

Passivity Based Stability Condition for Interfered Digital Filters

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Abstract

This paper presents a new passivity condition for fixed-point state-space interfered digital filters using saturation arithmetic. Passivity is a way to characterize input-output stability of a system, that is, supplied bounded input energy produces bounded output energy. The presented criterion also ensures asymptotic stability of the state-space digital filter with zero external disturbances. The result is expressed in terms of linear matrix inequality, and therefore, can be solved via existing numerical packages. A numerical example is arranged to validate the usefulness of the proposed method.

Keywords: Interfered Digital filter; Saturation arithmetic; Nonlinear effect; Nonlinear system; Passivity

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1. Introduction

The implementation of digital filters on a finite wordlength fixed point processor unavoidably has nonlinear effects because of quantization/overflow. Due to the existence of these nonlinearities, implemented digital filter may show unstable behavior [1, 2]. Therefore, many researchers have devoted significant efforts to study the stability properties of the state-space digital filters with nonlinearities [3-5] and references therein.

The implementation of digital filter of high-order in hardware can be achieved by cascading several biquad filters. External or mutual interference is unavoidable in high-order filter implementation. Such interferences may result in poor performance and malfunctioning of the implemented filters [6, 7]. Thus, analysis of the effects of the presence of external disturbances in digital filter is practical and significant problem.

In recent times, many results have been appeared for the stability analysis of interfered digital filters via saturation arithmetic [8-11]. Criterion for limit cycle free state-space interfered digital filters using saturation arithmetic via idea of H_∞ has been formulated in [8] Refs [9-11] present results for induced L_∞ stability of digital filters via saturation nonlinearities and external disturbance. The result reported in [12] discusses with problem of L_∞ performance analysis for time-varying delayed neural networks. The concept of $L_2 - L_\infty$ have been widely used in various applications such as discrete-time switched neural networks [13], Takagi-Sugeno fuzzy neural networks [14], FIR filters [15] etc. The study of digital filters in the existence of nonlinearities and external disturbance is significant and appealing problem.

Passivity is an important concept for analysis and design of controller based on input-output data for many complex systems [16, 17]. Passivity is closely associated to stability that is, if supplied energy to the system is bounded then it produces bounded output energy. Passivity analysis problem has attracted attention of researchers and several results have been reported in [16-21]. To deal with time-varying delays, passivity conditions for uncertain system have been established in [18]. Based on the concept of passivity a robust stability and passivity conditions have been established in [19]. Passivity is exploited to study hybrid systems in piecewise polynomial form in [20] and for stochastic neural networks with mixed time delays in [21]. However, no stability results are available for digital filters with nonlinearities and outward disturbances in the context of passivity.

Motivated by the above discussions, we present a criterion for very strict passivity in state-space digital filters via saturation arithmetic and external disturbance. The proposed criterion is established in linear matrix inequality (LMI) settings and therefore, one can solve the conditions by available numerical packages such as MATLAB LMI Toolbox. The rest of the paper is structured as follows. Section 2 gives the notation used and states the system under investigation. A new condition for very strict passivity of fixed-point state-space interfered digital filters with saturation arithmetic is brought out in Section 3. In Section 4, a numerical example is given to show the usefulness of the proposed result, and finally, conclusions are outlined in Section 5.

Throughout this paper, the superscript T means the transpose of a matrix; \mathcal{R}^k and $\mathcal{R}^{k \times k}$ denote the k -dimensional Euclidean space and the set of $k \times k$ real matrices, respectively; $G > \mathbf{0}$ ($G < \mathbf{0}$) indicates that G is positive (negative) definite symmetric matrix. The null matrix or null vector is denoted as $\mathbf{0}$. The notation used to represent identity matrix of appropriate dimension is I . Any vector or matrix norm is indicated by $\|\cdot\|$.

2. Problem Formulation

Consider the nonlinear system with external disturbance described by

$$\begin{aligned} \xi(q+1) &= \chi(\mathbf{v}(q)) + \boldsymbol{\omega}(q) \\ &= [\chi_1(v_1(q)) \ \chi_2(v_2(q)) \ \cdots \ \chi_n(v_n(q))]^T \\ &\quad + [\varpi_1(q) \ \varpi_2(q) \ \cdots \ \varpi_n(q)]^T \end{aligned} \quad (1a)$$

$$\mathbf{v}(q) = [v_1(q) \ v_2(q) \ \cdots \ v_n(q)]^T = A\xi(q) + \boldsymbol{\omega}(q) \quad (1b)$$

where $\xi(q) \in \mathcal{R}^k$ is the state vector, $\mathbf{v}(q) \in \mathcal{R}^k$ denotes the output vector, $\boldsymbol{\omega}(q) \in \mathcal{R}^k$ represents the external interference, and $A \in \mathcal{R}^{k \times k}$ is the known coefficient matrix. The normalized and symmetric saturation nonlinearities under considered in this paper are given by

$$\chi_i(v_i(q)) = \begin{cases} 1, & v_i(q) > 1, \\ v_i(q), & -1 \leq v_i(q) \leq 1, \ i = 1, 2, \dots, k \\ -1, & v_i(q) < -1, \end{cases} \quad (2)$$

where $\chi_i(v_i(q))$ is limited to sector $[0, 1]$ i.e.

$$\chi_i(0) = 0, \quad 0 \leq \frac{\chi_i(v_i(q))}{v_i(q)} \leq 1, \quad i = 1, 2, \dots, k \quad (3)$$

The motive of this paper is to design a filter (1) in order to satisfy the following conditions:

- 1) The system is said to be exponentially stable with $\boldsymbol{\omega}(q) = \mathbf{0}$.
- 2) A very strict passivity condition for all nonzero $\boldsymbol{\omega}(q)$, under the zero initial condition such that

$$\sum_{s=0}^q \boldsymbol{\omega}^T(s) \mathbf{v}(s) - \delta_0 \sum_{s=0}^q \mathbf{v}^T(s) \mathbf{v}(s) - \rho \sum_{s=0}^q \boldsymbol{\omega}^T(s) \boldsymbol{\omega}(s) + \beta \geq \sum_{s=0}^q \Omega(\xi(s)) \quad \forall q \geq 0 \quad (4)$$

where β , δ_0 , ρ are nonnegative constants and $\Omega(\xi(s)) \geq 0$ is storage function.

3. Novel Passivity Results

The following theorem establishes a sufficient passivity criterion for system (1)-(2).

Theorem 1. *The digital filter (1)-(2) is very strict passive from the external disturbance $\boldsymbol{w}(q)$ to the output $\boldsymbol{v}(q)$, if there exist matrices $\boldsymbol{P} = \boldsymbol{P}^T > \mathbf{0}$, $\boldsymbol{U} = \boldsymbol{U}^T > \mathbf{0}$, $\boldsymbol{N} = \text{diag}(n_1, n_2, \dots, n_n) > \mathbf{0}$ and scalars $\delta > 0$, $\delta_0 > 0$ and $\rho > 0$ satisfying:*

$$\boldsymbol{\Gamma} = \begin{bmatrix} (\delta + \delta_0)\boldsymbol{A}^T\boldsymbol{A} - \boldsymbol{U} - \boldsymbol{P} & \boldsymbol{A}^T\boldsymbol{N} & \left(\delta + \delta_0 - \frac{1}{2}\right)\boldsymbol{A}^T \\ \boldsymbol{N}\boldsymbol{A} & \boldsymbol{P} - \delta\boldsymbol{I} - 2\boldsymbol{N} & \boldsymbol{P} + \boldsymbol{N} \\ \left(\delta + \delta_0 - \frac{1}{2}\right)\boldsymbol{A} & \boldsymbol{P} + \boldsymbol{N} & \boldsymbol{P} + (\delta_0 + \delta + \rho - 1)\boldsymbol{I} \end{bmatrix} < \mathbf{0} \quad (5)$$

Proof Consider a Lyapunov function:

$$\boldsymbol{A}(\boldsymbol{\xi}(q)) = \boldsymbol{\xi}^T(q) \boldsymbol{P} \boldsymbol{\xi}(q) \quad (6)$$

Application of (6) to (1a) provides,

$$\begin{aligned} \Delta\boldsymbol{A}(\boldsymbol{\xi}(q)) &= \boldsymbol{A}(\boldsymbol{\xi}(q+1)) - \boldsymbol{A}(\boldsymbol{\xi}(q)) \\ &= [\boldsymbol{\chi}(\boldsymbol{v}(q)) + \boldsymbol{w}(q)]^T \boldsymbol{P} [\boldsymbol{\chi}(\boldsymbol{v}(q)) + \boldsymbol{w}(q)] - \boldsymbol{\xi}^T(q) \boldsymbol{P} \boldsymbol{\xi}(q) \\ &= \boldsymbol{\chi}^T(\boldsymbol{v}(q)) \boldsymbol{P} \boldsymbol{\chi}(\boldsymbol{v}(q)) + \boldsymbol{\chi}^T(\boldsymbol{v}(q)) \boldsymbol{P} \boldsymbol{w}(q) + \boldsymbol{w}^T(q) \boldsymbol{P} \boldsymbol{\chi}(\boldsymbol{v}(q)) \\ &\quad + \boldsymbol{w}^T(q) \boldsymbol{P} \boldsymbol{w}(q) - \boldsymbol{\xi}^T(q) \boldsymbol{P} \boldsymbol{\xi}(q) \end{aligned} \quad (7)$$

For a positive scalar δ , the following is true by (2)

$$\delta[\boldsymbol{v}^T(q)\boldsymbol{v}(q) - \boldsymbol{\chi}^T(\boldsymbol{v}(q))\boldsymbol{\chi}(\boldsymbol{v}(q))] \geq 0 \quad (8)$$

which also implies:

$$\delta[\boldsymbol{\xi}^T(q)\boldsymbol{A}^T\boldsymbol{A}\boldsymbol{\xi}(q) + \boldsymbol{w}^T(q)\boldsymbol{A}\boldsymbol{\xi}(q) + \boldsymbol{\xi}^T(q)\boldsymbol{A}^T\boldsymbol{w}(q) + \boldsymbol{w}^T(q)\boldsymbol{w}(q) - \boldsymbol{\chi}^T(\boldsymbol{v}(q))\boldsymbol{\chi}(\boldsymbol{v}(q))] \geq 0 \quad (9)$$

By means of (9) and adding and subtracting $[\boldsymbol{w}^T(q)\boldsymbol{v}(q) - \delta_0\boldsymbol{v}^T(q)\boldsymbol{v}(q) - \rho\boldsymbol{w}^T(q)\boldsymbol{w}(q)]$, Equation (7) yields:

$$\begin{aligned} \Delta\boldsymbol{A}(\boldsymbol{x}(q)) &\leq \boldsymbol{\chi}^T(\boldsymbol{v}(q)) \boldsymbol{P} \boldsymbol{\chi}(\boldsymbol{v}(q)) + \boldsymbol{\chi}^T(\boldsymbol{v}(q)) \boldsymbol{P} \boldsymbol{w}(q) + \boldsymbol{w}^T(q) \boldsymbol{P} \boldsymbol{\chi}(\boldsymbol{v}(q)) + \boldsymbol{w}^T(q) \boldsymbol{P} \boldsymbol{w}(q) \\ &\quad - \boldsymbol{\xi}^T(q) \boldsymbol{P} \boldsymbol{\xi}(q) - \boldsymbol{w}^T(q)\boldsymbol{A}\boldsymbol{\xi}(q) - \boldsymbol{w}^T(q)\boldsymbol{w}(q) + \delta_0\boldsymbol{\xi}^T(q)\boldsymbol{A}^T\boldsymbol{A}\boldsymbol{\xi}(q) + \delta_0\boldsymbol{w}^T(q)\boldsymbol{A}\boldsymbol{\xi}(q) \\ &\quad + \delta_0\boldsymbol{\xi}^T(q)\boldsymbol{A}^T\boldsymbol{w}(q) + \delta_0\boldsymbol{w}^T(q)\boldsymbol{w}(q) + \rho\boldsymbol{w}^T(q)\boldsymbol{w}(q) + \delta\boldsymbol{\xi}^T(q)\boldsymbol{A}^T\boldsymbol{A}\boldsymbol{\xi}(q) + \delta\boldsymbol{w}^T(q)\boldsymbol{A}\boldsymbol{\xi}(q) \\ &\quad + \delta\boldsymbol{\xi}^T(q)\boldsymbol{A}^T\boldsymbol{w}(q) + \delta\boldsymbol{w}^T(q)\boldsymbol{w}(q) - \delta\boldsymbol{\chi}^T(\boldsymbol{v}(q))\boldsymbol{\chi}(\boldsymbol{v}(q)) + \boldsymbol{w}^T(q)\boldsymbol{v}(q) - \delta_0\boldsymbol{v}^T(q)\boldsymbol{v}(q) \\ &\quad - \rho\boldsymbol{w}^T(q)\boldsymbol{w}(q) + \phi(q) - \phi(q) \\ &= \boldsymbol{\mu}^T(q)\boldsymbol{\Gamma}\boldsymbol{\mu}(q) + \boldsymbol{\xi}^T(q)\boldsymbol{U}\boldsymbol{\xi}(q) + \boldsymbol{w}^T(q)\boldsymbol{v}(q) - \delta_0\boldsymbol{v}^T(q)\boldsymbol{v}(q) - \rho\boldsymbol{w}^T(q)\boldsymbol{w}(q) - \phi(q) \end{aligned} \quad (10)$$

Where,

$$\boldsymbol{\mu}(q) = [\boldsymbol{\xi}^T(q) \quad \boldsymbol{\chi}^T(\boldsymbol{v}(q)) \quad \boldsymbol{w}^T(q)]^T \quad (11)$$

And,

$$\phi(q) = 2\boldsymbol{\chi}^T(\boldsymbol{v}(q))\boldsymbol{N}[\boldsymbol{v}(q) - \boldsymbol{\chi}(\boldsymbol{v}(q))] \quad (12)$$

which is nonnegative by considering (2).
When $\Gamma < \mathbf{0}$, Equation (10) satisfies

$$\Delta \mathbf{A}(\mathbf{x}(q)) < \xi^T(q) \mathbf{U} \xi(q) + \varpi^T(q) \mathbf{v}(q) - \delta_0 \mathbf{v}^T(q) \mathbf{v}(q) - \rho \varpi^T(q) \varpi(q) \quad (13)$$

Summation of both sides of (13) from 0 to q leads to

$$\begin{aligned} \sum_{s=0}^q \xi^T(s) \mathbf{U} \xi(s) + \sum_{s=0}^q \varpi^T(s) \mathbf{v}(s) - \delta_0 \sum_{s=0}^q \mathbf{v}^T(s) \mathbf{v}(s) - \rho \sum_{s=0}^q \varpi^T(s) \varpi(s) &> \sum_{s=0}^q \Delta \mathbf{A}(\xi(s)) \\ &= \mathbf{A}(\xi(q+1)) - \mathbf{A}(\xi(0)) \geq 0 \end{aligned} \quad (14)$$

which can be written as

$$\sum_{s=0}^q \varpi^T(s) \mathbf{v}(s) - \delta_0 \sum_{s=0}^q \mathbf{v}^T(s) \mathbf{v}(s) - \rho \sum_{s=0}^q \varpi^T(s) \varpi(s) + \beta \geq - \sum_{s=0}^q \xi^T(s) \mathbf{U} \xi(s) + \mathbf{A}(\xi(q)) \quad (15)$$

where $\beta = \mathbf{A}(\xi(0))$ and $\mathbf{A}(\xi(q)) \geq 0$ under the zero initial condition that assures (4). This completes the proof of Theorem 1.

Now, by choosing $\varpi(q) = \mathbf{0}$ one can achieve the following result.

Corollary 1. *The system (1)-(2) is said to be very strict passive for $\varpi(q) \neq \mathbf{0}$ and asymptotically stable when $\varpi(q) = \mathbf{0}$, if there exist $k \times k$ matrices $\mathbf{P} = \mathbf{P}^T > \mathbf{0}$, $\mathbf{U} = \mathbf{U}^T > \mathbf{0}$, $\mathbf{N} = \text{diag}(n_1, n_2, \dots, n_n) > 0$ and scalars $\delta > 0$, $\delta_0 > 0$ and $\rho > 0$ satisfying*

$$\begin{bmatrix} (\delta + \delta_0) \mathbf{A}^T \mathbf{A} - \mathbf{U} - \mathbf{P} & \mathbf{A}^T \mathbf{N} & \left(\delta + \delta_0 - \frac{1}{2} \right) \mathbf{A}^T \\ \mathbf{N} \mathbf{A} & \mathbf{P} - \delta \mathbf{I} - 2 \mathbf{N} & \mathbf{P} + \mathbf{N} \\ \left(\delta + \delta_0 - \frac{1}{2} \right) \mathbf{A} & \mathbf{P} + \mathbf{N} & \mathbf{P} + (\delta_0 + \delta + \rho - 1) \mathbf{I} \end{bmatrix} < \mathbf{0} \quad (16)$$

$$-\mathbf{U} + \delta_0 \mathbf{A}^T \mathbf{A} > \mathbf{0} \quad (17)$$

Proof For a digital filter given by (1) with $\varpi(q) = \mathbf{0}$, the forward difference of the Lyapunov function satisfies,

$$\begin{aligned} \Delta \mathbf{A}(\xi(q)) &< \xi^T(q) \mathbf{U} \xi(q) - \delta_0 \xi^T(q) \mathbf{A}^T \mathbf{A} \xi(q) \\ &= -\xi^T(q) [-\mathbf{U} + \delta_0 \mathbf{A}^T \mathbf{A}] \xi(q) \leq 0 \end{aligned} \quad (18)$$

which leads to:

$$\lim_{q \rightarrow \infty} \xi(q) = \mathbf{0} \quad (19)$$

The condition (19) confirms that the system (1)-(2) is asymptotically stable when $\varpi(q) = \mathbf{0}$. This completes the proof of Corollary 1.

Remark 1. Corollary 1 is in LMI settings, thus one may verify that by using MATLAB LMI toolbox [22, 23]

Remark 2. The system (1)-(2) for the objective (4) is said to be

- (i) passive when $\delta_0 = \rho = 0$,
- (ii) input strict passive when $\delta_0 = 0$ and $\rho > 0$,
- (iii) output strict passive when $\delta_0 > 0$ and $\rho = 0$.

Thus, passive, input strict passive and output strict passive are the special cases of very strict passive and conditions for them can be obtained from Theorem 1 as special cases.

Remark 3. Digital filters are most conveniently realized on a digital signal processor which may be a general purpose digital computer or a specially designed hardware. The realized digital filter is expected not to show any zero input limit cycle and delivers acceptable performance. The result reported in Theorem 1 is formulated by considering saturation arithmetic and external disturbance. One may find it interesting to extend the proposed idea for any combination of quantization/overflow nonlinearities and state-delays. Further, the proposed approach can also be extended for two-dimensional [26] systems which appears to be an appealing future work.

4. Numerical Results

To test our method and to validate the effectiveness of the proposed problem resolution, consider the system (1)-(2) with

$$A = \begin{bmatrix} 0.35 & -0.5 \\ 0.4 & 0 \end{bmatrix} \quad (20)$$

and external interference $\varpi(q)$ is

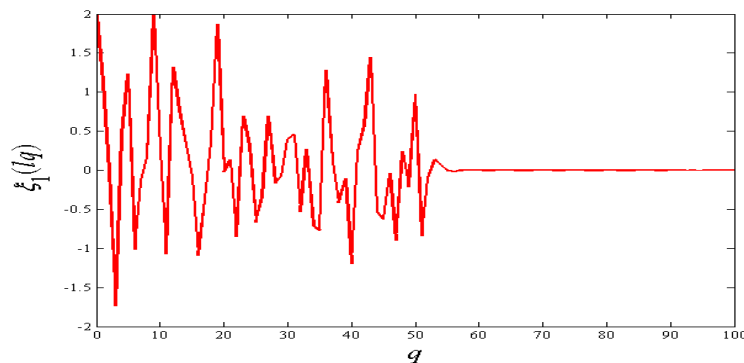
$$\varpi(q) = \begin{cases} [0.2\eta_1(q) & 0.25\eta_2(q)]^T, & 0 \leq q \leq 50, \\ [0 & 0]^T, & \text{otherwise,} \end{cases} \quad (21)$$

where $\eta_1(q)$ and $\eta_2(q)$ are white Gaussian sequences with $\mu=0$ and $\sigma=1$, where μ represents mean and σ shows the variance. By means of Corollary 1, a feasible solution to the LMIs (16) and (17) using MATLAB LMI toolbox [22, 23] is given by

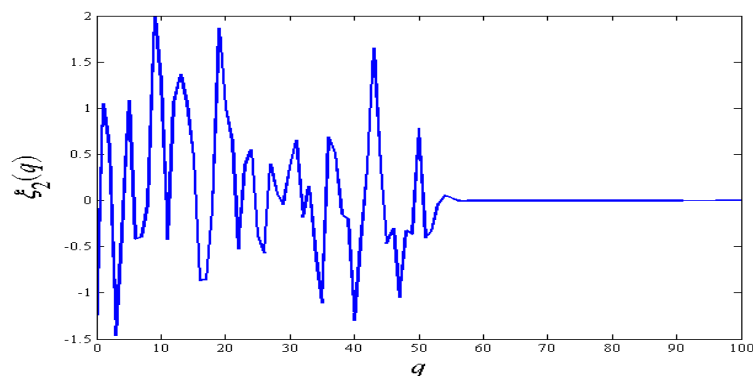
$$P = \begin{bmatrix} 0.1372 & -0.0136 \\ -0.0136 & 0.1334 \end{bmatrix}, \quad U = \begin{bmatrix} 0.0405 & -0.0330 \\ -0.0330 & 0.0339 \end{bmatrix} \\ N = \begin{bmatrix} 0.2252 & 0 \\ 0 & 0.2867 \end{bmatrix}, \quad \delta = 0.0992, \quad \delta_0 = 0.2107, \quad \rho = 0.0816 \quad (22)$$

Therefore, Corollary 1 confirms the digital filter is very strictly passive in the existence of external disturbance and asymptotically stable when external disturbance vanishes.

Figure 1 and 2 show the state trajectories for the states $\xi_1(q)$ and $\xi_2(q)$ for the initial condition $\xi(0)=[2 \ -1.5]^T$. This figure clears that $\xi(q)$ is restricted around the origin where $\varpi(q) \neq \mathbf{0}$ and converges to zero with $\varpi(q) = \mathbf{0}$.



The plot of $\xi_1(q)$

The plot of $\xi_2(q)$

5. Conclusion

A sufficient condition for very strict passivity for state-space interfered digital filters with saturation nonlinearities has been established. In the scenario of non existence of external disturbance, condition to confirm asymptotic stability of realized digital filter has been made available. The proposed result is computationally efficient and a mathematical example is provided to validate the given result.

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