Indonesian Journal of Electrical Engineering and Computer Science
Vol. 7, No. 1, July 2017, pp. 116 ~ 122
DOI: 10.11591/ijeecs.v7.i1.pp116-122

MRI Denoising Using Sparse Based Curvelet Transform with Variance Stabilizing Transformation Framework

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Abstract

We develop an efficient MRI denoising algorithm based on sparse representation and curvelet transform with variance stabilizing transformation framework. By using sparse representation, a MR image is decomposed into a sparsest coefficients matrix with more no of zeros. Curvelet transform is directional in nature and it preserves the important edge and texture details of MR images. In order to get sparsity and texture preservation, we post process the denoising result of sparse based method through curvelet transform. To use our proposed sparse based curvelet transform denoising method to remove rician noise in MR images, we use forward and inverse variance-stabilizing transformations. Experimental results reveal the efficacy of our approach to rician noise removal while well preserving the image details. Our proposed method shows improved performance over the existing denoising methods in terms of PSNR and SSIM for T1, T2 weighted MR images.

***Keywords**: Magnetic Resonance Imaging, Sparse Representation, Rician Noise, Curvelet Transform, Variance Stabilizing Transformation*

1. Introduction

Recently, Magnetic Resonance Imaging (MRI) has been extensively used in diagnosis of diseases and treatments. Due to the extensive use of MRI, the quality of MR images becomes an important issue [1]. However, during image acquisition process by different types of sensors, they may introduce noises and artifacts appear reducing the quality of image. In order to obtain the best possible diagnosis from MR images, it is required to denoise MR images without affecting its anatomical details [2]. Therefore, MRI denoising remains a challenge due to presence of noise and artifacts [3]. In general, the noise introduced in MRI usually follows rician distribution and hence it is modelled as rician noise [4].

In MRI literature, many denoising algorithms have been proposed to remove rician noise present in MR images. Henkelman et al., [5] and McGibney et al., [6] were the first to present their effort to reduce noise in MR images. A number of filtering techniques have also been used for MRI denoising such as adaptive smoothing [7], total variation based convex filtering [8], anisotropic wiener filtering [9], and anisotropic diffusion [10]. Generally, filtering techniques can successfully remove noise from corrupted MR images but it blurs the fine details. Hence, transformed based denoising methods have been introduced and it removes noise without bluing the image [11]. For example, denoising methods based on wavelet transforms which relies on statistical inference are used for MR image denoising [12-13]. Recently, square sparsifying transforms have been used with an over complete dictionary as compared to wavelet basis. Denoising algorithms with sparse representation based image denoising has shown its popularity [15]. To achieve sparse signal representation K-means singular value decomposition (KSVD) is used for adapting dictionaries [15]. KSVD based algorithms show very good results for gaussian noise removal [16].

In this paper, we use KSVD denoising technique for rician noise removal of brain MR images. As rician noise is signal dependent and it is observed that noise variance is not uniform. It is also observed that based MRI denoising algorithm is computationally inefficient. We use forward and inverse variance-stabilizing transformations instead of directly applying KSVD for rician noise removal. The optimal forward transform is applied to convert rician distributions into gaussian distribution with constant variance which is subsequently used with KSVD. Inverse

Received February 27, 2017; Revised May 16, 2017; Accepted June 1, 2017

variance stabilization transform is used with the obtained intermediate result to get back the original distributions. The proposed algorithm is stable and computationally efficient for MRI denoising. In addition, to enhance the robustness of the proposed denoising algorithm so as to retain fine details in an MR image we propose to use curvlet transform along with KSVD. We post process the denoising result of KSVD through curvelet transform in order to preserve the important edge and texture details of MR images. The proposed algorithm has been tested in a simulated environment with different MR images. The result shows that our proposed approach is efficient in removing rician noise while well preserving the texture details over the state of the art methods and there is a substantial improvement in the PSNR and SSIM measures for MR images containing edges.

The rest of the paper is organized as follows. In Section 2, we present a short overview of MRI model, KSVD based denoising, Curvelet transform for texture preservation and Variance-stabilizing transformation. We introduce a robust MRI denoising algorithm for rician noise removal Section 3. In section 4, we present experimental results and compare the denoising performance of our proposed method with some existing methods before concluding in Section 5.

2. Background

In this section, we present the theoretical and technical concept of important elements required for development of our proposed method.

2.1. MRI Model

MR images are usually computed from both real and imaginary components where both of the two components are corrupted by zero mean gaussian noise [1]. Normally, the reconstruction of MRI is performed by calculating inverse discrete fourier transform of the original data. Therefore, noise present in reconstructed MR image is complex white gaussian noise [2]. For computer and visual analysis of MRI, the reconstructed magnitude is basically used. The noisy in MRI follows a rician distribution and it is image dependent [3]. Hence, removal of rician noise in MRI is difficult. If the real and imaginary components are contaminated by gaussian noise with mean values A_R and A_I respectively and with standard deviation σ, the rician distribution will be described by:

$$P_{mag}(M) = \frac{M}{\sigma^2} e^{-(M^2+A^2)/2\sigma^2} I_0(AM/\sigma^2) \tag{1}$$

where I_0 denotes zeroth order Bessel function and A is given by $A = \sqrt{A_R^2 + A_I^2}$

2.2. KSVD in Medical Image Analysis

Sparse representation has shown its popularity in performing image denoising. The KSVD algorithm is based on sparse based image denoising technique [11]. In KSVD, the noisy image is partitioned in to a set of image patches [13]. For any given noisy image Y, we divide Y into L no of image patches. The sparse decomposition is defined as:

$$\min_{D, x_i} \| y - Dxi \|_2 \text{ subject to } \| y_i - Dx_i \|_2 < \varepsilon \tag{2}$$

Where, x_i represents the sparse matrix, D is the dictionary, ε represents sparsity threshold, $\|\cdot\|_0$ is the l_0 norm that represents number of non-zero coefficients. Here, the objective is to find dictionary D in such a way that it yields a sparse representation $\|X\|_0$ for noisy image Y .However, exact determination of $\|X\|_0$ proves to be a non-polynomial hard problem. Instead of exact determination, approximate solution may be considered [17]. In order to find such solutions, a simplest pursuit algorithm OMP is used. KSVD requires number of iterations to optimize D and X. Each iteration consists of two stages such as sparse coding stage to update the coefficients of X and dictionary update stage to optimize atoms or columns of dictionary D. Here, we focus on sparse decomposition of MR images and apply KSVD algorithm for denoising of MRI data sets.

2.3. Curvelet Transform for Texture Preservation

The curvelet transform has many directions and positions that represent edges and curve-singularities efficiently than wavelet. The limitations of wavelet based image denoising are avoided by the development of curvelet transform which is based on both multiscale analysis and geometrical ideas to obtain optimal rate of convergence [18]. In image processing, most natural images show curved edges instead of straight and it is not possible to obtain efficient representations using ridgelets alone. In order to capture curve edges, the image is first partitioned into sub-images and then, at sufficiently fine scales ridgelet transform is deployed to each of the obtained sub-images. This multiscale ridgelet transform is called as Curvelet transform [19].

In Discrete Curvelet Transform, the object y uses a dyadic sequence and a bank of sub-band filters $(p_0 y, \Delta_1 y, \Delta_2 y.........)$ with the frequency distribution such that the bandpass filter Δ_i is determined near the frequencies $[2^i, 2^{i+2}]$ as given:

$$\Delta_i(y) = \psi_{2i} * y, \hat{\psi}_{2i}(v) = \hat{\psi}(2^{-2i} v) \tag{3}$$

The basic idea of curvelet based image denoising is based on thresholding, where each curvelet coefficient is compared with a given threshold [20-21]. The curvelet coefficient is set to zero if it is found to be less than the given threshold. Otherwise it is kept as it is or slightly reduced in magnitude.

2.4. Variance-Stabilizing Transformations (VST)

In order to use standard image denoising algorithms for rician noise removal in MR images, variance-stabilizing transformations is developed [22]. It converts rician distribution with variable variance to gaussian distribution with constant variance. It provides a stable and fast iterative method for estimation of noise level in MRI. Generally, variance stabilizing transforms requires three steps for removal of rician noise in MR images. First, it is used as optimal forward transform to stabilize noise variance and converts rician distributions into gaussian distribution. In second step standard denoising method is applied. After denoising operation, the inverse VST is applied to the denoised output to get back the original distributions.

The mean and variance of rician distribution is given by:

$$M = \sigma\sqrt{\frac{\pi}{2}} L\left(-\frac{v^2}{2\sigma^2}\right) \tag{4}$$

$$\sigma_M^2 = 2\sigma^2 + v^2 - \frac{\pi\sigma^2}{2} L^2\left(-\frac{v^2}{2\sigma^2}\right) \tag{5}$$

From equation 5, it is observed that noise variance is not consistent for entire MR data. And, from equation 4, the expectation varies from the parameter of interest, specifically v. The first issue is solved by applying forward variance-stabilizing transformation to the MR data before applying denoising method, whereas the second issue is addressed by applying the inverse transformation after denoising, which provides a robust estimation of v out of the filtered transformed data.

3. Proposed Method: A Combined Approach of KSVD and Curvelet Transform with Variance Stabilizing Transformation Framework (KSVDCT)

The proposed MRI denoising method combines the advantages of variance stabilizing transformation and sparse representation based curvelet transform to reconstruct MR images corrupted with rician noise. Figure 1 shows the steps followed in our proposed method. First, the noisy MR image is processed by the variance stabilization transform so that the rician distribution is converted into gaussian distribution with constant variance. As variance stabilizing transformation removes the dependency of noise variance, it makes the standard denoising methods suitable for the transformed images. Then KSVD based denoising technique is applied on the stabilized data to remove the gaussian noise. The output of KSVD denoising is processed through Curvelet transform to preserve edge and texture of MR image. Finally, the processed result is converted back to its initial state by the inverse variance stabilization transformations and we get the denoised image.

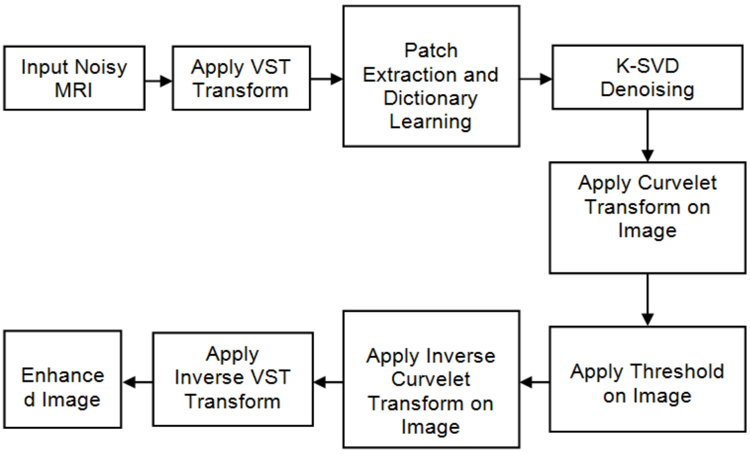


Figure 1. Block Diagram of Proposed Rician Noise Removal Method

KSVDCT Algorithm

The method consists of following steps

- *The rician distributed noisy MR image Y is converted to gaussian distribution by applying variance stabilising transformation.*
- *Then the gaussian distributed noisy MR image is divided into L no of overlapping image patches of size $r \times r$.*
- *Using K-SVD denoising, noisy MR image Y is decomposed into a sparse vector X (Equation 2).*
- *Compute threshold value of the distorted image X.*
- *Apply Discrete Curvelet Transform on sparse vector X to shift it from spatial domain to curvelet domain (Equation 2).*
- *Apply computed threshold on distorted image.*
- *Apply inverse Discrete Curvelet Transform on distorted image (followed by application of threshold on it) to shift image from curvelet domain to spatial domain.*
- *Finally, inverse variance stabilising transformation transform is applied to get back original distribution.*

4. Experimental Result and Discussion

In this section, we assess the visual and quantitative performance of our proposed method on noisy MR image. We compare the performance of our method with KSVD and KSVD denoising with VST framework (KSVD+VST). We perform experiments using simulated brain

MR images. For visual and quantitative performance, we consider two criteria which include PSNR and SSIM.Peak Signal to Noise Ratio (PSNR) provides quantitative results of various denoising methods [23]. PSNR is given by:

$$PSNR = 10\log_{10}\frac{L^2}{MSE} \tag{6}$$

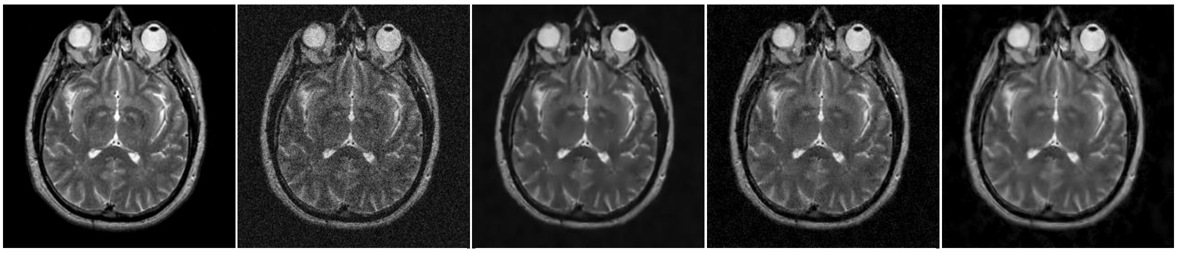
where L represents the highest pixel value and MSE represents the mean squared error.

Structural similarity (SSIM) index explores the structure information [24] and is defined as:

$$SSIM(x,r) = \frac{(2\mu_x\mu_r + c_1)(2\sigma_{xr} + c_2)}{(\mu_x^2 + \mu_r^2 + c_1)(\sigma_x^2 + \sigma_r^2 + c_2)} \tag{7}$$

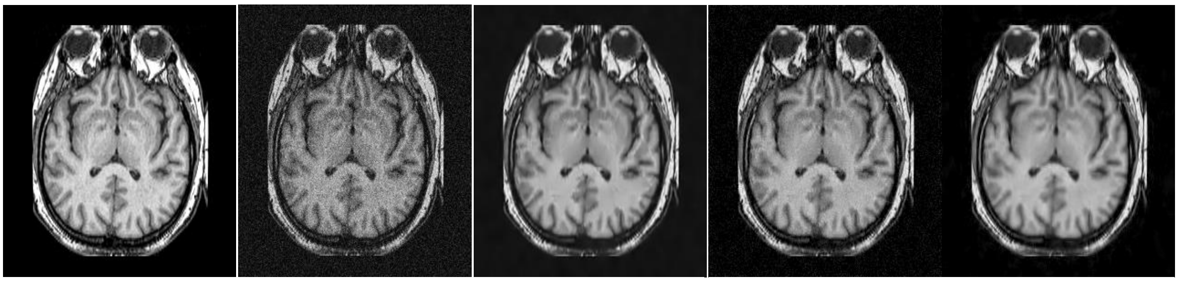
Where μ_x and μ_r represents the mean of images x and r respectively whereas σ_x and σ_r represents their corresponding standard deviations, σ_{xr} is the covariance, $c_1 = (k_1L)^2$ and $c_2 = (k_2L)^2$ are two constants. *L* is the dynamic range of the intensity values.

In this experiment, we have used simulated brain data from Harvard database. The simulated brain data consists of T1 and T2-weighted brain images of size 256x256. Here, we have taken the noisy versions of the above images contaminated with rician noise using Equation1 having different noise variance. To be more specific, the noise varying from 9% to 15% with an increment of 2% is used in the experiments. In our experiments, we set the parameters for MRI based denoising as the dictionaries in use are of size 64x256, the sparsity limit T_0 and the number of iteration is 10. In order to prove the effectiveness of sparsity based denoising technique to reduce rician noise, we add rician noise with different standard deviation to T1-weighted brain MR image. Figure 1 and 2 shows the visual evaluation of different methods on the T1w and T2w brain MR image with noise percentage of 9% respectively. Figure 2(a) shows the original T1w MR image, while Figure 2(b) shows the equivalent noisy MR image. Figure 2(c) shows the denoised result by KSVD, Figure 2(d) shows the denoised result by KSVD+VST and Figure 2(e) shows the denoised result by our proposed method. Similarly, Figure 3(a), (b), (c), (d) and (e) show the original T2w MRI image, corresponding noisy image, denoised result by KSVD, denoised result by KSVD+VST and the denoised result by our proposed method respectively. It is observed from both Figure 2 and 3 that the KSVD method applied directly on rician noisy MR image blurs the output image. However, KSVD with VST framework reduce rician noise effectively. But, our proposed method KSVDCT suppresses rician noise more while retaining phantom details.



(a) (b) (c) (d) (e)

Figure 2. (a) Original T1w MR image (b) Noisy T1w MR image (c) Denoised T1w MR image using KSVD (d) Denoised T1w MR image using KSVD with VST framework (e) Denoised T1w MR image using Proposed method KSVDCT



(a) (b) (c) (d) (e)

Figure 3. (a) Original T2w MR image (b) Noisy T2w MR image (c) Denoised T2w MR image using KSVD (d) Denoised T2w MR image using KSVD with VST framework (e) Denoised T2w MR image using Proposed method KSVDCT

Table 1. PSNR and SSIM Comparison of Different Methods for T1w and T2w MR Data

	Noise Level	Percentage Noise=9%		Percentage Noise=11%		Percentage Noise=13%		Percentage Noise=15%	
	Methods	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM
T1w	Noisy Image	19.318	0.4311	17.575	0.3844	16.126	0.3433	14.884	0.3073
	KSVD	21.260	0.5364	19.539	0.5060	18.095	0.4747	16.841	0.4420
	KSVD+VST	25.617	0.5760	23.914	0.5257	22.539	0.4865	21.375	0.4519
	Proposed Method	**27.403**	**0.8107**	**26.085**	**0.7479**	**24.966**	**0.6960**	**24.042**	**0.6597**
T2w	Noisy Image	19.313	0.3684	17.577	0.3183	16.136	0.2761	14.903	0.2405
	KSVD	21.334	0.5025	19.592	0.4631	18.119	0.4230	16.832	0.3816
	KSVD+VST	25.726	0.5258	24.101	0.4743	22.691	0.4289	21.380	0.3865
	Proposed Method	**27.989**	**0.7282**	**26.667**	**0.6779**	**25.518**	**0.6261**	**24.392**	**0.5647**

Table 1 shows the comparative results of PSNR and SSIM values of three different methods on T1w and T2w brain MR images. From the Table, it can be found that KSVD with VST framework based denoising improves PSNR values more than KSVD applied directly on rician noisy MR image. However, our post-processing method KSVDCT performs best result for entire range of noise.

5. Conclusion

We introduced an efficient denoising technique KSVDCT based on sparse representation and curvelet transform for noise removal in MRI. This method employs sparse coefficients of the image, seeking the most similar patches in the whole image. Also, we preserve the texture and important details of MR images using curvelet transform. We conduct experiments on the simulated brain images to estimate the visual and quantitative performance of our proposed method. The denoising results show that the propose rician noise removal technique performs better than some of the state-of-the-art image denoising methods. It provides better texture preservation and noise attenuation. Also, our approach can preserve singularities along lines and edges in an efficient way. Our proposed denoising method can be used widely in MRI applications.

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