts are made to determine the above five parameters.

3. OBJECTIVE FUNCTION

The objective function, also named as performance index is to solve a mathematical problem such that a system satisfies a performance specification expressed depends on the requirement of control engineer. The commonly used performance indexes are [1]:

- Integral of squared error (ISE)
- Integral of time multiplied squared error (ITSE)
- Integral of absolute error (IAE)
- Integral of time multiplied absolute error (ITAE)

Since in this problem, the response of adaptive model will match the experimental data changes with respect to time, the two objective functions ITAE and ITSE are considered for comparisons.

$$\text{ITAE} = \int_0^{\infty} t * \text{abs}(\text{expt. data point} - \text{adaptive data point}) dt \quad (4)$$

$$\text{ITSE} = \int_0^{\infty} t * (\text{expt. data point} - \text{adaptive data point})^2 dt \quad (5)$$

4. NELDER-MEAD "SIMPLEX" DIRECT SEARCH METHOD

In the mid-1960s, two English statisticians invented the Nelder-Mead "simplex" direct search method used for solving the unconstrained optimization problem [9]. The Nelder-Mead method iteratively generates a sequence of interested vertex points which converge to an optimal vertex point of objective function $f(x)$ [10]. At each iteration, the vertices x_i are ordered according to the objective function values

$$f(x_1) \leq f(x_2) \leq f(x_3) \leq \dots \leq f(x_{n+1}) \quad (6)$$

where x_1 is the best vertex and x_{n+1} is the worst vertex. The algorithm uses four possible operations: reflection, expansion, contraction and shrink, each being associated with a scalar parameter: α (reflection), β (expansion), γ (contraction), and δ (shrink). The values of α , β , γ and δ are lying in the range of >0 , >1 and 0 to 1 in both γ and δ respectively.

The one iteration of Nelder-Mead algorithm is as follows [10]:

- Find out worst vertices using equation- let it be x_{n+1}
- Compute the reflection point

$$x_r = \bar{x} + \alpha(\bar{x} - x_{n+1}) \text{ where } \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \tag{7}$$

evaluate

$$f_r = f(x_r) \text{ if } f_1 \leq f_r < f_n, \text{ replace } x_{n+1} \text{ with } x_r$$

c. Compute the expansion point if $f_r < f_1$

$$x_e = \bar{x} + \beta(x_r - \bar{x}) \tag{8}$$

evaluate $f_e = f(x_e)$ if $f_e < f_r$ replace x_{n+1} with x_e otherwise by x_r

d. Compute the outside contraction point if $f_n \leq f_r < f_{n+1}$

$$x_{oc} = \bar{x} + \gamma(x_r - \bar{x}) \tag{9}$$

evaluate $f_{oc} = f(x_{oc})$ if $f_{oc} \leq f_r$ replace x_{n+1} with x_{oc} otherwise go to step-6

e. Compute the inside contraction point if $f_r > f_{n+1}$

$$x_{ic} = \bar{x} - \gamma(x_r - \bar{x}) \tag{10}$$

f. shrink: for $2 \leq i \leq n + 1$
define

$$x_i = x_1 + \delta(x_i - x_1) \tag{11}$$

and proceed to the next iteration.

5. RESULTS AND DISCUSSION

The parameters of separately excited dc motor can be determined by performing the various experiments in laboratory. The dc motor specification is given in machine specification data sheet. But, actually in some complicated system, it is difficult to ascertain the exact value of parameters because of inaccuracy of measuring instruments. Therefore, the adaptive method is the best solution for determination of unknown parameters of the system.

The data of speed encoder and current sensor corresponding to time are collected by running the dc motor at rated voltage and with load torque of 5N-m by using the speed encoder and current sensor. Two look up tables are formed based on the speed and current response data in MATLAB simulink environment. The transfer function of DC motor is also placed in same MATLAB simulink file. The transfer function of dc motor model will act as adaptive model because of unknown parameters are updated on each iteration. The responses of lookup table model will act as reference model. The current responses of reference and adaptive are stored in the workspace y_out1 , whereas the speed responses are stored in y_out . The model should be stored in certain name. By calling this name the responses are passed to the objective function. The objective function is built in m-file of the MATLAB [11-12]. The simulink model is shown in Figure 1.

The optimised adaptive values are found out by considering the two objective functions.

$$f=ITAE=\int_0^\infty t|e(t)| dt \text{ or } ITSE=\int_0^\infty t(e(t))^2 dt \tag{12}$$

where, $|e(t)|$ is the difference of reference speed and adaptive speed and t is the time at that instant.

Another two cases are also performed by adapting both current response and voltage response for considering both objective functions.

$$f_1 \text{ and } f_2=ITAE=\int_0^\infty t|e_1(t)| dt \text{ and } \int_0^\infty t|e_2(t)| dt \text{ or } ITSE=\int_0^\infty t(e_1(t))^2 dt \text{ and } \int_0^\infty t(e_2(t))^2 dt \tag{13}$$

$$f = \sqrt{f_1 * f_2} \tag{14}$$

where, $|e_1(t)|$ and $|e_2(t)|$ is the difference of reference speed and adaptive speed and reference armature current and adaptive current respectively. The results of 4 cases are compared and given in tabular form in Table 1. One of the optimisation results are given in appendix for reference. The program of objective function of ITAE used for to adapt the reference response and main calling function are also given in appendix.

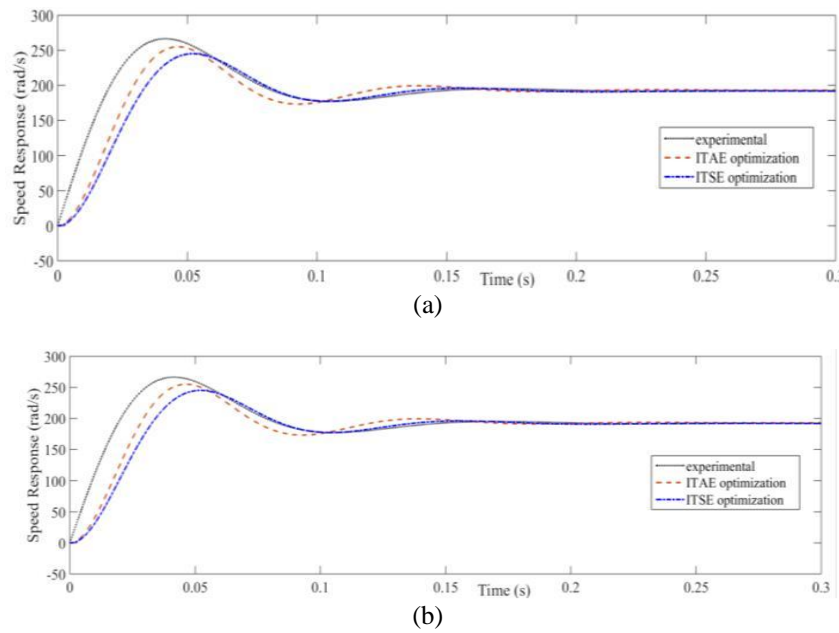


Figure 2. (a, b) Adaptive Speed and current responses using ITAE and ITSE criteria for adaption of speed response to experimental data only

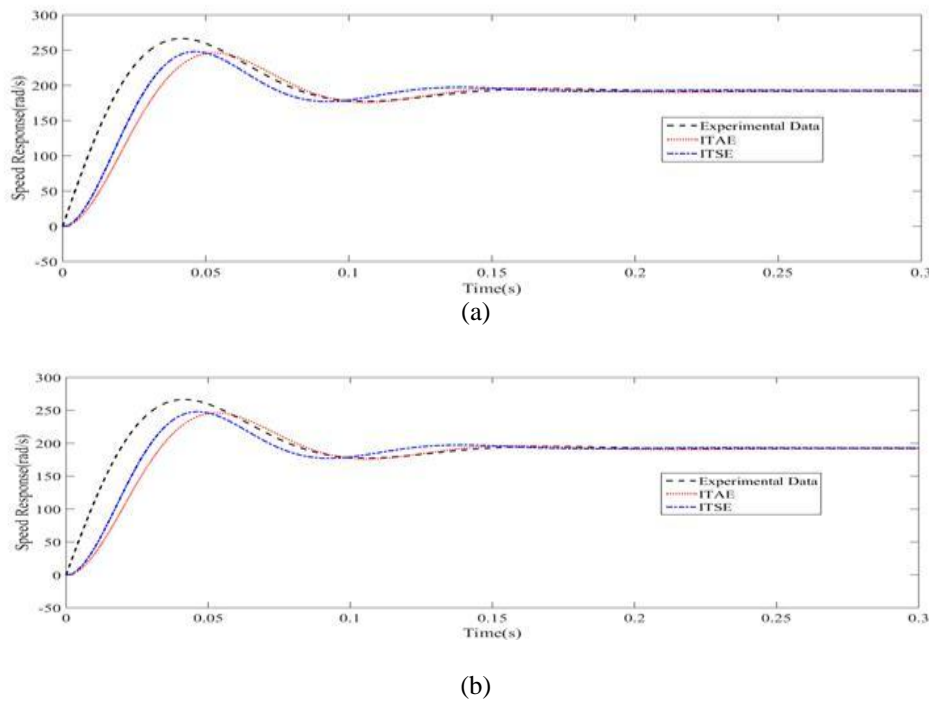


Figure 3. (a, b) Adaptive Speed and current responses using ITAE and ITSE criteria for adaption of both speed and current responses with the experimental data

Table 1. Initial values ($R_a=0.1$, $L_a=1$, $K_b=1.8$, $J=0.09$, $B=0.02$)
simulation increment =0.00005 second Time of
simulation=.03second

Parameters Values	Machine Data sheet	Only speed response (adaptive values)		Both Speed and Armature current response (adaptive values)	
		ITAE	ITSE	ITAE	ITSE
R_a (Ω)	0.5	0.0222	0.0033	0.0108	0.0067
L_a (H)	0.01	1.0380	0.1571	0.5102	0.3748
K_b	1.23	1.1904	1.2254	1.2311	1.2590
J	0.05	0.0154	0.0889	0.0342	0.0434
B	0.02	0.0423	0.1085	0.0250	-0759

6. CONCLUSIONS

In this paper a novel approach for estimation the parameters of separately excited DC motor is presented for enhance the students' knowledge. The approach is the adaption of response of transfer function of DC motor with unknown parameters with the experimental responses. Two well known objective functions are considered for error estimations. Nelder–Mead simplex direct search method is used to minimisation of objective function. Since the results will not match with the machine data sheet and also the armature current response is also deviated from the experimental data in both chosen objective function, the efforts are made to adapt both current and speed responses. The adaptive responses are shown in Figure 3 (a and b) confirms the accuracy of adapting if both current and speed responses are adapted, particularly in ITAE optimisation. The accuracy can be enhanced by using the other well known optimisation techniques. The content of this paper is not for accurate estimation, but to explain the procedures for adaption. The results of the adaptive algorithms are quite encouraging and therefore, suggest that it is a helpful not only for electrical engineer students but for other students, who works on finding the unknown parameters of their system.

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APPENDIX

Objective Function:

```
% Objectivefunction for finding out TF of machine using ITAE objective
function
%ITAE=int of 0 to inf[(t*abs(e(t)))*dt]
%simpson 1/3 rule is used for integration
function f=byam_tf_obj(x,t_end,h)
global a
global b
```

```

global c
global d
global e
global y_out % From scope which to be improved scope name y_out
global ti_me % From scope of time variable as ti_me
a=x(1)
b=x(2)
c=x(3)
d=x(4)
e=x(5)
tt=(0:h:t_end);
[t,y]=sim('byamdcdrives_tfl',tt); % model name
f=0;
l=length(ti_me);
f=(ti_me(l)*abs(y_out(l,1)-y_out(l,2)));
% scope output 1 and 2 (1,1) means
%1st element of the 1st output. (2,3) represents 2nd element of the 3rd
%output
for i=2:2:l-1
f=f+4*((ti_me(i)*abs(y_out(i,1)-y_out(i,2))));
end
for i=3:2:l-2
f=f+2*((ti_me(i)*abs(y_out(i,1)-y_out(i,2))));
end
f=h/3*f;
Main program
%%Main program
global a
global b
global c
global d
global e
global y_out
global ti_me
t_end=0.4;
h=.00005;
x0=[0.1,1,1.8,0.09,0.02];
[X,FVAL,EXITFLAG,OUTPUT]=fminsearch(@byam_tf_obj,x0,[],t_end,h)
options = optimset('Display','iter','MaxIter',%20);

```

Optimisation Results:

$R_a=0.0222$; $L_a=1.0380$; $K_p=1.1904$; $J=0.0154$ $B=0.0423$

FVAL = 0.1185

EXITFLAG = 1

OUTPUT= iterations: 154, funcCount: 261 algorithm: 'Nelder-Mead simplex direct search'

message: 'Optimization terminated: the current x satisfies the termination criteria using OPTIONS.TolX of 1...'