Indonesian Journal of Electrical Engineering and Computer Science
Vol. 9, No. 2, February 2018, pp. 526~538
ISSN: 2502-4752, DOI: 10.11591/ijeecs.v9.i2.pp526-538

Performance of Full-Duplex One-Way and Two-Way Cooperative Relaying Networks

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Article Info

Article history:

Received Oct 5, 2017
Revised Nov 18, 2017
Accepted Jan 2, 2018

Keywords:

AF protocol.
Full-duplex
Physical layer network coding (PNC)
Two-way Relay

ABSTRACT

The wireless research requires concurrent transmission and reception in a single time/frequency channel with good spectral efficiency. The Full duplex system is the alternate for the conventional half duplex systems. An investigation on the need for a full duplex two way (FD-TWR) and one way relaying (FD-OWR) to improve the performance of outage probability and average rate employing amplify-and-forward (AF) and decode-and-forward (DF) protocol is considered. Further the relaying systems performance under the network coding schemes is taken into consideration. The outage probability and average rate of FD-TWR and FD-OWR using a physical layer network coding was performed. In contrast to “straightforward” network coding which performs arithmetic function on digital bit streams after information have been received. The result shows the DF protocol achieves better outage probability and average rate, when compared to the AF protocol. And comparing the full duplex schemes like FD-TWR and FD-OWR, it is found that the FD-TWR achieves better outage probability and average rate, when compared to the FD-OWR. The performance was extended with different loop interference among the relay antennas. The performance show that FD-TWR performs well even in spite of loop interference.

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1. INTRODUCTION

Cooperative communication is one of the luring research titles which offer a better result for the battery life crisis and improving the transmission capacity. Cooperative diversity can be defined as a numerous antenna technique proposed to improve the whole network channel capacities intended for any specified set of bandwidths. In the wireless multi-hop networks the used diversity can be further developed by the combination of relayed signal and the direct signal that is being received.

Compared to the half duplex relaying, full duplex achieve higher capacity in both transmission and reception on the same carrier frequency. The capacity tradeoff between AF based full duplex with self-interference and half duplex under absence of fading in the source-relay and self-interference channels was studied [1]. The capacity tradeoff between DF based full duplex with self-interference and half duplex under absence of fading in the source-relay and self-interference channels was analyzed [2].

Further the capacity tradeoff between Amplify and Forward (AF) based full duplex and half-duplex relaying was given [3] with an assumption that the source-relay channel was under fading. The outage probability was derived using the assumption that there was no direct link between the source and destination nodes. Two gain control schemes for the AF based full duplex protocol maximizing the Signal-to Interference- plus-Noise Ratio (SINR) and decreased transmit power was obtained [4]. The outage analysis for a DF based FD-OWR under the assumption that there was no direct link between the source and destination nodes gives the conditions to guarantee superior full-duplex against half-duplex mode [5].

The achievable rate for the DF based full-duplex multiple input multiple-output (MIMO) one-way relaying. And then, residual self-interference, direct link, limited transmitter/receiver dynamic range and imperfect channel state information (CSI), were also taken into consideration [6]. In [7] a combination of opportunistic full-duplex/half-duplex mode selection and transmitted power adaptation for maximizing the spectrum efficiency was analyzed.

A comparison on the outage probability and system throughput for a two-way half-duplex to one-way full-duplex relaying was carried out and the FD-OWR could outperform bidirectional half-duplex relaying, even in the presence of self-interference [8-9].

The throughput and outage probability of a full-duplex block Markov relaying scheme with self interference at the relay under independent non-identically distributed Nakagami-m fading [10]. The pair-wise error probability, bit error rate (BER) and diversity performance of the AF based full-duplex linear relaying and dual-hop systems, under the effect of residual self-interference [11]. In [12], the virtual full-duplex relaying by means of two half-duplex relays which was a good alternative before standardizing full-duplex technology. In the same work, self-interference is replaced by inter-relay interference in this virtual version is considered.

The outage probability of a variable-gain AF based FD-OWR with direct link to half-duplex counterpart and proposed a highly exact approximation to the outage probability. FD-TWR can further improve system capacity by achieving bidirectional data transmission and reception on the same carrier frequency simultaneously [13-14].

The achievable rate region for FD-TWR without residual self-interference. Also derived this achievable rate region but they assumed the existence of residual self-interference and the resource efficiency of two-way and full-duplex relaying systems [15-18]. Then the diversity-multiplexing tradeoff of FD-TWR and proposed a compress and forward strategy to achieve the optimal diversity-multiplexing tradeoff. In [19], the outage probability of the AF based FD-TWR with residual self-interference, in case of the perfect and imperfect CSI and derived approximate closed form expressions.

An optimal max-min relay selection scheme of the AF based relaying and studied its BER, ergodic capacity and outage probability [20]. In the same work, an optimal power allocation and duplex mode selection to minimize the outage probability was also presented. In [21], the Degree of Freedom (DoF) of the K-pair-user with a MIMO relay. In this, a full duplex PNC, in which the relay used detect-and-forward technique and the maximum likelihood (ML) based joint detection to eliminate the multiple access interference [22].

The Full Duplex system can be analyzed by using the Physical layer network coding and the performance of outage probability and average rate are improved. The rest of the paper is organized as follows: The system model having M relays was discussed in section II. Section III gives the performance evaluation of the Nth best relay selection scheme over the AF and DF Channels. The simulation results are presented in the section IV and conclusion is discussed in the section V.

2. SYSTEM MODEL

A three-node FD-TWR model which consists of two nodes A and B, and a relay, is considered in with FD-OWR and FD-TWR. In each time slot, FD-OWR can achieve unidirectional data transmission and reception between nodes source(S) and destination (D) via the relay on the same carrier frequency, while FD-TWR can achieve bidirectional data transmission and reception. This means that FD-TWR can further multiplex the transmitting and receiving time, compared with FD-OWR. Moreover, only the relay in FD-OWR works in full-duplex, whereas all the nodes in FD-TWR operate in this mode. Therefore, FD-TWR would suffer from more severe self-interference, also called Loop Interference (LI), caused by the co-channel transmission and imperfect interference cancellation, compared with FD-OWR. Furthermore, FD-TWR is similar to half-duplex two-way relaying [7] and still consists of the multiple access (MAC) and broadcast (BC) stages. But, these stages in FD-TWR can be performed in parallel, in the same time slot and thus, all the nodes work in full-duplex mode and suffer from residual self-interference. In cellular networks, the node A, relay and node B, are denoted as the User Equipment (UE), Relay Node (RN) and Base Station (BS), respectively.

The involved channels are node S to relay (SR), relay to node A (RS), node B to relay (DR), relay to node B (RD), and residual self-interference in node S, relay and node D. The corresponding channel coefficients are denoted as h_{SR}, h_{RS} , h_{DR} , h_{RD} , h_{SS} , h_{RR} , h_{DD} . Thus the instantaneous noise signal-to-noise ratio (SNR), γ is an exponential random variable (RV) with probability density function (PDF),

$f_{\overline{\gamma}}(\gamma) = (1/\overline{\gamma})e^{-\gamma/\overline{\gamma}}$

where, $\overline{\gamma}$ is an instantaneous SNR.

The instantaneous channel SNR is $\gamma = |h|^2 P/\sigma^2$
where, h is channel coefficient, σ^2 is noise power. The normalized transmitted powers of a node S, relay, node D are $P_S = 1$,$P_R = 1$, $P_D = 1$ respectively and the residual self interference channels are assumed to be identical, i.e. $\overline{\gamma}_{SS} = \overline{\gamma}_{RR} = \overline{\gamma}_{DD} = \overline{\gamma}_{LI}$. For FD-TWR, the relay simultaneously receives signals from both source nodes A and B, and the residual self-interference caused by its co-channel transmission signal and then forwards them to the corresponding destination nodes B and A.

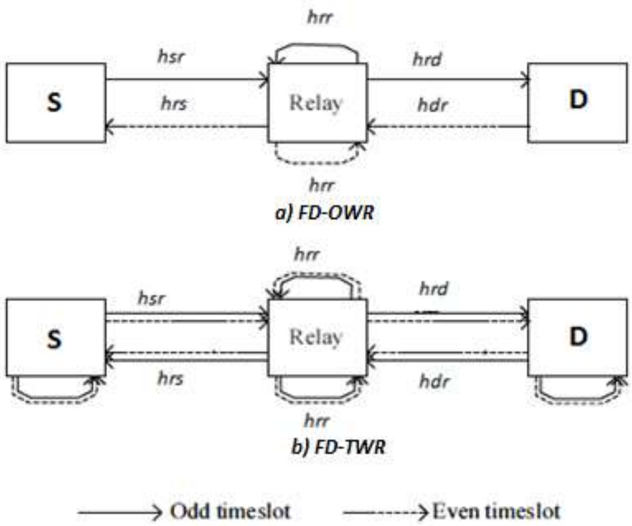

Figure 1. System Model of Full-Duplex Two Way and One Way Relaying

The destination nodes B and A simultaneously receive signals forwarded by the relay and residual self-interference created by their co-channel transmitted signals.In the k-th time slot, the signals received at the relay(R), nodes D and S can be expressed as,

$$y_R[k] = h_{SR}x_S[k] + h_{DR}x_D[k] + h_{RR}t_R[k] + n_R[k] \quad (1)$$

$$y_D[k] = h_{RD}t_R[k] + h_{DD}t_D[k] + n_D[k] \quad (2)$$

$$y_S[k] = h_{RS}t_R[k] + h_{SS}t_S[k] \quad (3)$$

where $t_R[k]$, $t_D[k]$ and $t_S[k]$ are the transmitted signals of the relay, nodes D and S respectively.

3. PERFORMANCE MODEL

The performance of the DF based Full Duplex and AF based FD relay is presented here.

3.1 DF based FD -TWR

The DF based FD-TWR with PNC, the relay decodes the signals received from both source nodes S and D, and then it implements PNC to recode the decoded data and forwards the recoded data to the destination nodes D and S. After receiving the network coded signals from the relay, the nodes D and S perform decoding to obtain their desired data, respectively.

$$\gamma_R = \frac{\gamma_{SR}+\gamma_{DR}}{\bar{\gamma}_{RR}+1}\,, \quad \gamma_D = \frac{\gamma_{RD}}{\bar{\gamma}_{DD}+1}\,, \quad \gamma_S = \frac{\gamma_{RS}}{\bar{\gamma}_{SS}+1}$$

For the DF based FD-TWR with PNC, in *k*-th time slot, the signal transmitted at the relay can be expressed as,

$$t_R[k] = x_S[k-\tau] \oplus x_D[k-\tau] \tag{4}$$

Then, the instantaneous SNR of the signal received at the relay can be expressed as,

$$y_R = \frac{\varepsilon\{|h_{SR}x_S[k]|^2\}+\varepsilon\{|h_{DR}x_D[k]|^2\}}{\varepsilon\{|h_{RR}t_R[k]|^2\}+\varepsilon\{|n_R[k]|^2\}}$$

$$= \frac{\gamma_{SR}+\gamma_{DR}}{\bar{\gamma}_{RR}+1} \tag{5}$$

Substitute equation (4) in (2) and (3)

$$y_D[k] = h_{RD}(x_S[k-\tau] \oplus x_D[k-\tau]) + h_{DD}t_D[k] + n_D[k] \tag{6}$$

$$y_S[k] = h_{RS}(x_S[k-\tau] \oplus x_D[k-\tau]) + h_{SS}t_S[k] + n_S[k] \tag{7}$$

Since both destination nodes D and S know their preciously transmitted data, they can subtract the back-propagating self interference in (6) and (7) after decoding, through bit-level XOR operation. The instantaneous SNRs of signals received at nodes D and S can be respectively expressed as,

$$y_D = \frac{\varepsilon\{|h_{RD}x_S[k-\tau]|^2\}}{\varepsilon\{|h_{DD}t_D[k]|^2\}+\varepsilon\{|n_D[k]|^2\}}$$

$$y_D = \frac{\gamma_{RD}}{\bar{\gamma}_{DD}+1} \tag{8}$$

Similarly at node S,

$$y_S = \frac{\varepsilon\{|h_{RS}x_D[k-\tau]|^2\}}{\varepsilon\{|h_{SS}t_S[k]|^2\}+\varepsilon\{|n_S[k]|^2\}}$$

$$y_S = \frac{\gamma_{RS}}{\bar{\gamma}_{SS}+1} \tag{9}$$

3.1.1 Average Rate

The average rate for the DF based Full-Duplex two-way relaying equals the average of the minimum of the rate for the source-relay and relay-destination channels in ,

$$\bar{\mathrm{R}} = \varepsilon\{\min(\log_2(1+\gamma_{\mathrm{R}}), \min(\log_2(1+\gamma_{\mathrm{S2R}}), \log_2(1+ \quad \gamma_{\mathrm{R2D}})) + \min(\log_2(1+\gamma_{\mathrm{D2R}}), \log_2(1+\gamma_{\mathrm{R2S}}))))\}$$

$$\bar{R} \le min(\varepsilon\{log_2(1+\gamma_R)\}, \varepsilon\{log_2(1+min(\gamma_{S2R},\gamma_{R2D}))\}, +\varepsilon\{log_2(1+min(\gamma_{D2R},\gamma_{R2S}))\}) \tag{10}$$

Applying Jensen's inequality in above equation,

$$\varepsilon\{log_2(1+\gamma_R)\} = \varepsilon\left\{log_2\left(1+\frac{\gamma_{SR}+\gamma_{DR}}{\bar{\gamma}_{RR}+1}\right)\right\} \tag{11}$$

$$\varepsilon\{log_2(1+\gamma_R)\} = \frac{1}{ln2}\left\{\int_0^\infty ln\left(1+\frac{x+y}{\bar{\gamma}_{RR}+1}\right)\frac{e^{-y/\bar{\gamma}_{DR}}}{\bar{\gamma}_{DR}}dy + \int_0^\infty\int_0^\infty \frac{1}{\bar{\gamma}_{RR}+1+x+y} e^{-x/\bar{\gamma}_{SR}}\frac{e^{-y/\bar{\gamma}_{DR}}}{\bar{\gamma}_{DR}}dxdy\right\} \tag{12}$$

In order to derive the closed form- expression conveniently, we first define a random variable X as the minimum of γ_{S2R} and γ_{R2D} ,

$$X = min(\gamma_{S2R}, \gamma_{R2D})$$

$$= min\left(\frac{\bar{\gamma}_{SR}}{\bar{\gamma}_{RR}+1}, \frac{\bar{\gamma}_{DR}}{\bar{\gamma}_{RR}+1}\right) \quad (13)$$

Then, we deduce its cumulative distributive function (CDF),

$$F_X(x) = P(X \le x) = P(min(\gamma_{S2R}, \gamma_{R2D}) \le x)$$
$$\begin{cases} 1 - e^{-\frac{(\bar{\gamma}_{SR}(\bar{\gamma}_{DD}+1)+\bar{\gamma}_{RD}(\bar{\gamma}_{RR}+1))x}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}}, & x > 0 \\ 0, & x \le 0 \end{cases} \quad (14)$$

Based on CDF of X, $\varepsilon\{\log_2(1 + \min(\gamma_{S2R}, \gamma_{R2D}))\}$ is derived as,

$$\bar{R}_D^{DF} = \varepsilon\{log_2(1 + min(\gamma_{S2R}, \gamma_{R2D}))\}$$
$$\bar{R}_D^{DF} = \int_0^\infty log_2(1+x)\, dF_X(x)$$

$$\bar{R}_D^{DF} = \frac{1}{ln2} e^{\frac{(\bar{\gamma}_{SR}(\bar{\gamma}_{DD}+1)+\bar{\gamma}_{RD}(\bar{\gamma}_{RR}+1))}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}} \times E_1\left(\frac{(\bar{\gamma}_{SR}(\bar{\gamma}_{DD}+1)+\bar{\gamma}_{RD}(\bar{\gamma}_{RR}+1))}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}\right) \quad (15)$$

Similarly, $\varepsilon\{\log_2(1 + \min(\gamma_{D2R}, \gamma_{R2S}))\}$ is represented as,

$$\bar{R}_S^{DF} = \varepsilon\{log_2(1 + min(\gamma_{D2R}, \gamma_{R2S}))\}$$

$$\bar{R}_S^{DF} = \frac{1}{ln2} e^{\frac{(\bar{\gamma}_{DR}(\bar{\gamma}_{SS}+1)+\bar{\gamma}_{RS}(\bar{\gamma}_{RR}+1))}{\bar{\gamma}_{DR}\bar{\gamma}_{RS}}} \times E_1\left(\frac{(\bar{\gamma}_{DR}(\bar{\gamma}_{SS}+1)+\bar{\gamma}_{RS}(\bar{\gamma}_{RR}+1))}{\bar{\gamma}_{DR}\bar{\gamma}_{RS}}\right) \quad (16)$$

Substitute equation (12), (14) and (15) in (10),The average rate of for the DF based FD-TWR is expressed as,

$$\bar{R}_{sum}^{DF,PNC} \le min \frac{\bar{\gamma}_{DR} e^{\frac{\bar{\gamma}_{RR}+1}{\bar{\gamma}_{DR}}} E_1\left(\frac{\bar{\gamma}_{RR}+1}{\bar{\gamma}_{DR}}\right) - \bar{\gamma}_{SR} e^{\frac{\bar{\gamma}_{RR}+1}{\bar{\gamma}_{SR}}} E_1\left(\frac{\bar{\gamma}_{RR}+1}{\bar{\gamma}_{SR}}\right)}{(ln2)(\bar{\gamma}_{DR}-\bar{\gamma}_{SR})},$$

$$\frac{1}{ln2} e^{\frac{\bar{\gamma}_{SR}(\bar{\gamma}_{DD}+1)+\bar{\gamma}_{RD}(\bar{\gamma}_{RR}+1)}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}} E_1\left(\frac{\bar{\gamma}_{SR}(\bar{\gamma}_{DD}+1)+\bar{\gamma}_{RD}(\bar{\gamma}_{RR}+1)}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}\right)$$

$$+\frac{1}{ln2} e^{\frac{\bar{\gamma}_{DR}(\bar{\gamma}_{SS}+1)+\bar{\gamma}_{RS}(\bar{\gamma}_{RR}+1)}{\bar{\gamma}_{DR}\bar{\gamma}_{RS}}} E_1\left(\frac{\bar{\gamma}_{DR}(\bar{\gamma}_{SS}+1)+\bar{\gamma}_{RS}(\bar{\gamma}_{RR}+1)}{\bar{\gamma}_{DR}\bar{\gamma}_{RS}}\right) \quad (17)$$

According to [(2),(5),(6)], the average rate for the DF based FD-OWR can be expressed as,

$$\bar{R} = \varepsilon\{log_2(1 + (\gamma_R,\ \gamma_D))\} \quad (18)$$

$$\bar{R} = \frac{1}{ln\,2}\ e^{\frac{(\gamma_{SR}+\gamma_{DR}(\gamma_{RR}+1))}{\gamma_{SR}\gamma_{DR}}} E_1\left(\frac{\gamma_{SR}+\gamma_{DR}(\gamma_{RR}+1)}{\gamma_{SR}\gamma_{DR}}\right) \quad (19)$$

3.1.2 Outage Probability

The outage probability of the DF based FD-TWR from[33,(14),(15)],

$$P_{out}^{DF} = 1 - P\left(\left\{\{\gamma_{S2R}^{DF} \ge \gamma_{th}\} \cap \{\gamma_{D2R}^{SIC} \ge \gamma_{th}\} \cap \{\gamma_{R2S}^{DF} \ge \gamma_{th}\} \cap \{\gamma_{R2D}^{DF} \ge \gamma_{th}\}\right\} \cup \left\{\{\gamma_{D2R}^{DF} \ge \gamma_{th}\} \cap \{\gamma_{S2R}^{SIC} \ge \gamma_{th}\} \cap \{\gamma_{R2S}^{DF} \ge \gamma_{th}\} \cap \{\gamma_{R2D}^{DF} \ge \gamma_{th}\}\right\}\right) \quad (20)$$

$$\gamma_{S2R}^{DF} = \frac{\gamma_{SR}}{\gamma_{DR}+\bar{\gamma}_{RR}+1}\ ,\ \gamma_{S2R}^{SIC} = \frac{\gamma_{SR}}{\bar{\gamma}_{RR}+1}\ , \gamma_{D2R}^{DF} = \frac{\gamma_{DR}}{\gamma_{SR}+\bar{\gamma}_{RR}+1}$$
$$\gamma_{D2R}^{SIC} = \frac{\gamma_{DR}}{\bar{\gamma}_{RR}+1}, \gamma_{R2S}^{DF} = \frac{\gamma_{RS}}{\bar{\gamma}_{SS}+1}, \gamma_{R2D}^{DF} = \frac{\gamma_{RD}}{\bar{\gamma}_{DD}+1} \quad (21)$$

Substitute (21) in (20),

CASE 1: $\gamma_{th} \geq 1$,

$$P_{out}^{DF,PNC} = 1 - \int_{\gamma_{th}(\overline{\gamma}_{LI}+1)}^{\infty} \int_{\gamma_{th}(y+\overline{\gamma}_{LI}+1)}^{\infty} \frac{1}{\overline{\gamma}_{SR}} e^{-x/\overline{\gamma}_{SR}} \frac{1}{\overline{\gamma}_{DR}} e^{-y/\overline{\gamma}_{DR}} dxdy -$$

$$\int_{\gamma_{th}(\overline{\gamma}_{LI}+1)}^{\infty} \int_{\gamma_{th}(y+\overline{\gamma}_{LI}+1)}^{\infty} \frac{1}{\overline{\gamma}_{SR}} e^{-x/\overline{\gamma}_{SR}} \frac{1}{\overline{\gamma}_{DR}} e^{-y/\overline{\gamma}_{DR}} dxdy \quad (22)$$

CASE 2: $\gamma_{th} \in (0,1)$

$$P_{out}^{DF,PNC} 1 - \frac{\overline{\gamma}_{SR}}{\overline{\gamma}_{SR}+\overline{\gamma}_{DR}\gamma_{th}} e^{-\frac{\gamma_{th}(\overline{\gamma}_{LI}+1)(\overline{\gamma}_{SR}+\overline{\gamma}_{DR}+\overline{\gamma}_{DR}\gamma_{th})}{\overline{\gamma}_{SR}\overline{\gamma}_{DR}}} -$$

$$\frac{\overline{\gamma}_{DR}}{\overline{\gamma}_{DR}+\overline{\gamma}_{SR}\gamma_{th}} e^{-\frac{\gamma_{th}(\overline{\gamma}_{LI}+1)(\overline{\gamma}_{SR}+\overline{\gamma}_{DR}+\overline{\gamma}_{SR}\gamma_{th})}{\overline{\gamma}_{SR}\overline{\gamma}_{DR}}} + \frac{(1-\gamma_{th})^2\overline{\gamma}_{SR}\overline{\gamma}_{DR}}{(\gamma_{th}\overline{\gamma}_{SR}+\overline{\gamma}_{DR})(\overline{\gamma}_{SR}+\gamma_{th}\overline{\gamma}_{DR})} e^{-\frac{\gamma_{th}(\overline{\gamma}_{LI}+1)(\overline{\gamma}_{SR}+\overline{\gamma}_{DR})}{(1-\gamma_{th})\overline{\gamma}_{SR}\overline{\gamma}_{DR}}} \quad (23)$$

Here the outage probability of the DF based FD-TWR with PNC, is given in equation (24).

$$\begin{cases} 1 - \frac{\overline{\gamma}_{SR}}{\overline{\gamma}_{SR}+\overline{\gamma}_{DR}\gamma_{th}} e^{-\frac{\gamma_{th}(\overline{\gamma}_{LI}+1)(\overline{\gamma}_{SR}+\overline{\gamma}_{DR}\gamma_{th})}{\overline{\gamma}_{SR}\overline{\gamma}_{DR}}} - \frac{\overline{\gamma}_{DR}}{\overline{\gamma}_{DR}+\overline{\gamma}_{SR}\gamma_{th}} e^{-\frac{\gamma_{th}(\overline{\gamma}_{LI}+1)(\overline{\gamma}_{SR}+\overline{\gamma}_{DR}+\overline{\gamma}_{SR}\gamma_{th})}{\overline{\gamma}_{SR}\overline{\gamma}_{DR}}} , \gamma_{th} \geq 1 \\ 1 - \frac{\overline{\gamma}_{SR}}{\overline{\gamma}_{SR}+\overline{\gamma}_{DR}\gamma_{th}} e^{-\frac{\gamma_{th}(\overline{\gamma}_{LI}+1)(\overline{\gamma}_{SR}+\overline{\gamma}_{DR}\gamma_{th})}{\overline{\gamma}_{SR}\overline{\gamma}_{DR}}} - \\ \frac{\overline{\gamma}_{DR}}{\overline{\gamma}_{DR}+\overline{\gamma}_{SR}\gamma_{th}} e^{-\frac{\gamma_{th}(\overline{\gamma}_{LI}+1)(\overline{\gamma}_{SR}+\overline{\gamma}_{DR}+\overline{\gamma}_{SR}\gamma_{th})}{\overline{\gamma}_{SR}\overline{\gamma}_{DR}}} + \frac{(1-\gamma_{th}{}^2)\overline{\gamma}_{SR}\overline{\gamma}_{DR}}{(\gamma_{th}\overline{\gamma}_{SR}+\overline{\gamma}_{DR})(\overline{\gamma}_{SR}+\gamma_{th}\overline{\gamma}_{DR})} e^{-\frac{\gamma_{th}(\overline{\gamma}_{LI}+1)(\overline{\gamma}_{SR}+\overline{\gamma}_{DR})}{(1-\gamma_{th})\overline{\gamma}_{SR}\overline{\gamma}_{DR}}} , \\ for\ \gamma_{th} \in (0,1) \end{cases} \quad (24)$$

According to [(5), (3)] the outage probability of DF based FD-OWR is,

$$P_{out}^{DF} = 1 - \left(1 - \int_{0}^{\gamma_{th}(\gamma_{RR}+1)} \frac{1}{\gamma_{SR}} e^{\frac{-x}{\overline{\gamma}_{SR}}} dx\right) \times \left(1 - \int_{0}^{\gamma_{th}} \frac{1}{\overline{\gamma}_{RD}} e^{\frac{-y}{\overline{\gamma}_{RD}}} dy\right)$$

$$P_{out}^{DF} = 1 - e^{-\frac{\gamma_{th}(\overline{\gamma}_{SR}+\overline{\gamma}_{RD}(\overline{\gamma}_{RR}+1))}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}} \quad (25)$$

The comparison of (24) and (25) reveals that the outage probability of the DF based FD-TWR with PNC is higher than that in the DF based FD-OWR, because residual self interference, generated at the destination nodes due to their co-channel transmission, deteriorates the SNRs of the received signal.

3.2 AF based FD -TWR

In the AF based FD-OWR, in the k-th time slot, the signal transmitted by the relay is the amplification of the prior received signal and it can be expressed as,

$$t_R[k] = \beta y_R[k-\tau] \quad (26)$$

Where β is the amplification factor, which depends on the channel coefficients, and τ is the processing delay. Sub equation (1) in (26)

$$t_R[k] = \beta(h_{SR}x_S[k-\tau] + h_{DR}x_D[k-\tau] + h_{RR}t_R[k-\tau] + n_R[k-\tau]) \quad (27)$$

The instantaneous transmitted power is expressed as,

$$\varepsilon\{|t_R[k]|^2\} = \beta^2(|h_{SR}|^2 + |h_{DR}|^2 + |h_{RR}|^2\varepsilon\{|t_R[k-\tau]|^2\} + \sigma_R^2) \quad (28)$$

Considering the power constraint of P_R at the relay and assuming that its transmitting power is $\varepsilon\{|t_R[k]|^2\} = P_R = 1$. Then,

$$\beta^2 = \frac{1}{|h_{SR}|^2+|h_{DR}|^2+|h_{RR}|^2+\sigma_R^2}$$

$$\beta = [|h_{SR}|^2 + |h_{DR}|^2 + |h_{RR}|^2 + \sigma_R^2]^{-1/2} \quad (29)$$

where h_{RR} is residual self-interference after interference cancellation. The received signal at node D can be expressed as,

Sub equation (27) in (2)

$$\begin{aligned} y_D[k] &= h_{RD}[\beta(h_{SR}x_S(k-\tau) + h_{DR}x_D(k-\tau) + h_{RR}t_R(k-\tau) + n_R(k-\tau))] + h_{DD}t_D[k] + n_D[k] \\ &= \beta h_{RD}\big(h_{SR}x_S(k-\tau) + h_{DR}x_D(k-\tau) + h_{RR}t_R(k-\tau) + n_R(k-\tau)\big) + h_{DD}t_D[k] + n_D \end{aligned} \tag{30}$$

Since the node D know their transmitted symbols, the back-propagating self-interference can be subtracted.

$$y_D[k] = \beta h_{RD}\big(h_{SR}x_S(k-\tau) + h_{RR}t_R(k-\tau) + n_R(k-\tau)\big) + h_{DD}t_D[k] + n_D[k]$$

The instantaneous power received at these nodes are expressed as,

$$\varepsilon\{|y_D[k]|^2\} = \beta^2|h_{RD}|^2(|h_{SR}|^2 + |h_{RR}|^2\varepsilon\{|t_R[k-\tau]|^2\} + \sigma_R^2) + |h_{DD}|^2 + \sigma_D^2$$

Here the power constraint $\varepsilon\{|t_R[k]|^2\} = P_R = 1$

$$\varepsilon\{|y_D[k]|^2\} = \beta^2|h_{RD}|^2(|h_{SR}|^2 + |h_{RR}|^2 + \sigma_R^2) + |h_{DD}|^2 + \sigma_D^2 \tag{31}$$

Then the instantaneous SNR at the node D can be expressed as,

$$\begin{aligned} \gamma_D &= \frac{\beta^2|h_{RD}|^2|h_{SR}|^2}{\beta^2|h_{RD}|^2(|h_{RR}|^2+\sigma_R^2)+|h_{DD}|^2+\sigma_D^2} \\ &= \frac{|h_{RD}|^2|h_{SR}|^2}{(|h_{RR}|^2+\sigma_R^2)+\frac{|h_{DD}|^2+\sigma_D^2}{\beta^2}} \end{aligned} \tag{32}$$

Similarly the instantaneous SNR at the node S can be expressed as,

$$\begin{aligned} \gamma_S &= \frac{\beta^2|h_{RS}|^2|h_{DR}|^2}{\beta^2|h_{RS}|^2(|h_{RR}|^2+\sigma_R^2)+|h_{SS}|^2+\sigma_S^2} \\ &= \frac{|h_{RS}|^2|h_{DR}|^2}{|h_{RS}|^2(|h_{RR}|^2+\sigma_R^2)+\frac{|h_{SS}|^2+\sigma_S^2}{\beta^2}} \end{aligned} \tag{33}$$

Then substitute equation (29) in (31) and (33). The instantaneous SNR of the AF based FD-TWR at the node S and D can be expressed as,

$$\begin{aligned} \gamma_S &= \frac{|h_{RS}|^2|h_{DR}|^2}{|h_{RS}|^2(|h_{RR}|^2+\sigma_R^2)+\frac{|h_{SS}|^2+\sigma_S^2}{[[|h_{SR}|^2+|h_{DR}|^2+|h_{RR}|^2+\sigma_R^2]^{-1/2}]^2}} \\ \gamma_S &= \frac{\gamma_{RS}\gamma_{DR}}{\gamma_{RS}(\gamma_{RR}+1)+(\gamma_{SS}+1)(\gamma_{SR}+\gamma_{DR}+\gamma_{RR}+1)} \end{aligned} \tag{34}$$

Similarly at node D,

$$\gamma_D = \frac{\gamma_{RD}\gamma_{SR}}{\gamma_{RD}(\gamma_{RR}+1)+(\gamma_{DD}+1)(\gamma_{SR}+\gamma_{DR}+\gamma_{RR}+1)} \tag{35}$$

Equation (34) and (35) indicate that FD-TWR has more residual self-interference compared to FD-OWR because all the nodes in FD-TWR operate in full-duplex mode, while only the relay in FD-OWR operates in this mode. Thus, FD-TWR deteriorates the SNR of the received end-to-end signal.

The average rate for the AF based FD-TWR is defined as,

$$\bar{R} = \varepsilon\{\log_2(1+\gamma_S)\} + \varepsilon\{\log_2(1+\gamma_D)\} \tag{36}$$

Sub equation (36) in (34)

$$\varepsilon\{\log_2(1+\gamma_S)\} = \frac{1}{ln2}\left(\int_0^\infty\int_0^\infty \frac{x+\bar{\gamma}_{SS}+1}{xy+x(\bar{\gamma}_{RR}+1)+(\bar{\gamma}_{SS}+1)(x+y+\bar{\gamma}_{RR}+1)}\frac{1}{\bar{\gamma}_{RS}}e^{-\frac{x}{\bar{\gamma}_{RS}}}e^{-\frac{y}{\bar{\gamma}_{DR}}}dxdy\right) - \int_0^\infty\int_0^\infty \frac{\bar{\gamma}_{SS}+1}{x(\bar{\gamma}_{RR}+1)+(\bar{\gamma}_{SS}+1)(x+y+\bar{\gamma}_{RR}+1)}\frac{1}{\bar{\gamma}_{RS}}e^{-\frac{x}{\bar{\gamma}_{RS}}}e^{-\frac{y}{\bar{\gamma}_{DR}}}dxdy \quad (37)$$

From the above equation $I_{1,1}$ and $I_{2,2}$is,

$$I_{1,1} = \int_0^\infty\int_0^\infty \frac{x+\bar{\gamma}_{SS}+1}{xy+x(\bar{\gamma}_{RR}+1)+(\bar{\gamma}_{SS}+1)(x+y+\bar{\gamma}_{RR}+1)}\frac{1}{\bar{\gamma}_{RS}}e^{-\frac{x}{\bar{\gamma}_{RS}}}e^{-\frac{y}{\bar{\gamma}_{DR}}}dxdy$$

$$I_{1,2} = \int_0^\infty\int_0^\infty \frac{1}{\bar{\gamma}_{RS}}e^{-\frac{x}{\bar{\gamma}_{RS}}}e^{-\frac{y}{\bar{\gamma}_{DR}}}\frac{\bar{\gamma}_{SS}+1}{x(\bar{\gamma}_{RR}+1)+(\bar{\gamma}_{SS}+1)(x+y+\bar{\gamma}_{RR}+1)}dxdy$$

Then, $I_{1,1}$ can be simplified in equation (38) in the bottom of the page.

$$I_{1,1} = \int_0^\infty\int_0^\infty \frac{x+\bar{\gamma}_{LI}+1}{(x+\bar{\gamma}_{LI}+1)+2(x+\bar{\gamma}_{LI}+1)(\bar{\gamma}_{LI}+1)-(\bar{\gamma}_{LI}+1)^2}\frac{1}{\bar{\gamma}_{RS}}e^{-\frac{x}{\bar{\gamma}_{RS}}}e^{-\frac{y}{\bar{\gamma}_{DR}}}dxdy \quad (38)$$

In order to obtain a tightly lower bound easily, the constant term $-(\bar{\gamma}_{LI}+1)^2$ can be discard in the denominator.

$$I_{1,1} = \int_0^\infty\int_0^\infty \frac{x+\bar{\gamma}_{LI}+1}{x+\bar{\gamma}_{LI}+1}\times\frac{1}{y+2(\bar{\gamma}_{LI}+1)}\frac{1}{\bar{\gamma}_{RS}}e^{-\frac{x}{\bar{\gamma}_{RS}}}e^{-\frac{y}{\bar{\gamma}_{DR}}}dxdy$$

$$I_{1,1} = e^{\frac{2(\bar{\gamma}_{LI}+1)}{\bar{\gamma}_{DR}}}E_1\left(\frac{2(\bar{\gamma}_{LI}+1)}{\bar{\gamma}_{DR}}\right) \quad (39)$$

Then,

$$I_{1,2} = \int_0^\infty\int_0^\infty \frac{\bar{\gamma}_{SS}+1}{x(\bar{\gamma}_{RR}+1)+(\bar{\gamma}_{SS}+1)(x+y+\bar{\gamma}_{RR}+1)}\frac{1}{\bar{\gamma}_{RS}}e^{-\frac{x}{\bar{\gamma}_{RS}}}e^{-\frac{y}{\bar{\gamma}_{DR}}}dxdy$$

Here, the residual self interference is assumed to be identical, then $\bar{\gamma}_{SS} = \bar{\gamma}_{RR} = \bar{\gamma}_{DD} = \bar{\gamma}_{LI}$

$$I_{1,2} = \int_0^\infty\int_0^\infty \frac{1}{\bar{\gamma}_{RS}}e^{-\frac{x}{\bar{\gamma}_{RS}}}e^{-\frac{y}{\bar{\gamma}_{DR}}}\frac{\bar{\gamma}_{LI}+1}{x(\bar{\gamma}_{LI}+1)+(\bar{\gamma}_{LI}+1)(x+y+\bar{\gamma}_{LI}+1)}dxdy$$

$$I_{1,2} = \frac{1}{\bar{\gamma}_{RS}}\int_0^\infty e^{-\frac{x}{\bar{\gamma}_{RS}}}e^{\frac{2x+\bar{\gamma}_{LI}+1}{\bar{\gamma}_{DR}}}E_1\left(\frac{2x+\bar{\gamma}_{LI}+1}{\bar{\gamma}_{DR}}\right)dx$$

$$I_{1,2} = \frac{\bar{\gamma}_{DR}}{2\bar{\gamma}_{RS}-\bar{\gamma}_{DR}}\left(e^{\frac{(\bar{\gamma}_{LI}+1)}{2\bar{\gamma}_{RS}}}E_1\left(\frac{\bar{\gamma}_{LI}+1}{2\bar{\gamma}_{RS}}\right) - e^{\frac{(\bar{\gamma}_{LI}+1)}{\bar{\gamma}_{DR}}}E_1\left(\frac{\bar{\gamma}_{LI}+1}{\bar{\gamma}_{DR}}\right)\right) \quad (40)$$

Sub equation (39) and (40) in (37). The average rate of FD-TWR from destination to source

$$\{\log_2(1+\gamma_S)\} > \frac{1}{ln2}\left(e^{\frac{2(\bar{\gamma}_{LI}+1)}{\bar{\gamma}_{DR}}}E_1\left(\frac{2(\bar{\gamma}_{LI}+1)}{\bar{\gamma}_{DR}}\right) - \frac{\bar{\gamma}_{DR}}{2\bar{\gamma}_{RS}-\bar{\gamma}_{DR}}\left(e^{\frac{\bar{\gamma}_{LI}+1}{2\bar{\gamma}_{RS}}}E_1\left(\frac{\bar{\gamma}_{LI}+1}{2\bar{\gamma}_{RS}}\right) - e^{\frac{\bar{\gamma}_{LI}+1}{\bar{\gamma}_{DR}}}E_1\left(\frac{\bar{\gamma}_{LI}+1}{\bar{\gamma}_{DR}}\right)\right)\right) \quad (41)$$

The average rate of FD-TWR from source to destination,

$$\varepsilon\{\log_2(1+\gamma_D)\} > \frac{1}{ln2}\left(e^{\frac{2(\bar{\gamma}_{LI}+1)}{\bar{\gamma}_{SR}}}E_1\left(\frac{2(\bar{\gamma}_{LI}+1)}{\bar{\gamma}_{SR}}\right) - \frac{\bar{\gamma}_{SR}}{2\bar{\gamma}_{RD}-\bar{\gamma}_{SR}}\left(e^{\frac{\bar{\gamma}_{LI}+1}{2\bar{\gamma}_{RD}}}E_1\left(\frac{\bar{\gamma}_{LI}+1}{2\bar{\gamma}_{RD}}\right) - e^{\frac{\bar{\gamma}_{LI}+1}{\bar{\gamma}_{SR}}}E_1\left(\frac{\bar{\gamma}_{LI}+1}{\bar{\gamma}_{SR}}\right)\right)\right) \quad (42)$$

Then substitute equation (41) and (42) in (36).The average rate for the AF based FD-TWR is,

$$\bar{R}^{AF}_{sum} \geq \frac{1}{\ln 2}\left(e^{\frac{2(\bar{\gamma}_{LI}+1)}{\gamma_{DR}}}E_1\left(\frac{2(\bar{\gamma}_{LI}+1)}{\bar{\gamma}_{DR}}\right) + e^{\frac{2(\bar{\gamma}_{LI}+1}{\bar{\gamma}_{SR}}}E_1\left(\frac{2(\bar{\gamma}_{LI}+1)}{\bar{\gamma}_{SR}}\right) - \frac{\bar{\gamma}_{DR}}{2\bar{\gamma}_{SR}-\bar{\gamma}_{DR}}\left(e^{\frac{\bar{\gamma}_{LI}+1}{2\bar{\gamma}_{SR}}}E_1\left(\frac{\bar{\gamma}_{LI}+1}{2\bar{\gamma}_{SR}}\right) - e^{\frac{\bar{\gamma}_{LI}+1}{\bar{\gamma}_{DR}}}E_1\left(\frac{\bar{\gamma}_{LI}+1}{\bar{\gamma}_{DR}}\right)\right) - \frac{\bar{\gamma}_{SR}}{2\bar{\gamma}_{RD}-\bar{\gamma}_{SR}}\left(e^{\frac{\bar{\gamma}_{LI}+1}{2\bar{\gamma}_{RD}}}E_1\left(\frac{\bar{\gamma}_{LI}+1}{2\bar{\gamma}_{RD}}\right) - e^{\frac{\bar{\gamma}_{LI}+1}{\bar{\gamma}_{SR}}}E_1\left(\frac{\bar{\gamma}_{LI}+1}{\bar{\gamma}_{SR}}\right)\right)\right) \quad (43)$$

The average rate for the AF based FD-OWR is,

$$\bar{R}^{AF}_{FD-OWR} = \frac{\gamma_{th}(\gamma_{RR}+1)e^{\frac{\gamma_{RR}+1}{\gamma_{SR}}}E_1(\frac{\gamma_{RR}+1}{\gamma_{SR}})}{(\ln 2)(\gamma_{DR}(\gamma_{RR}+1)-\gamma_{SR})} - \frac{\gamma_{SR}e^{1/\gamma_{DR}}E_{1(\frac{1}{\gamma_{DR}})}}{(\ln 2)(\gamma_{DR}(\gamma_{RR}+1)-\gamma_{SR})} \quad (44)$$

The AF based FD-TWR cannot achieve full time multiplexing gain, compared with FD-OWR, because it also suffers from the residual self-interference at the two destination nodes.

3.2.2 Outage probability

Let $\gamma_{th} = 2^{R_{th}} - 1$, where γ_{th} and R_{th} are the outage SNR and rate thresholds, respectively. Thus, the outage probability of FD-TWR is defined as,

$P_{out}^{AF} = P\{\min(\log_2(1+\gamma_S), \log_2(1+\gamma_D)) < R_{th}\}$

For the AF based FD-TWR, the integral domain for its outage probability consist of

$D_1 = \{(x,y)|0 < x < \infty, 0 < y < \gamma_{th}(\bar{\gamma}_{LI}+1)\}$

$D_3 = \{(x,y)| \frac{(\bar{\gamma}_{LI}+1)(3\gamma_{th}+(9\gamma_{th}{}^2+4\gamma_{th})^{1/2})}{2} \le x < \infty, \frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(x+\bar{\gamma}_{LI}+1)}{x-2\gamma_{th}(\bar{\gamma}_{LI}+1)} \le y < \frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(2x+\bar{\gamma}_{LI}+1)}{x-\gamma_{th}(\bar{\gamma}_{LI}+1)}\}$

Then, the outage probability of AF based FD-TWR is given in (45).

$$P_{out}^{AF} = \int_0^{\gamma_{th}(\bar{\gamma}_{LI}+1)} \frac{1}{\bar{\gamma}_{DR}} e^{-\frac{y}{\bar{\gamma}_{DR}}} \int_0^{\infty} \frac{1}{\bar{\gamma}_{SR}} e^{-\frac{x}{\bar{\gamma}_{SR}}} dxdy + \int_{\gamma_{th}(\bar{\gamma}_{LI}+1)}^{\infty} \frac{1}{\bar{\gamma}_{DR}} e^{-\frac{y}{\bar{\gamma}_{DR}}} \int_0^{\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(2y+(\bar{\gamma}_{LI}+1))}{y-\gamma_{th}(\bar{\gamma}_{LI}+1)}} \frac{1}{\bar{\gamma}_{SR}} e^{-\frac{x}{\bar{\gamma}_{SR}}} dx\, dy + \int_{\frac{(\bar{\gamma}_{LI}+1)(3\gamma_{th}+(9\gamma_{th}^2+4\gamma_{th})^{\frac{1}{2}})}{2}}^{\infty} \int_{\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(x+\bar{\gamma}_{LI}+1)}{x-2\gamma_{th}(\bar{\gamma}_{LI}+1)}}^{\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(2x+\bar{\gamma}_{LI}+1)}{x-\gamma_{th}(\bar{\gamma}_{LI}+1)}} \frac{1}{\bar{\gamma}_{SR}} e^{-\frac{x}{\bar{\gamma}_{SR}}} \frac{1}{\bar{\gamma}_{DR}} e^{-\frac{y}{\bar{\gamma}_{DR}}} dx\, dy \quad (45)$$

From the equation (45) $I_{2,1}$ and $I_{2,2}$ is represented as,

$$I_{2,1} = \int_0^{\gamma_{th}(\bar{\gamma}_{LI}+1)} \frac{1}{\bar{\gamma}_{DR}} e^{-\frac{y}{\bar{\gamma}_{DR}}} \int_0^{\infty} \frac{1}{\bar{\gamma}_{SR}} e^{-\frac{x}{\bar{\gamma}_{SR}}} dxdy + \int_{\gamma_{th}(\bar{\gamma}_{LI}+1)}^{\infty} \frac{1}{\bar{\gamma}_{DR}} e^{-\frac{y}{\bar{\gamma}_{DR}}} \int_0^{\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(2y+(\bar{\gamma}_{LI}+1))}{y-\gamma_{th}(\bar{\gamma}_{LI}+1)}} \frac{1}{\bar{\gamma}_{SR}} e^{-\frac{x}{\bar{\gamma}_{SR}}} dxdy \quad (46)$$

$$I_{2,2} = \int_{\frac{(\bar{\gamma}_{LI}+1)(3\gamma_{th}+(9\gamma_{th}^2+4\gamma_{th})^{\frac{1}{2}})}{2}}^{\infty} \int_{\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(x+\bar{\gamma}_{LI}+1)}{x-2\gamma_{th}(\bar{\gamma}_{LI}+1)}}^{\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(2x+\bar{\gamma}_{LI}+1)}{x-\gamma_{th}(\bar{\gamma}_{LI}+1)}} \frac{1}{\bar{\gamma}_{SR}} e^{-\frac{x}{\bar{\gamma}_{SR}}} \frac{1}{\bar{\gamma}_{DR}} e^{-\frac{y}{\bar{\gamma}_{DR}}} dxdy \quad (47)$$

Equation (46) can be written as,

$$I_{2,1} = 1 - 2\left(\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(\bar{\gamma}_{SR}+2\bar{\gamma}_{DR})}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}\right)^{\frac{1}{2}} \times e^{-\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(\bar{\gamma}_{SR}+2\bar{\gamma}_{DR})}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}} \times K_1\left(2\left(\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(\bar{\gamma}_{SR}+2\bar{\gamma}_{DR})}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}\right)^{\frac{1}{2}}\right) \quad (48)$$

Equation (47) can be written as,

$$I_{2,2} = 2\left(\frac{\gamma_{th}(2\gamma_{th}+1)(\bar{\gamma}_{LI}+1)^2}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}\right)^{\frac{1}{2}} e^{-\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(\bar{\gamma}_{SR}+2\bar{\gamma}_{DR})}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}} K_1\left(2\left(\frac{\gamma_{th}(2\gamma_{th}+1)(\bar{\gamma}_{LI}+1)^2}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}\right)^{\frac{1}{2}}\right) - \frac{1}{\bar{\gamma}_{SR}} \int_0^{\frac{(\bar{\gamma}_{LI}+1)(-\gamma_{th}+(9\gamma_{th}^2+4\gamma_{th})^{\frac{1}{2}})}{2}} e^{-\left(\frac{z}{\bar{\gamma}_{SR}}+\frac{\gamma_{th}(2\gamma_{th}+1)(\bar{\gamma}_{LI}+1)^2}{\bar{\gamma}_{DR}z}+\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(\bar{\gamma}_{SR}+2\bar{\gamma}_{DR})}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}\right)} dz - 2\left(\frac{\gamma_{th}(2\gamma_{th}+1)(\bar{\gamma}_{LI}+1)^2}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}\right)^{\frac{1}{2}} e^{-\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(2\bar{\gamma}_{SR}+\bar{\gamma}_{DR})}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}} K_1\left(2\left(\frac{\gamma_{th}(2\gamma_{th}+1)(\bar{\gamma}_{LI}+1)^2}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}\right)^{\frac{1}{2}}\right) + \frac{1}{\bar{\gamma}_{SR}} \int_0^{\frac{(\bar{\gamma}_{LI}+1)(\gamma_{th}+(9\gamma_{th}^2+4\gamma_{th})^{\frac{1}{2}})}{2}} e^{-\left(\frac{z}{\bar{\gamma}_{SR}}+\frac{\gamma_{th}(2\gamma_{th}+1)(\bar{\gamma}_{LI}+1)^2}{\bar{\gamma}_{DR}z}+\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(2\bar{\gamma}_{SR}+\bar{\gamma}_{DR})}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}\right)} dz \quad (49)$$

From the equation (49) $I_{2,2,1}$ and $I_{2,2,2}$ is represented as,

$$I_{2,2,1} < e^{-\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(\bar{\gamma}_{SR}+2\bar{\gamma}_{DR})}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}} \times \left(1 - e^{-\frac{(\bar{\gamma}_{LI}+1)(-\gamma_{th}+(9\gamma_{th}^2+4\gamma_{th})^{\frac{1}{2}})}{2}}\right) \quad (50)$$

$$I_{2,2,2} < e^{-\frac{\gamma_{th}(\bar{\gamma}_{LI}+1)(2\bar{\gamma}_{SR}+\bar{\gamma}_{DR})}{\bar{\gamma}_{SR}\bar{\gamma}_{DR}}} \times \left(1 - e^{-\frac{(\bar{\gamma}_{LI}+1)(\gamma_{th}+(9\gamma_{th}^2+4\gamma_{th})^{\frac{1}{2}})}{2}}\right) \quad (51)$$

The outage probability of the AF based FD-TWR can be tightly upper bounded by,

Sub $I_{2,1}$ and $I_{2,2}$ in equation (45)

$$P_{out}^{AF} \leq 1 - 2\left(\frac{\gamma_{th}(2\gamma_{th}+1)(\overline{\gamma}_{LI}+1)^2}{\overline{\gamma}_{SR}\overline{\gamma}_{DR}}\right)^{\frac{1}{2}} \times e^{-\frac{\gamma_{th}(\overline{\gamma}_{LI}+1)(2\overline{\gamma}_{SR}+\overline{\gamma}_{DR})}{\overline{\gamma}_{SR}\overline{\gamma}_{DR}}} K_1\left(2\left(\frac{\gamma_{th}(2\gamma_{th}+1)(\overline{\gamma}_{LI}+1)^2}{\overline{\gamma}_{SR}\overline{\gamma}_{DR}}\right)^{\frac{1}{2}}\right) + \left(1 - e^{\frac{(\overline{\gamma}_{LI}+1)(\gamma_{th}+(9\gamma_{th}^2+4\gamma_{th})^{\frac{1}{2}}}{2\overline{\gamma}_{SR}}}\right) - e^{-\frac{\gamma_{th}(\overline{\gamma}_{LI}+1)(\overline{\gamma}_{SR}+2\overline{\gamma}_{DR})}{\overline{\gamma}_{SR}\overline{\gamma}_{DR}}}\left(1 - e^{\frac{(\gamma_{LI}+1)(-\gamma_{th}+(9\gamma_{th}^2+4\gamma_{th})^{\frac{1}{2}}}{2\overline{\gamma}_{SR}}}\right) \quad (52)$$

Where $K_v(.)$ is the modified Bessel function of the second kind.The outage probability of AF based FD-OWR is,

$$P_{out}^{AF,FD-OWR} = 1 - 2\left(\frac{\gamma_{th}(\gamma_{th}+1)(\overline{\gamma}_{RR}+1)}{\overline{\gamma}_{SR}\overline{\gamma}_{DR}}\right)^{\frac{1}{2}} \times e^{-\frac{\gamma_{th}(\overline{\gamma}_{SR}+\overline{\gamma}_{RD}(\overline{\gamma}_{RR}+1)))}{\overline{\gamma}_{SR}\overline{\gamma}_{DR}}} \times K_1\left(2\left(\frac{\gamma_{th}(\gamma_{th}+1)(\overline{\gamma}_{RR}+1)}{\overline{\gamma}_{SR}\overline{\gamma}_{DR}}\right)^{\frac{1}{2}}\right) \quad (53)$$

The outage probability of the AF based FD-TWR is higher than that in FD-OWR. This is because the residual self-interference generated at the destination nodes in FD-TWR deteriorates the SNR of the received signals. This also reveal that time multiplexing can help to improve the average rate, but simultaneously it also leads to a loss in the outage performance.

4. SIMULATION RESULTS

In this section, the performance of the FD-TWR scheme is presented using MATLAB simulations. The average rate and Outage probability of FD-TWR scheme are presented.

In Figure 2 the outage probability of the DF based FD-TWR and FD-OWR with PNC under the outage rate threshold, $R_{th} = 1$ b/s/Hz is shown. In this FD-TWR achieves better performance than the FD-OWR, because the DF based FD-TWR suffers from more severe residual self-interference than FD-OWR. It is also shown that PNC can improve the outage performance of the DF based FD-TWR, because it enables the relay to forward the signals with maximum power without performing power allocation, which improves the quality of the relaying link. In this the loop interference can be varied with respect to 3 dB, 6 dB, and 10 dB.

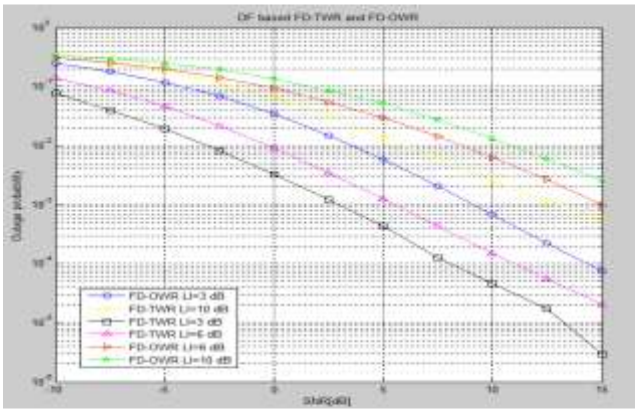

Figure 2. Outage Probability of the DF Based FD-TWR and FD-OWR

Figure 3 compares the outage probability of the AF based FD-TWR and FD-OWR. It is evident that FD-TWR achieves better performance than the FD-OWR with PNC under the outage rate threshold of R_{th} =1 b/s/Hz.

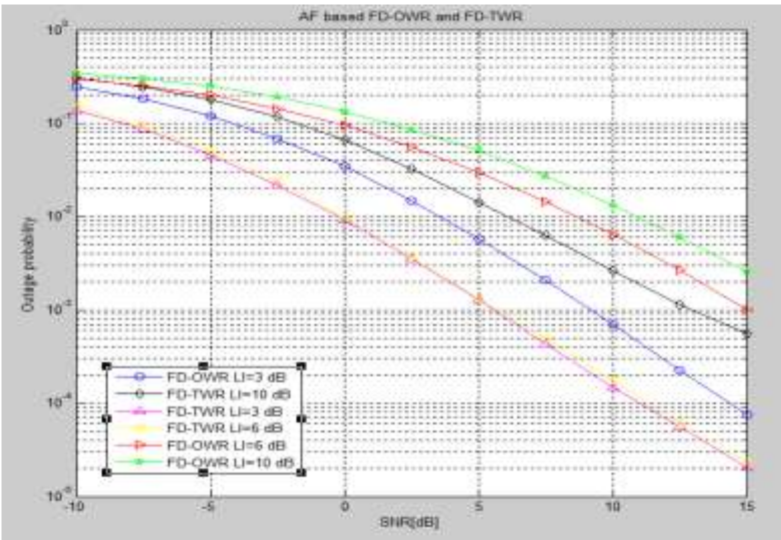

Figure 3 Outage Probability of the AF based FD-TWR and FD-OWR

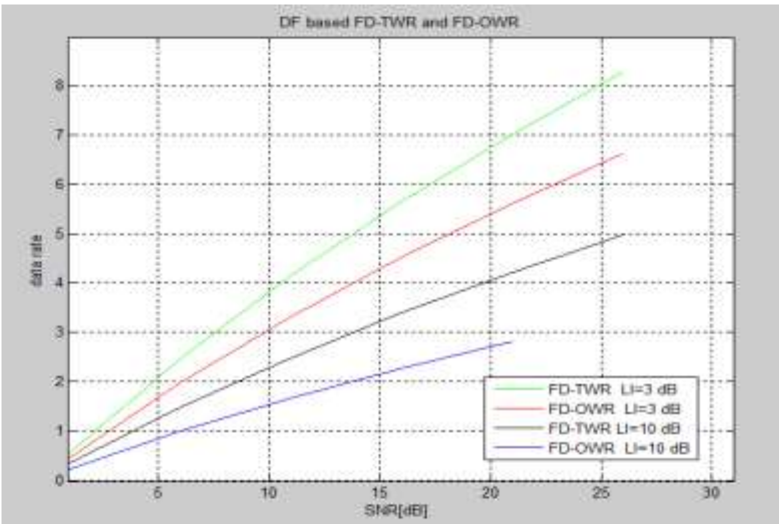

Figure 4 Average rate of the DF based FD-TWR and FD-OWR

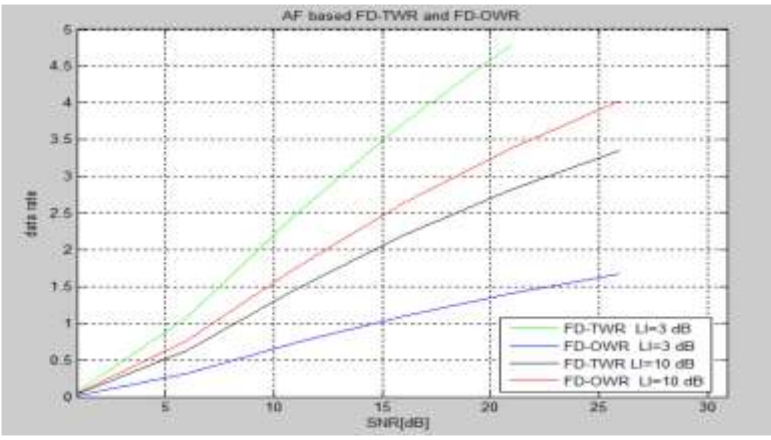

Figure 5 Average rate of the DF based FD-TWR and FD-OWR

Figure. 4 compares the average rate of the DF based FD-TWR and FD-OWR with physical layer network coding. The results show that the DF based FD-TWR can achieve higher rate than FD-OWR. Besides, PNC can improve the rate for the DF based FD-TWR in the low SNR region. In this the loop interference can be varied with respect to LI=3 dB and LI=10 dB. At 10 dB FD-TWR transmits 2.3 b/s and FD-OWR transmits 1.6 b/s. Then at 3 dB FD-TWR transmits 3.8 b/s and FD-OWR transmits 3 b/s. The average rate of the AF based FD-TWR and FD-OWR with physical layer network coding is compared in Figure 5. The DF based FD-TWR can achieve higher rate than FD-OWR. The loop interference is varied with respect to LI=3 dB and LI=10 dB and the performance is evaluated. At 10 dB FD-TWR transmits 1.6 b/s and FD-OWR transmit 0.7 b/s. Then at 3 dB FD-TWR transmits 2.4 b/s and FD-OWR transmits 1.6 b/s.

5. CONCLUSION

The outage probability and average rate of FD-TWR and FD-OWR using a physical layer network coding was analytically derived. The performance evaluation was done for relaying protocols like DF and AF schemes employing Physical network coding. The outage probability of the DF based FD-TWR and FD-OWR with PNC under the outage rate threshold, $R_{th} = 1$ b/s/Hz. In this FD-TWR achieves better performance than the FD-OWR, because the DF based FD-TWR suffers from more severe residual self-interference than FD-OWR. It is also shown that PNC can improve the outage performance of the DF based FD-TWR, because it enables the relay to forward the signals with maximum power without performing power allocation, which improves the quality of the relaying link. The outage probability of the AF based FD-TWR and FD-OWR shows the FD-TWR achieves better performance than the FD-OWR with PNC under the outage rate threshold. The results show that the outage probability of the AF based FD-TWR is higher than that in FD-OWR. The AF based FD-TWR suffers from the residual self-interference not only at the relay but also at the destination nodes, which deteriorates the SNR of the end-to-end link. The average rate of the AF based FD-TWR and FD-OWR with physical layer network coding shows that the DF based FD-TWR can achieve higher rate than FD-OWR.

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