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# On-line Assessment of Voltage Stability using Synchrophasor Technology

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## Abstract

Series of blackouts encountered in recent years in power system have been occurred because either of voltage or angle instability or both together was not detected within time and progressive voltage or angle instability further degraded the system condition, because of increase in loading. This paper presents the real-time assessment methodology of voltage stability using Phasor Measurement Unit (PMU) with observability of load buses only in power network. PMUs are placed at strategically obtained location such that minimum number of PMU's can make all load buses observable. Data obtained by PMU's are used for voltage stability assessment with the help of successive change in the angle of bus voltage with respect to incremental load, which is used as on-line voltage stability predictor (VSP). The real-time voltage phasors obtained by PMU's are used as real time voltage stability indicator. The case study has been carried out on IEEE-14 bus system and IEEE-30 bus systems to demonstrate the results.

Keywords: voltage stability, voltage stability predictor, phasor measurement unit, optimal placement

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## 1. Introduction

The power system is critical interdependent infrastructure and it is necessary to transmit electrical power from generation to load end reliably, keeping generation to load balance at every time, such that its performance should be effective. Also, power system supports the world economy to a great extent. So, the stability and security of electric power network is an important issue for planning and operation engineer [1]. For this purpose, it is important to analyze the system for every instant of time, so use of real-time measurements are necessary. A Synchrophasor based wide area measurement system (WAMS) gives the real-time measurements of voltage and current phasors with the help of PMUs. These PMUs are strategically placed at different locations in the power network and gives the voltage and current phasors of the bus where it is installed. Voltage stability is one of the most important areas for engineers to maintain the operation of power system within contractual and steady voltage limits before and after any disturbances. These disturbances are, may be, sudden loss of generation or lines, or changing loads, which affects the operating point of system and frequency. So, it is necessary to rapidly monitor and adjust to system changes to attain a new operating point or an equilibrium point keeping generation to load balance. This ability of system is the goal of voltage stability assessment and control [2].

Series of blackouts encountered in recent years in power system, have been occurred because either of voltage or angle instability or both together was not detected within time and progressive voltage or angle instability further degraded the system condition, because of increase in loading [3, 4]. So, synchronized phasor measurements are very important for wide area measurement systems used in advanced power system monitoring, protection, and control applications. Phasor measurement unit (PMU) becomes more and more attractive to power engineers because it can provide time synchronized measurements of voltage and currents phasor [5]. Synchronization is achieved by same-time sampling of voltage and current waveforms using timing signals from the Global Positioning System Satellite (GPS). The present and possible future applications of phasor measurement units have been well documented [6]. Several algorithms and approaches have been published in the literature for the optimal placement of PMUs in power system. Initiating work in PMU development and utilization is done by Phadke et al. [5, 7]. In reference [8], a strategic PMU placement algorithm is developed to

(1)

(2)

improve the bad data processing capability of state estimation by taking advantage of PMU technology. Providing selected buses with PMUs can make the entire system observable. The authors in [9, 10] developed an optimal placement algorithm for PMUs by using integer linear programming. Moreover, modeling of zero injection buses has been presented in [10]. In reference [11], a generalized integer linear programming formulation and solution approach for placement of PMUs is proposed. Communication Infrastructure and Islanding based optimal placement of PMUs have been documented in [12-14] and [15, 16] respectively. Voltage stability ranking for OPP has been proposed in [17]. Authors in [18] presented a multiobjective function to optimize the number of PMUs and maximize the observability by using Binary Gravitational Search Algorithm (BGSA). To verify the effectiveness of BGSA, different contingency have been considered in [18]. In this paper, a voltage stability index has been proposed as an application of PMUs. Several voltage stability index have been published in the literature. A static voltage stability index has been proposed in [19] which has been used the synchrophasor technology to early detection of impending voltage instability. In [20], a simple newton raphson algorithm based voltage stability analysis has been presented to identify the behavior of power system under various voltage stability conditions. Authors in [21, 22] discussed the some more index regarding voltage stability.

In this paper, PMUs are placed at strategically obtained location such that minimum number of PMU's can make all load buses observable and provide the on-line voltage phasors to predict the VSP. For this purpose, a formulation of optimal PMU placement problem is done by mixed integer linear programming (MILP). These PMUs provide voltage phasors of all load buses at very small intervals. Thus the rate of change of voltage angle due to change in load with respect to successive intervals of time and this can be used as the VSP to estimate the voltage stability. As the PMU gives real time voltage and current phasors, thus voltage stability predictor can be used for on-line voltage stability prediction. The voltage stability assessment problem using PMU is more efficient and can be used in practice. The main contributions of this paper are as under:

- a) Proposed an algorithm to optimize the number of PMUs and maximize the observability of the load buses of the power system.
- b) Proposed an on-line voltage stability predictor method which monitor the system and prevent the blackouts in power system.

The paper is systemized as follows: Section 2 states the problem formulation of minimum PMU placement and voltage stability prediction. Section 3 explains the solution methodology of proposed method. Finally, the test results are given in Section 4, and Section 5 concludes the paper.

## 2. Problem Formulation of Proposed Method 2.1. Optimal PMU Placement Methodology

 $f = A.Z \ge 1$ 

In this paper, a novel objective function has been proposed to optimize the number of PMUs and provides the maximum observability of the system. The proposed objective function can be written as follows:

Minimize

 $\sum_{i=1}^{n} w_i z_i = \sum_{i=1}^{n} \frac{z_i}{(c_{cl} * NC_i) + c_p}$ 

Subject to:

Connectivity matrix (*A*) defines the interconnection of system buses by transmission lines. The entries in *A* are defined as follows:

$$A_{ij} = \begin{cases} 1 & \text{if } i = j \\ 1 & \text{if } i \text{ and } j \text{ are connected} \\ 0 & \text{otherwise} \end{cases}$$
(3)

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where  $z_i$  is the elements of vector Z, which represents the status of the installation of a PMU at bus i. if  $z_i = 1$ , it means PMU is installed at bus i, otherwise  $z_i = 0$ . p is the total number of PMUs.  $c_{cl}$  and  $c_p$  represent the cost of channel and cost of PMU respectively.  $NC_i$  is the numbers of channel at bus *i*. In reference [11], authors used the multiobjective function to optimize the number of PMUs having maximum observability. In this paper, equation (1) provides both objectives in a single objective function which is the advantage over previous methods. In equation (1),  $w_i$  is the weight factor and element of column vector W, which represents the inverse of the cost of PMU with respect to number of channels (branches) connected to bus i.  $w_i$  is defined as follows:

$$w_i = \frac{1}{(c_{cl} * NC_i) + c_p}$$
(4)

If W is unity vector matrix, it means weights of all the buses are same, and this value of W has been used for without considering maximum observability position.

#### 2.2. Voltage Stability Predictor (VSP)

The voltage stability problem is mostly reactive power related problem. The reactive power deficit in stressed power system, either by incapability of voltage source to inject the required reactive power to the system or by inability of transmission line to transfer the demanded reactive power to the weak buses, causes voltage instability in power system. If during any one of these conditions, any change will occurs in the system, like increase in load, transmission line outage or outage of generation, further degrades the voltage stability of the system and may cause decline in voltage at load bus. The successive degradation in system condition may cause voltage collapse, also. So, it is important to monitor voltage of all load buses.

Let the load at any bus changed and because of this the voltage  $V_{jk}$  at any load bus j at instant k is changed to  $Vj_{(k+1)}$  at  $(k+1)^{th}$  instant. So, the successive change in voltage with respect to a successive time step due to change in load is equal to:

$$S_{j(k+1)} = \frac{V_{j(k+1)} - V_{jk}}{(t_{(k+1)} - t_k)}$$
(5)

where  $S_{j(k+1)}$  is successive change in voltages from instant 'k' to '(k+1)' at bus 'j' with respect to change in time of the system at these instants. When load is further changed at next instant i.e. at instant (k+2) then  $S_{j(k+1)}$  is changed to  $S_{j(k+2)}$  due to change in voltage  $V_j$  at bus j. Numerator,  $(t_{(k+1)} - t_k)$  is the successive time step. The change in successive change in voltage angle with respect to time due to change in load gives the voltage stability predictor (VSP) for assessment of voltage stability of the system. VSP should be calculated at all load buses of the network and is given by,

$$VSP_{j(k+1)} = \frac{\tan^{-1}(S_{j(k+1)})}{-\pi/2}$$
(6)

In equation (6), numerator is in radian and it's maximum value is  $(-\pi/2)$ . Therefore, to find out the VSP within 0 to 1,  $(-\pi/2)$  is used in denominator. Now the value of VSP in equation (6) vary between 0 to 1. The maximum value of VSP at instant (k+1) gives the prediction of voltage stability of the network at that instant.

$$VSP_{(k+1)} = \max\left(VSP_{j(k+1)}\right) \tag{7}$$

Thus, this value of  $VSP_{(k+1)}$  indicates the proximity of voltage collapse. When the value of  $VSP_{(k+1)}$  is closer to one then the system is more nearer to the instability point.

# 3. Solution Methodology

# 3.1. PMU Placement

In order to form the constraint set, the binary connectivity matrix A, whose entries are defined in (3), will be formed first. Matrix A can be directly obtained from the bus admittance matrix by transforming its entries into binary form.



Figure 1. 5-bus system

Consider the 5-bus system and its measurement configuration shown above. Building the A matrix for the 5-bus system of Figure 1 yields:

	0	0	0	0	0
	0	0	0	0	0
A =	0	0	0	0	0
	0	0	0	1	1
	0	0	0	1	1

The constraints for this case can be formed as:

$$f(X) = \begin{cases} x_4 + x_5 & \ge 1\\ x_4 + x_5 & \ge 1 \end{cases}$$
(9)

The use of 1 in the right hand side of the inequality ensures that at least one of the variables appearing in this will be non-zero. The constraint  $f_4 \ge 1$  implies that at least one PMU must be placed at either one of buses 4 or 5 (or both) in order to make bus 4 observable.

## 3.2. Voltage Stability Predictor

For the 5-bus system shown in Figure 1 there are two load buses (bus 4 and bus 5) and three generator buses (bus 1, slack bus, and bus 2 and bus 3). So for the load bus 4 and bus 5, the values of VSP have been examined using equation (10) and (11) as shown below, from Equation (5),

$$S_{4(k+1)} = \frac{V_{4(k+1)} - V_{4k}}{t_{k+1} - t_k}$$
(10)

And, 
$$S_{5(k+1)} = \frac{V_{5(k+1)} - V_{5k}}{t_{k+1} - t_k}$$
 (11)

Equation (10) and (11) represent the rate of change of voltage at bus with respect to time. Now from equation (6), VSP at bus 4 and bus 5 are as follows:

$$VSP_4 = \frac{\tan^{-1}(S_{4(k+1)})}{(-\pi/2)}$$
, and (12)

$$VSP_5 = \frac{\tan^{-1}(S_{5(k+1)})}{(-\pi/2)}$$
(13)

From Equation (7), most critical bus of the 5-bus system has been calculated, which is as follows:

$$VSP = \max\left(VSP_4, VSP_5\right) \tag{14}$$

Maximum value of VSP represents the most critical bus in the system which moves towards the instability of the system.

## 4. Voltage Stability Predictor

The proposed formulation has been tested on IEEE-14 bus and IEEE-30 bus test systems. However, MILP under MATLAB has been used to solve the proposed problem expressed by equations (1) and (2). IEEE 14-bus and IEEE 30-bus systems are shown in Figure 2 & Figure 3. The Information of the IEEE 14-bus and IEEE 30-bus test systems and load buses of respective systems are given in the Table 1. The results for PMU placement have been displayed in Tables 2.

In this paper, voltage stability predictor method discussed in section III is used to check the voltage stability of different load buses. The effect of loading at any load bus is assessed at increasing the load on that bus itself, to its neighboring bus connect to the bus under consideration and to the other weak load buses using  $VSP_k$  predictor. The results are shown in Figure 4 to Figure 10.





Figure 2. IEEE 14-bus test system

Figure 3. IEEE 30-bus test system

Table 1. System information of IEEE bus test systems							
Test System	Number of branches	Number of load buses	Load buses				
IEEE 14-bus	20	9	4,5,7,9,10,11,12,13,14				
IEEE 30-bus	41	24	3,4,6,7,9,10,12,14,15,16,17,18,19, 20,21,22,23, 24,25,26,27,28,29,30				

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Table 2. Simulation results for the test systems									
System	Number of PMUs	Location of PMUs							
IEEE 14-bus	3	4, 10,13							
IEEE 30-bus	8	4,6,10,12,18,24,25,27							

Figure 4 shows the voltage variation at some of the critical buses of IEEE 14-bus due to increase in load (active ad reactive power) at bus 11. As shown in Figure 5, value of VSP is varying from a value nearer to zero to one. The voltage of bus 11 is approaching to the collapse point, due to continuous increase in load at bus 11, the value of predictor VSP changes from 0 to 0.96. Variation in predictor VSP is slow in beginning as comparison with the variation in predictor nearer to the collapse point. Figure 6 shows the closer view of VSP of Figure 5. Again Figure 7 shows the voltage variation at some critical buses of IEEE 14-bus system due to increase in loads at all the load buses. It is clear from Figure 7 that the rate of change of voltage is maximum at bus 5. Therefore value of VSP at bus 5 is approaches nearer to 1 which is shown in Figure 8 and Figure 9 shows the closer variation of VSP to better understanding the variation at some of the critical buses.







Figure 6. Closer view of VSP of Figure 5



Figure 5. VSP plot for load increase at bus 11



Figure 7. Critical bus voltages due to increase loads at all the load buses



Figure 8. VSP plot for load increase at all the buses

Bus 19, -14, -18 and -20 for IEEE 30-bus test system

Time (in sec)



Time (in sec)

The variation of voltage of bus 19 with the increase in loading of bus 19, in IEEE-30 bus system, is shown in Figure 10. Moreover, the variation in values of VSP of buses -14, -18, -19 and -20 are also shown in the Figure 10. The change in the value of VSP is fastest for bus 19 and respectively lower for other buses as bus 20, bus 18 and bus 14 as the loading of bus 19 increases. It shows that the neighboring buses, like bus 20 or bus 18, of bus 19 is much more affected than the far buses as bus 14 by increase in loading of bus 19. Figure 11 shows the closer variation in voltage and VSP as shown in Figure 10.



Figure 11. Closer view of Figure 10

2 48F

2 488

# 5. Conclusion

This paper proposed a simple algorithm of optimal placement of PMUs in power system to provide the maximum observability of load buses of the network for on-line voltage stability assessment. The OPP problem is formulated using topology based algorithm and solved using mixed integer linear programming. The voltage stability assessment problem is formulated as VSP and solved using MATLAB programming. Simulation results on IEEE 14-bus and IEEE 30bus test systems indicate that the proposed placement method with voltage stability assessment is satisfactorily provides observable system measurements with minimum number of PMUs and, also, the index to determine the stability of power network.

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