Power System State Estimation by Novel Approach of Kalman Filter

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Abstract
The electrical network measurements by measuring device Phasor Measurement Device (PMU) are usually sent to the control centers using data acquisition system and other communication protocols available. However, these measurements contain uncertainties due to the measurements and communication noise (errors), incomplete metering or unavailability of some of measurements. The overall aim of state estimation is to calculate the state variables of the power system by minimizing errors available at the control center. Due to generate desired quantities by optimal estimate which is given the set of measurements, Kalman filters are widely used. This paper discusses the application of an Extended Kalman Filter (EKF) algorithm, the Unscented Kalman Filter (UKF) algorithm, and New EKF+M and UKF+M estimator algorithm,those are modification of EKF and UKF for enhance accuracy and elapse time is less. The effectiveness and performance of EKF+M and UKF+M Estimator over another Filtration algorithm is shown. These state estimation techniques applied on IEEE-30 bus, 14 bus and 9 bus test system.

Keywords: power system state estimation (PSSE), dynamic state estimation (DSE), extended kalman filter (EKF), unscented kalman filter (UKF)

1. Introduction

Due to the load variations, increment of load demand the power system is on rapid growth and so there is wide increment in complexity so power system is dynamic system. It makes the monitoring and control of power systems a very complex and significant issue. The Energy Management Systems (EMS) at the control centers is responsible for this task of monitoring, quality enhancement and control of the power system. The state estimator provides an optimum real time data of the system state based on the available measurements on the assumed system model. The resulting dynamic changes need to be monitored continuously and therefore the power system state estimation needs to be performed in short interval of time. The efficiency and accuracy of the state estimator output is very crucial as it forms the basis for the EMS functions. Thus, the concept of state estimation plays a major role in ensuring the secure and economic operation of the power systems in large-scale interconnected power grids. Depending on the desired states, power system state estimation can be formulated as a static or dynamic estimation problem.

Traditional state estimators, which are based on steady state system model, cannot capture the system dynamics very well due to the less updating rate of SCADA systems. Phasor Measurement Unit (PMU)-based Systems (WAMS) introduced in 1980 [1-3]. The introduction of this more accurate and high speed measurement system, featured with synchronous sampling, has overturned the way state estimation process is being performed. This generate the cause of development of Dynamic State Estimation (DSE) proficiencies.The introduction of highly accurate angular measurement data by means of these high updating rate synchronized measurement devices play an important role in the modern day energy management systems [4]-[9]. The state estimators are used both voltage and current measurements together or separately for estimating system states [10-12].

Power system Dynamic State Estimation has been implemented by different Kalman filters with modifications. The most plebeian application of the Kalman filter (KF) [9] to nonlinear systems is in the form of Extended Kalman filter (EKF) [10-11], which linearizes all nonlinear transformations and substitutes Jacobian matrices for the linear transformations in KF

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equations. Power system DSE has been implemented by EKF [12, 13]. EKF works well only in a 'mild' nonlinear environment due to the first-order Taylor series approximation for nonlinear functions [14]. The linearization can be applied only if the Jacobian matrix exists.

Even if the Jocobian matrix exists, calculating it can be a difficult and error-prone. The unscented transformation (UT) [15] was developed to address the deficiencies of linearization by providing a more direct and denotative mechanism for transforming mean and covariance information. Based on UT, the unscented Kalman filter (UKF) as a derivative free alternative to EKF has been proposed in the framework of state estimation [16–18]. The UKF has been applied to power system DSE, for which no linearization or calculation of Jacobian matrices is needed [19, 20]. A linear UKF was developed for solve linear problems [21].

Therefore, even if classic UKF has good performance only in small systems. When the estimation error covariance is propagated, it sometimes cannot maintain the positive semi definiteness, thus making its square-root unable to be calculated. Secondly, its calculating time and estimation error is less but not perfect. So here we proposed new Kalman filter which elapsed time is less and more accurate.Algorithm robustness can increase on the basis of an adaptive weight assignment function [22]. For Enhance Numerical stability, a algorithm proposed by Junjian Qi in 2016 and presented a new toolbox, UKF-GPS [23].

2. State Estimation

The state estimation process for a power network is determining the best estimate of the present state of the system. The vector consisting of bus voltage magnitudes and bus voltage phase angles is called the static vector of an electric power system. The real time measurement data gathered from the network and used in the estimation process includes power injections, power flows on the transmission lines and voltage magnitudes at each bus of the system. Figure 1 shows the basic functioning of the power system for calculating best estimated states.
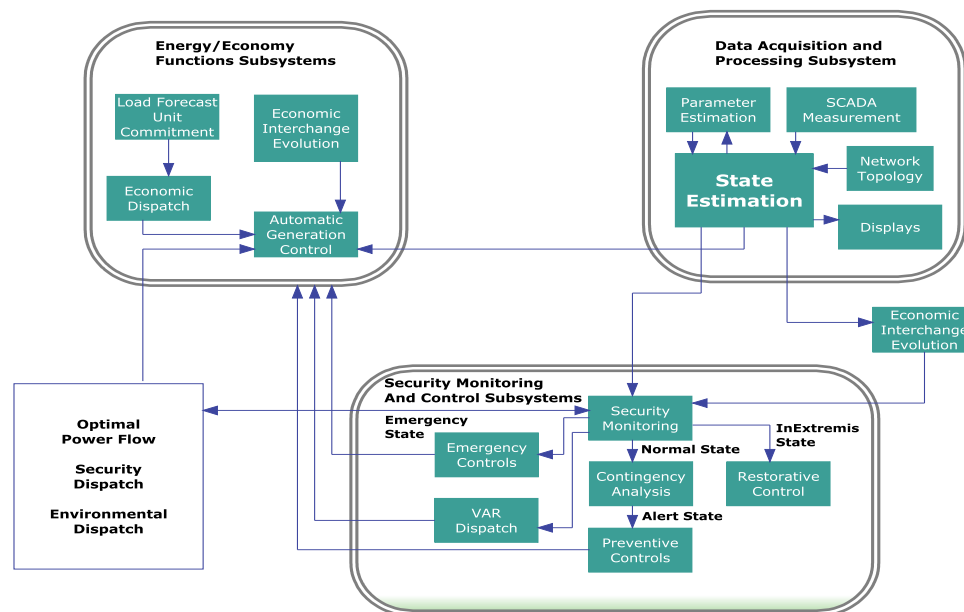

Figure 1. Basic Function of Power System State Estimation

2.1. Static State Estimation

The SSE uses only voltage magnitude real and reactive power flow injections and SCADA measurements. If the state vector is obtained for the current instant of time k from the set of measurement data received at the same instant of time k, then such an estimation method is called as Static State Estimation (SSE).

In static state estimation, the snap-shot of the measurements are taken, processed and the estimate of the state vector variables is obtained at the same point of time.

2.2. Dynamic State Estimation

Dynamic State Estimation (DSE)monitor the continuous dynamic changes in power systems. By using the actual physical modeling of the time varying nature of the power system, DSE algorithm predicts the system state at the next instant of time k + 1 along with the state estimates obtained at the previous instant of time k. The forecasting ability of the DSE algorithm plays a major role in the improvement of the overall energy management system operation and control.

DSE has mainly two objectives:
1. Prediction the state of power system at the next time period and
2. State Estimation based on both sets of measured and predicted data from PMU.

The main feature provides the additional time to the power system operator for making control decisions and analyzing the security of operating system. The second feature would significantly improve the performance and results of DSE.

The basic framework of Estimation Theory consists of:
1. Modeling the system, measurement and all the noise characteristics.
2. A criterion to match or mix the model output with the measurements.
3. A numerical algorithm for the above task and consequently obtain the estimates and the uncertainty of the estimated quantities and
4. An internal consistency checks to ensure that all the above steps are consistent and if not shows the need for modification.

The first aspect requires some 'a priori' knowledge about the system under investigation. The second needs the matching of model output with the measurements and it can be based either on deterministic or probabilistic criterion. The third step is the selection of a numerical algorithm to satisfy the above criterion. The last one is the process of model validation.

The above four aspects help to evolve a suitable mathematical model of the system by properly estimating the unknown parameters and other noise characteristics based on the input and measurement data. Subsequently such a model can be utilized in further studies like prediction, control and system optimization [24-28].

3. Kalman Filtering

The Kalman filter is the most widely used Bayesian-based method. It was named after Rudolf Kalman, who published his famous recursive method to estimate dynamic states [29]. The Kalman filter is that it matches the model and the measurement and in the process improves both by suppressing the noise in the measurement improves the accuracy of the state and the parameters in it. There could be many ways or criteria of combining the model and the measurements. Each one could give different results but the criterion to accept any result is that the estimates should be meaningful, reasonable, acceptable and useable.

3.1. Problem Formulation and Extended Kalman Filter Equations

One source of information is the state differential equations and the other source of information that captures the above change is the measurements made on the system. The correction to the state is provided by the measurements based on a proper criterion leading to reduced uncertainty of the state variables. Such a criterion is provided by a probabilistic weighted linear addition of the predicted state and the actual measurement data.

Consider the following nonlinear filtering problem defined for discrete time instants given by k = 1, 2..N

$$x_k = f\left(x_{k-1}, \Theta,\ u_{k-1}\right) + w_k \tag{1}$$

$$x_k = f\left(x_{k-1}, \Theta,\ u_{k-1}\right) + w_k \tag{2}$$

where 'x' is the state vector of size n × 1, 'u' is the control input and 'Z' is the measurement vector of size m × 1. The 'f' and 'h' are nonlinear functions of state and measurement equations respectively. The injected process noise, $w_k \square N$ (0, Q) and the

injected measurement noise, $v_k \; \square \; R\,(0,\ R)$ are assumed to be zero mean Additive White Gaussian Noise (AWGN), and are identically and independently distributed. The 'N' represents Normal or Gaussian distribution and

$$E\left[w_k w_j^T\right] = Q\delta(k-j)\ \&\ E\left[w_k\right] = 0 \tag{3}$$

$$E\left[v_k v_j^T\right] = R\delta(k-j)\ \&\ E\left[v_k\right] = 0 \tag{4}$$

$$E\left[w_k v_j^T\right] = 0\ \forall j,k = 1,2,\dots N \tag{5}$$

Where N is the total number of sampling instants. E [ ] is the expectation operator, δ is Kronecker delta function defined as

$$\delta(k-j) = \begin{cases} 0 & if\ k \neq j; \\ 1 & if\ k = j; \end{cases} \tag{6}$$

The parameter vector 'Θ' of size p×1 is augmented as additional states,

$$\begin{bmatrix} x_k \\ \Theta_k \end{bmatrix} = \begin{bmatrix} f\left(x_{k-1},\ \Theta_{k-1}, u_{k-1}\right) \\ \Theta_{k-1} \end{bmatrix} + \begin{bmatrix} w_k \\ 0 \end{bmatrix} \tag{7}$$

The nonlinear filtering problem is now defined as

$$X_k = f\left(X_{k-1}\right) + w_k \tag{8}$$

$$Z_k = h\left(X_k\right) + v_k \tag{9}$$

where 'X' and 'w' are respectively the augmented state and process noise vector is of size (n + p) × 1 and thus wk∼ N.

The estimation in the Kalman filter is known as adaptive filter tuning. The ghost of filter tuning chases every possible formulation or any variant of the Kalman filter be it Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF). The best possible tuning is necessary if one desires to get near optimal solutions. Here we are modifyingEKF and UKF by EKF+M and UKF+M Filter.

3.2 Different Types of Filtering Techniques
3.2.1 The EKF Algorithm

The EKF algorithm works on the discrete nonlinear system model, where w_k and v_k are Gaussian distribution noises with known parameters. The extended algorithm is almost similar with KF algorithm, but in this case there was introduced one local linearization step of the model equations [30]-[31].

The steps of the EKF state estimation algorithm is presented below:

0. Initialization step at k=0.

Initial estimated state vector $\tilde{\underline{x}}_0 = E\{\underline{x}_0\}$

Initial covariance matrix: $\tilde{P}_{0,x} = E\{(\underline{x}_0 - \tilde{\underline{x}}_0).(\underline{x}_0 - \tilde{\underline{x}}_0)^T\}$

I. Local linearization step (k >=1): linearizing the nonlinear model functions F () and G (), calculate the following matrices:

$$\varnothing_k = \begin{bmatrix} \dfrac{\partial F_1}{\partial x_1} & \cdots & \dfrac{\partial F_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial F_n}{\partial x_1} & \cdots & \dfrac{\partial F_n}{\partial x_n} \end{bmatrix}_{x = \tilde{\underline{x}}_{k-1}}$$

$$C_k = \begin{bmatrix} \dfrac{\partial G_1}{\partial x_1} & \cdots & \dfrac{\partial G_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial G_p}{\partial x_1} & \cdots & \dfrac{\partial G_p}{\partial x_n} \end{bmatrix}_{x = \tilde{x}_{k-1}}$$

(10)

The linearization method utilizes just the first term in the Taylor expansion of the nonlinear functions:

Prediction step: Calculation of the predicted state mean and covariance (time update)

$$\tilde{\underline{x}}_{\bar{k}} = F\left(\tilde{\underline{x}}_{k-1}, \underline{u}_{k-1}\right)$$
$$\tilde{P}_{\bar{x},k} = \varnothing_k . \tilde{P}_{x,k} . \varnothing_k^T + Q$$

(11)

Calculation of the filter gain vector:

$$K_k = \tilde{P}_{\bar{x},k} . C_k^T . (C_k . \tilde{P}_{\bar{x},k} . C_k^T + R)^{-1}$$

(12)

Correction step: The estimates are updated with latest observation (measurement update)

$$\tilde{\underline{x}}_k = \tilde{\underline{x}}_{\bar{k}} + K_k . \left(y_k - G\left(\tilde{\underline{x}}_{\bar{k}} . u_k\right)\right)$$
$$\tilde{P}_{x,k} = \left(I - K_k . C_k\right) . \tilde{P}_{\bar{x},k}$$

(13)

The EKF gives an approximation of the optimal estimate. The non-linearities of the system's dynamics are approximated by a linearized version of the non-linear system model around the last estimated state. This algorithm can be divergent if the consecutive linearization is not a good approximation of the non-linear model.

3.2.2. The UKF Algorithm

The state distribution is also represented by Gaussian random variables, but this method is using a minimal set of carefully chosen sample points. These points are called sigma points and they are completely capture the true mean and covariance of the system states and are propagated through the nonlinearity. For calculating the statistics of a random variable which undergoes a nonlinear transformation we can use the unscented transformation (UT) [32-33].

The standard UKF state estimation algorithm, with additive (zero mean) noise, is presented:

0. Initialization step at k=0:

initial estimated state vector: $\tilde{\underline{x}}_0 = E\{\underline{x}_0\}$

initial covariance matrix: $\tilde{P}_{x,0} = E\{(\underline{x}_0 - \tilde{\underline{x}}_0).(\underline{x}_0 - \tilde{\underline{x}}_0)^T\}$

I. Sigma points' calculation for k>=1

$$A_{k-1} = \left[\tilde{x}_{k-1} \tilde{x}_{k-1} + \gamma . \sqrt{\tilde{P}_{x,k-1}} \tilde{x}_{k-1} - \gamma . \sqrt{\tilde{P}_{x,k-1}} \right] \tag{14}$$

II. Propagation of the sigma points:

transform the sigma points through the state-update function: $\quad A_k^* = F(A_{k-1}. u_{k-1})$

calculate the apriori state estimate and apriori covariance, where the weights $W_i^{(m)}$ and $W_i^{(c)}$ are defined in accordance with relations:

$$\tilde{x}_{\bar{k}} = \sum_{i=0}^{2n} W_i^{(m)}.(A_k^*)_i \quad \tilde{P}_{x,k} = \sum_{i=0}^{2n} W_i^{(c)}.((A_k^*)_i - \tilde{x}_{\bar{k}}).((A_k^*)_i - \tilde{x}_{\bar{k}})^T + Q \tag{15}$$

III. Update of the output vectors:

transform the sigma points through the measurement-update function: $Y_k^* = G(A_{k-1}, u_k)$

calculate the mean and covariance of the measurement vector:

$$\tilde{y}_{\bar{k}} = \sum_{i=0}^{2n} W_i^{(m)}.(Y_k^*)_i \quad \tilde{P}_{\bar{y},k} = \sum_{i=0}^{2n} W_i^{(c)}.((Y_k^*)_i - \tilde{y}_{\bar{k}}).((A_k^*)_i - \tilde{x}_{\bar{k}})^T + R \tag{16}$$

IV. Calculate the cross covariance matrix:

$$\tilde{P}_{x,y,k} = \sum_{i=0}^{2.n} W_i^{(c)}.((x_k^*)_i - \tilde{x}_{\bar{k}}).((y_k^*)_i - \tilde{y}_{\bar{k}})^T \tag{17}$$

V. Calculation of the Kalman filter gain vector

$$K_k = \tilde{P}_{x,y,k}.(\tilde{P}_{\bar{y},k})^{-1} \tag{18}$$

VI. Calculate the estimated state and the covariance in accordance with the standard Kalman filter algorithm:

$$\underline{\tilde{x}}_k = \underline{\tilde{x}}_{\bar{k}} + K_k.(y_k - \tilde{y}_{\bar{k}}) \tag{19}$$

The UKF principle is simple and easy to implement because it does not require the calculation of Jacobians at each time step. The most computationally intensive operation in the UKF corresponds to calculating the new set of sigma points at each time update [34]-[36].

3.2.3. The EKF + M Filter and the UKF+M Filter

The EKF and UKF, both are best filtration methods available at present time. But in the account, we have to consider some assumptions also. The extended Kalman filter has been gradually applied in power system dispatching centers all over the world, while the unscented Kalman filter is a rather new theory. However compared to the extended Kalman filter, the superiority of the unscented Kalman filter resides in the fact that it simplifies the process of calculation by avoiding the Jacobian matrix. Instead, it only needs to compute a set of Sigma points along with the regular Kalman filter algorithm. In addition, the unscented Kalman filter shows better accuracy and faster response than the extended Kalman filter as the extended Kalman filter omits quadratic and higher order terms while the UKF does not. So for enhancing their accuracy and for reducing their elapsed time, we are modified in these algorithms. In addition, we have modify in EKF and UKF by adding all previous effect in absolute exponential from and that can reduce the effect of gain.

3.3. Advantages of EKF+M filter
1. Estimation time is less as compare to EKF.
2. The accuracy is improved.

3.4. Limitations of EKF+M filter
1. For large scale and complex systems, the Jacobian matrix calculation is too complicated to be executed in real time.
2. For large system, system does not give accurate result.

4. Results and Analysis

Because each nonlinear filtering algorithm often has its advantages and drawbacks, it is important to set up a specific case in a power system for comparison. In the addition of this we are considering standard IEEE 30 bus, IEEE 14 bus and IEEE 9 bus test system's data and applied on this filtration algorithm and take the state estimation output. On the comparison aspects Figure 2 shows the state estimation output of each algorithm on given filtration algorithm and compared with without filter output. The state variables for the system are voltage of the bus, real and reactive power of the bus, real and reactive power flows in the lines. J denotes the Jacobian matrix data in per unit. Standard deviation is 1e-5 for EKF and EKF+M and 1e-4 for UKF and UKF+M filters.

IEEE 30 bus system data-
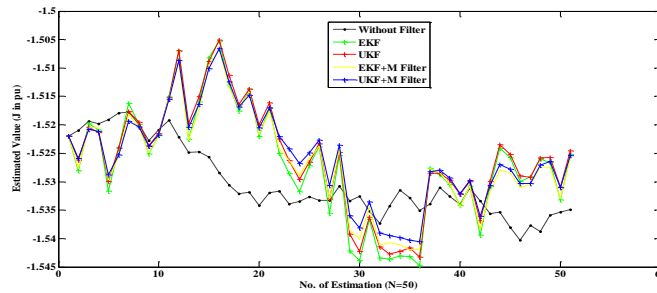
IEEE 14 bus system data-
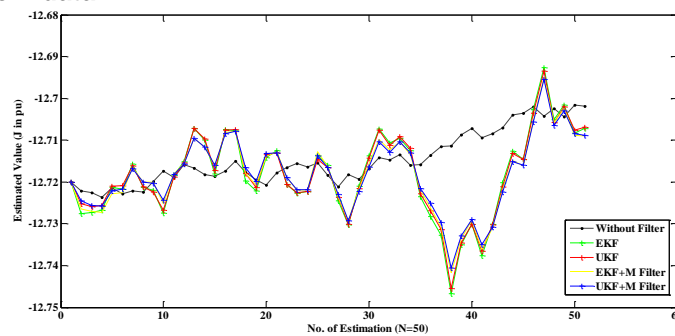
IEEE 9 bus system data-
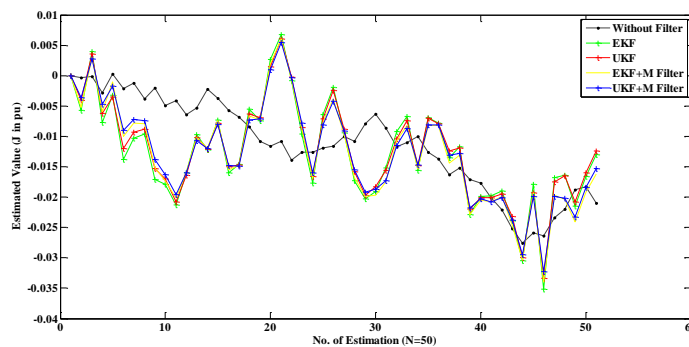
Figure 2. Comparison of estimated value with and without filters

Figure 3 shows the Performance Variation of each filter as respect to without filter by the dividing the values of filtered output matrix by without filter output matrix.

Figure 4 shows the Performance Difference of each filter as respect to without filter by subtracting the without filter data from filtered data.

IEEE 30 bus system data-
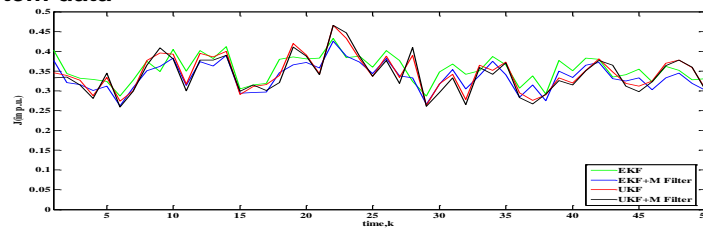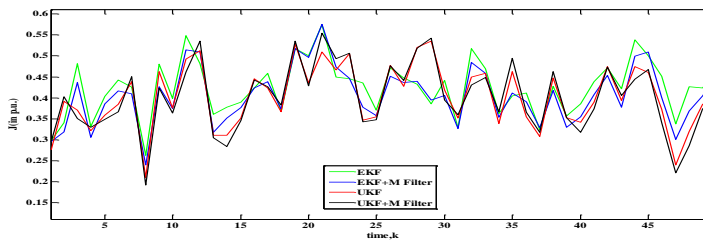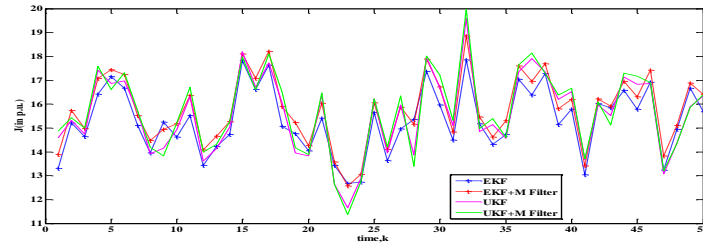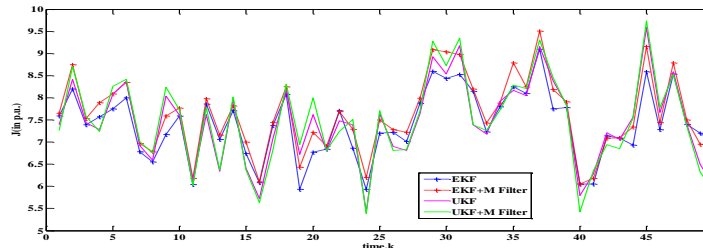

IEEE 14 bus system data-
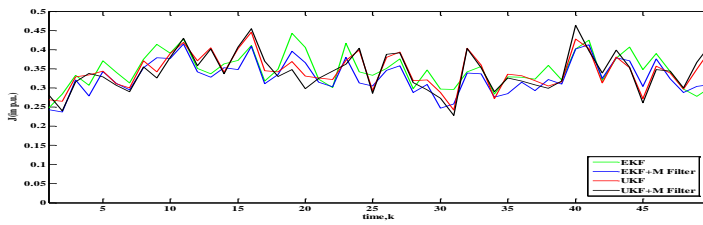
IEEE 9 bus system data-

Figure 3. Performance Variation of each filter as respect to without filter

IEEE 30 bus system data-

IEEE 14 bus system data-

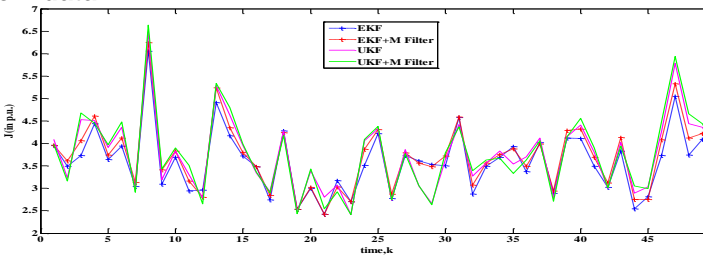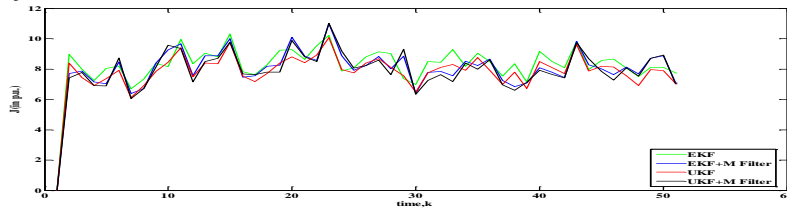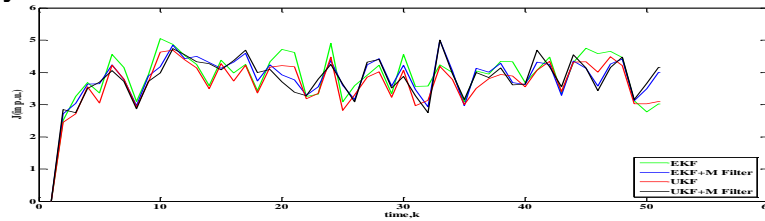**IEEE 9 bus system data-**

Figure 4. Performance Difference of each filter as respect to without filter

Figure 5 shows the Average Error data that is difference of Filtered Estimated matrix to Without Filtered Matrix. Figure 6 shows the Average output of Estimated Matrix.

IEEE 30 bus system data-

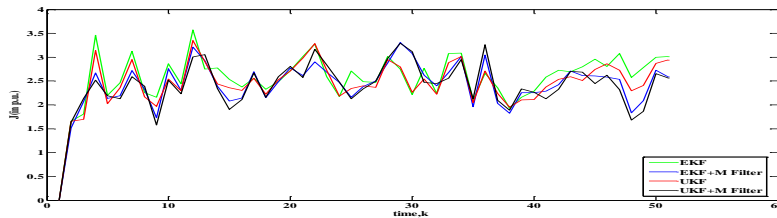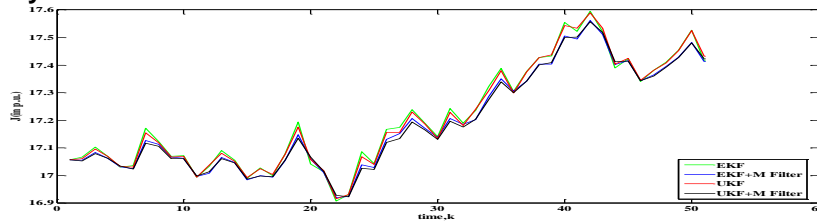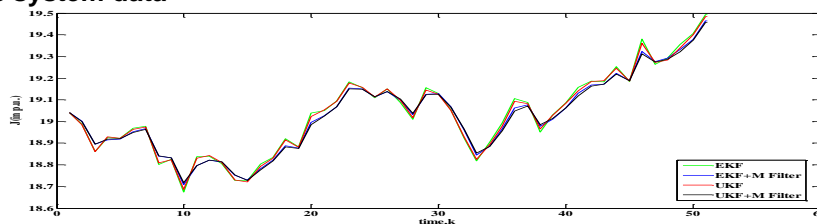**IEEE 14 bus system data-**

IEEE 9 bus system data-

Figure 5. Comparison of estimated matrix after filtration

IEEE 30 bus system data-

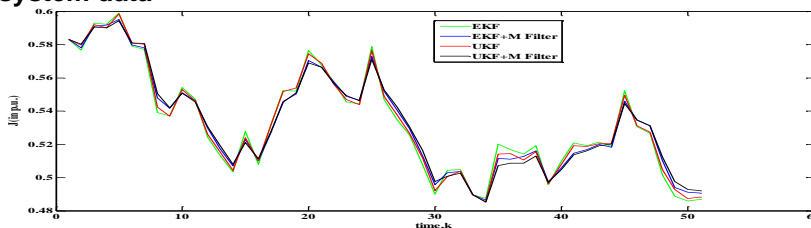**IEEE 14 bus system data-**

IEEE 9 bus system data-

Figure 6. Average Output of Estimated Matrix

According to previous results UKF+M estimator filter is giving better results when we applied on different bus system. Those are shown in tables shown below from Table 1 to Table 5. After apply all the filter techniques in the different bus systems, they show different results for different bus system as shown in various Tables 1-5.

Table 1. Time Elapsed by different Filters

	IEEE 30 Bus	IEEE 14 Bus	IEEE 9 Bus
EKF	4.046173	1.494122	0.464652
UKF	4.101987	1.330903	0.643924
EKF+M Filter	2.600231	0.897911	0.444882
UKF+M Filter	3.875465	0.760284	0.390884

The analytical view of different filters is shown in Figure 7. Here the time elapsed by UKF+M estimation is minimum as compare to previous EKF and UKF.
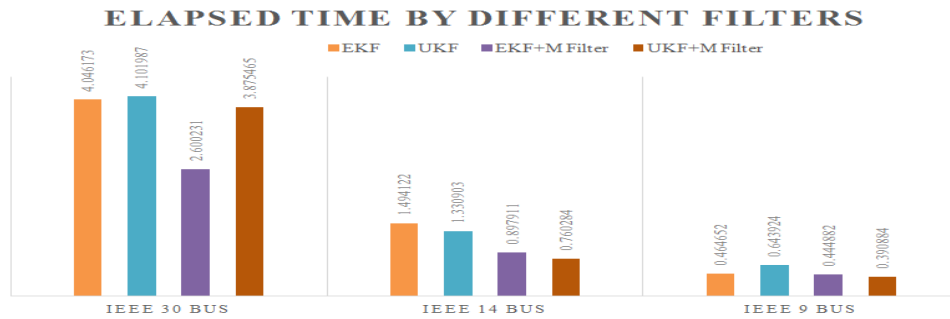

Figure 7. Elapsed Time by Different Filters

Table 2 contains the data are showingmean performance variation as compare to without filter output. Here the changes variation is about to minimum in UKF+M Estimator.The performance variation can be calculate by mean summation of the array of the performance matrix of filter output divided by the performance matrix of without filter. The analytical view of Table 2 is show in Figure 8 the variation is minimum in UKF+M Estimator.

Table 2. Mean Performance Variation of each filter as respect to without filter

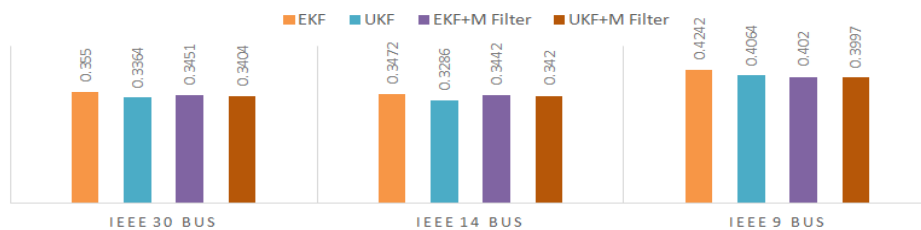|	IEEE 30 Bus	IEEE 14 Bus	IEEE 9 Bus
EKF	0.355	0.3472	0.4242
UKF	0.3364	0.3286	0.4064
EKF+M Filter	0.3451	0.3442	0.402
UKF+M Filter	0.3404	0.342	0.3997

Figure 8. Performance Variation of each filter as respect to without filter

The Table 3 contains the data of mean performance difference of each filter that can be calculate by mean summation of the array of the performance matrix of filter output minus the performance matrix of without filter. That is approximately less in UKF+M estimator as compare to other Kalman filters. The analytical view of Table 3 is shown in Figure 9:

Table 3. Mean Performance Difference of each filter as respect to without filter

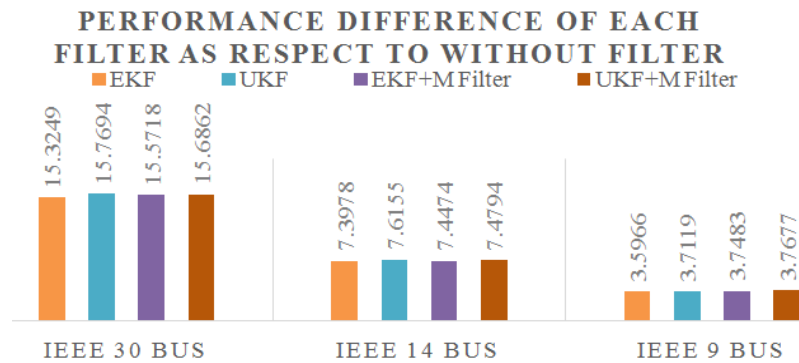|	IEEE 30 Bus	IEEE 14 Bus	IEEE 9 Bus
EKF	15.3249	7.3978	3.5966
UKF	15.7694	7.6155	3.7119
EKF+M Filter	15.5718	7.4474	3.7483
UKF+M Filter	15.6862	7.4794	3.7677

Figure 9. Average Performance variation of each filter

The Table 4 contains the data of error in each filter that is measuring error and Gaussian error from without filter. That can be calculated by average summation of the array of error Kalman filter matrix of different Kalman filter. The analytical view of Table 4 is shown in Figure 10.

Table 4. Average Error in Each filter from without Filter

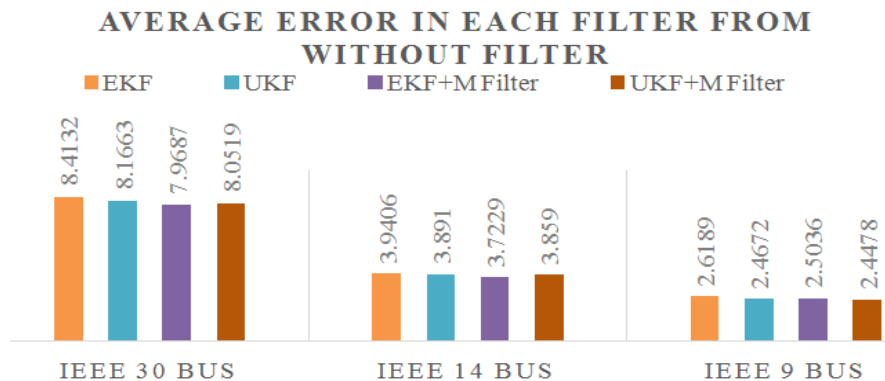|	IEEE 30 Bus	IEEE 14 Bus	IEEE 9 Bus
EKF	8.4132	3.9406	2.6189
UKF	8.1663	3.891	2.4672
EKF+M Filter	7.9687	3.7229	2.5036
UKF+M Filter	8.0519	3.859	2.4478

Figure 10. Average error in each filter from without filter

The table 5 contains the median of all estimated data. Here the number of estimated data is N=50 so the median of that for all filters are shown. The analytical view of Table 5 is shown in Figure 11, the median of estimated data is minimum in UKF+M estimator.

Table 5. Median of the estimated data

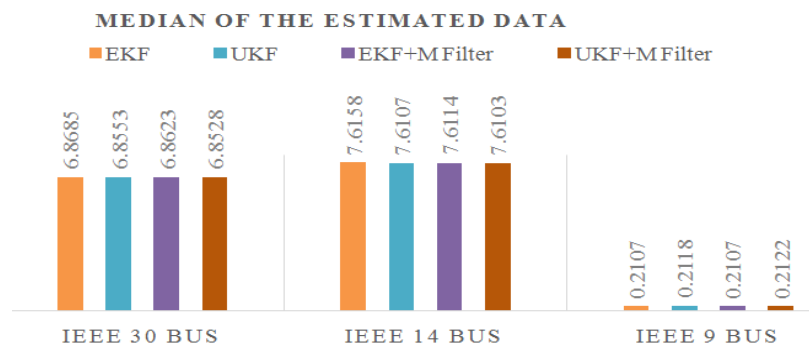|	IEEE 30 Bus	IEEE 14 Bus	IEEE 9 Bus
EKF	6.8685	7.6158	0.2107
UKF	6.8553	7.6107	0.2118
EKF+M Filter	6.8623	7.6114	0.2107
UKF+M Filter	6.8528	7.6103	0.2122

Figure 11. Median of the estimated data

5. Conclusion

Accurate information about dynamic states is critical to efficient control of a power system, especially with the increasing complexity resulting from uncertainties variations introduced by intermittent renewable energy sources, natural disasters, responsive loads, mobile consumption of plug-in vehicles, and new market designs. Using a statistical framework, this paper compares the performance of an EKF, UKF, EKF+M and UKF+M Estimator for the purpose of estimating dynamic states from Data available of IEEE test system. To summarize the observations from the Mat lab coding using this data, Table 1 to 4 is constructed for quick comparison.

Ongoing and future work includes dynamic state estimation methods at system levels and sensitivity studies to determine how parameter errors may influence the state estimation, as well as efficient, accurate, and flexible methods for estimating the states in both real time and offline environments.

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