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Image Reconstruction Based on Combination of Inverse Scattering Technique and Total Variation Regularization Method

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Abstract

The Forward-Backward Time-Stepping (FBTS) had proven its potential to reconstruct images of buried objects in inhomogeneous medium with useful quantitative information about its size, shape, and locality. The Total Variation regularization method was incorporated with the FBTS algorithm to deal with the ill-posedness or ill-conditionedness of the inverse problem. The effectiveness of the proposed technique is confirmed by numerical simulations. The numerical method was carried out on simple object detection through FBTS with and without TV regularization method. The detection and reconstruction of relative permittivity and conductivity of the simple object have shown an improvement as TV regularization method applied whereas it smoothed the vibrations of the images and gave a better estimation of the image's boundaries.

Keywords: *inverse scattering, regularization, total variation regularization*

1. Introduction

Microwave imaging is a technique used to sense an object or objects by means of analyzing microwave signals. Extensive researches have been conducted on microwave imaging and it had proven its potential utility and effectiveness for wide range of applications such as in medical imaging, remote sensing of earth, and non-destructive testing and evaluation. For example, in remote sensing, Synthetic Aperture Radar (SAR) is a form of radar system that has the capability to produce a high resolution imagery of bio and geophysical parameter of the Earth's surface [1-2]. In Non-Destructive Testing and Evaluation (NDT & E), high frequency electromagnetic energy is utilized to determine material characterization and its structural integrity. Pulsed Eddy Current (PEC) is among the popular technique applied in NDT & E due to its effectiveness of quantifying defects in multilayer structures. Some applications involving non-destructive testing and evaluation are identifying hidden defect characterization in some complex structures [3], and defect detection of riveted structures of aging aircraft [4]. Microwave imaging applications for medical purposes had pulled a great attention and continuously investigated by many researchers. Medical imaging is the visualization of a biological system for clinical diagnosis and medical intervention proposes. Medical imaging modalities examples are X-ray, Ultrasound, Computerized Tomography (CT), Positron Emission Tomography (PET) and Magnetic Resonance Imaging (MRI). In microwave tomography, transmitting antennas are used to illuminate the biological part in microwave region and the scattered field data are then collected by receiving antennas. The scattered field information will be processed and analysed to reconstruct the dielectric properties of the biological part. The dielectric properties concern are the relative permittivity, ε_r, the capacitance of a unit volume of matter and conductivity, σ, the conductivity of a unit volume of matter. Microwave tomography

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can be obtained through single-frequency, multi-frequency or time domain approach [5-6]. Microwave tomography is preferable compare to other conventional medical imaging modalities because it is known for its non-ionizing detection, more comfortable for patients, inexpensive, no side effect for human body and potentially increases the sensitivity of cancer detection [7].

Recently, there are a positive growth in research about medical tomography for cancer detection due to the significant contrast of dielectric properties of healthy and malignant or abnormalities tissue at microwave frequencies. For similar reason, a group of researchers focus on microwave imaging technique for breast cancer detection [8-10]. Takenaka, et al., [11] introduced Forward Backward Time-Stepping (FBTS) to solve electromagnetic inverse scattering problems in time domain. Time domain approach in FBTS means allowing a broad spectrum of frequencies to be utilized in a single optimization, thus, this approach will give more useful quantitative information of electrical parameter profiles and increase in accuracy for the reconstructed images. FBTS has shown a strong ability to reconstruct images providing the electric properties of the objects together with its size, shape, position of the scatters or objects. In recent studies, the reconstructed of breast imagery with FBTS technique had shown a significant disparity of the dielectric properties in which enabling the differentiation and recognition of normal and abnormal breast tissue. It is reported that FBTS technique capable of detecting as small as 4mm malignant tumour in its reconstructed image [12]. Another advancement of FBTS is reconstruction of breast imagery in three dimensional form [13], and incorporating low pass filter with several cut-off frequencies in the FBTS algorithm to avoid nonlinearity problem [14]. Beside FBTS technique, a genetic algorithm [15] shows a an ability to reconstruct a highly contrasted forms of dielectric properties

However, the process of developing the reconstructed images is dependent greatly on its quantitative information. Although the quantitative information can be attainable by solving inverse scattering problem, it is quite challenging and generally ill-posed or ill-conditioning. A small perturbation in the scattering data will bring a large perturbation in the solution. Ill-posed problems can be handled either by incorporating a-priori information through the use of transformation or by incorporating an appropriate numerical methods, called regularization techniques [16]. Regularization method can be applied to improve the conditioning and determine a useful approximation of the actual reconstruction image. Some of the commonly known and used methods of regularization are Singular-Value Decomposition (SVD) as in [17-18], and Tikhonov regularization as in [19]. SVD and Tikhonov regularization perform successfully in image enhancement, noise removal and deblurring however, these types of regularization tend to produce smoothed or over smoothed solution which eliminates sharpness at object boundaries on the resulting solution. The TV regularization was introduced by Rudin, Osher and Fatemi in [20], is a constrained optimization type of numerical approach for image denoising while preserved the edge information. Unlike other conventional smoothing filter that has a greater tendency to blur the discontinuities, TV regularization is well suited for edge preserving as it is not discriminating against smoothness in an image and tends to preserve edges of the reconstructed image [21]. Due to efficacy and robustness of TV regularization method, it has been considered to be utilized in image denoising, deblurring, inpainting, edge preserving and restoration. TV regularization method has been applied in image denoising [22], reconstruction for images that missing some pixels due to impulsive noise [23], and reconstruction of super resolution images [24]. There are also researches on combination of regularization methods for example, hybrid of Tikhonov and TV regularization method in [25-26] to compensate both methods' weaknesses thus improving the outcome. In this paper, the Total Variation (TV) regularization scheme will be incorporated in the FBTS inverse scattering algorithm to improve the reconstructed image by giving information such as its shapes and location, and also reconstructing sharp edges in the enclosed region that should give a better estimation of the image's boundaries.

2. Research Method

In FBTS, the nonlinear inverse scattering data are being formulated in the time domain by applying Finite-Difference Time-Domain (FDTD) scheme. The FDTD scheme is used to calculate the scattering signals in forward time-stepping and adjoint field in backward time-stepping. Convolution Perfectly Matched Layer (CPML) is used for the FDTD solution space for set up of simulation boundaries. The particular of CPML applied is considered as in [27]. Errors between measured and simulated microwave scattering measurement are compared and minimized by applying Polak- Ribiète-Polyak conjugate gradient method [12]. FBTS is then extended by adding a TV regularization method and the cost functional *F*(**p**) to be minimized is stated as follows.

$$\begin{aligned} F(\mathbf{p}) &= Q(\mathbf{p}) + Q_{TV}(\mathbf{p}) \\ &= \int_0^T \sum_{m=1}^{M} \sum_{n=1}^{N} \left| v_m(\mathbf{p}; \mathbf{r}_n, t) - \tilde{v}_m(\mathbf{r}_n, t) \right|^2 dt \\ &\quad + \lambda_{\varepsilon_r} \int_\Omega \sqrt{\left| \nabla \varepsilon_r(\mathbf{r}) \right|^2 + \alpha_{\varepsilon_r}}\, d\mathbf{r} + \lambda_\sigma \int_\Omega \sqrt{\left| \nabla \sigma(\mathbf{r}) \right|^2 + \alpha_\sigma}\, d\mathbf{r} \end{aligned} \quad (1)$$

Where $Q(\mathbf{p})$ is the first term and $Q_{TV}(\mathbf{p})$ is the second term or TV regularization term of the equation. *T* is the duration of measurement, *M* and *N* representing the transmitting and receiving antenna, $v_m(\mathbf{p};\mathbf{r}_n,t)$ is the calculated electromagnetic fields for an estimated medium **p**, and $\tilde{v}_m(\mathbf{r}_n,t)$ is the measured electromagnetic fields with respect to the m^{th} source. As for the second term, λ_{ε_r} and λ_σ are the regularization parameters, α_{ε_r} and α_σ are the small parameters, and Ω is an estimation region surrounding unknown objects. The medium parameter of vector function is given by:

$$\mathbf{p}(\mathbf{r}) = (\varepsilon_r(\mathbf{r}), \sigma(\mathbf{r})) \quad (2)$$

The TV of the second term is defined as follows.

$$\begin{aligned} Q_{TV}(\mathbf{p}) &= \lambda_{\varepsilon_r} Q_{TV_{\varepsilon_r}}(\varepsilon_r) + \lambda_\sigma Q_{TV_\sigma}(\sigma_r) \\ Q_{TV_{\varepsilon_r}}(\varepsilon_r) &= \int_\Omega \sqrt{|\nabla \varepsilon_r(\mathbf{r})|^2 + \alpha_{\varepsilon_r}}\, d\mathbf{r} \\ Q_{TV_\sigma}(\sigma) &= \int_\Omega \sqrt{|\nabla \sigma(\mathbf{r})|^2 + \alpha_\sigma}\, d\mathbf{r} \end{aligned} \quad (3)$$

Determining an optimal value of regularization parameters is an important criterion and should be chosen carefully for a good solution. The regularization parameters in Equation (1) are the smoothing agent for the TV gradient, if it too large for the solution, the reconstructed image will be over smoothed. The regularization parameters in this research were considered by means of numerical experiments with references as in [28-30]. The ratio of the actual image over the reconstruction image are calculated in term of Improvement in Signal-to-Noise Ratio (ISNR) metric. A larger ISNR value indicates a greater quality of the reconstructed image, as it also yields a low Mean Square Error (MSE). For this research, the optimal value of regularization parameters are $\lambda_{\varepsilon_r} = 0.1$ and $\lambda_\sigma = 0.012$.

The gradient of the $Q_{TV_{\varepsilon_r}}(\varepsilon_r)$ with respect to ε_r and that of the $Q_{TV_\sigma}(\sigma)$ with respect to σ are given by solving Frechet differential of $Q_{TV}(\mathbf{p})$.

$$g_{TV_{\varepsilon_r}}(\mathbf{r}) = \nabla \cdot \left(\frac{\nabla \varepsilon_r(\mathbf{r})}{\sqrt{|\nabla \varepsilon_r(\mathbf{r})|^2 + \alpha_{\varepsilon_r}}} \right) \quad g_{TV_\sigma}(\mathbf{r}) = \nabla \cdot \left(\frac{\nabla \sigma(\mathbf{r})}{\sqrt{|\nabla \sigma(\mathbf{r})|^2 + \alpha_\sigma}} \right) \quad (4)$$

The small parameters α_{ε_r} and α_σ in Equation (4) are included to avoid division by zero. The small parameters value should be small enough and not affect the TV gradient. In this research, the values of the small parameters are $\alpha_{\varepsilon_r} = \alpha_\sigma = 1.0 \times 10^{-24}$. The gradient of conjugate gradient method which is utilized for FBTS as in [12] is considered incorporating of TV method as in Equation (5).

$$g_{\varepsilon_r} = \int_0^T \sum_{m=1}^{M} \sum_{n=1}^{N} w_{mi}(\mathbf{p};\mathbf{r},t) \frac{d}{dt} v_{mi}(\mathbf{p};\mathbf{r},t) dt + \boldsymbol{\beta} g_{TV_{\varepsilon_r}}$$
$$g_{\sigma} = \int_0^T \sum_{m=1}^{M} \sum_{n=1}^{N} w_{mi}(\mathbf{p};\mathbf{r},t) v_{mi}(\mathbf{p};\mathbf{r},t) dt + \boldsymbol{\beta} g_{TV_{\sigma}} \quad (5)$$

Whereas $w_{mi}(\mathbf{p};\mathbf{r},t)$ and $v_{mi}(\mathbf{p};\mathbf{r},t)$ are the i^{th} component of electromagnetic fields $v_m(\mathbf{p};\mathbf{r},t)$ and adjoint fields $w_m(\mathbf{p};\mathbf{r},t)$ calculated in the reconstruction region, **r**. The weightage parameter **β** is to balance TV gradient term.

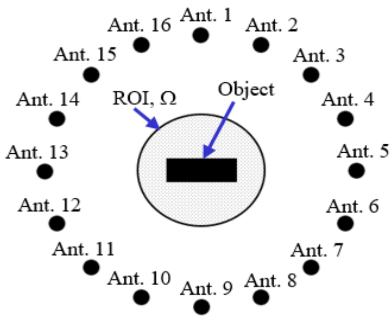
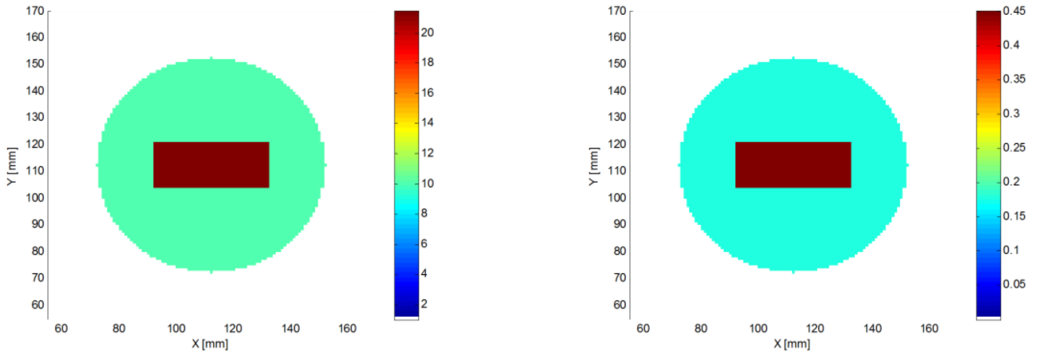


Figure 1. Antennas configuration of the problem

Figure 1 shows a typical configuration of an active microwave tomography setup for FBTS inverse scattering problem. The antennas act as transmitter and receiver sequentially whereas only a transmitter transmits a signal at a time. Other remaining antennas will act as a receiver collecting scattering signals. Each optimization requires a complete dataset of all antennas transmitting and receiving scattering signals. A sinusoidal modulated Gaussian pulse is used as a source signal. The source signal is centered at 2 GHz frequency and its bandwidth is 1.3 GHz. The test is carried out on a simple object detection in free space with 100 iterations and using 16 antennas. The object is a rectangle shaped object embedded inside the region of interest (ROI) with the length of 20mm and width of 8mm. While the ROI is a circular shaped with 40mm of diameter. The object is assume to be unknown shape and located in center of the ROI. The relative permittivity of 21.45 and the conductivity of 0.45 S/m are the electrical properties for the object. As for the ROI, the relative permittivity was set to be 9.98 and conductivity is 0.18 S/m. Due to nonlinearity of the inverse problem, conjugate gradient method does not always find its global minimum and its result is dependent on a proper initial guess [10]. A proper initial guess will avoid optimization to be trapped in local minimum. The initial guess chosen for relative permittivity is 13.7 and the conductivity is 0.20 S/m.

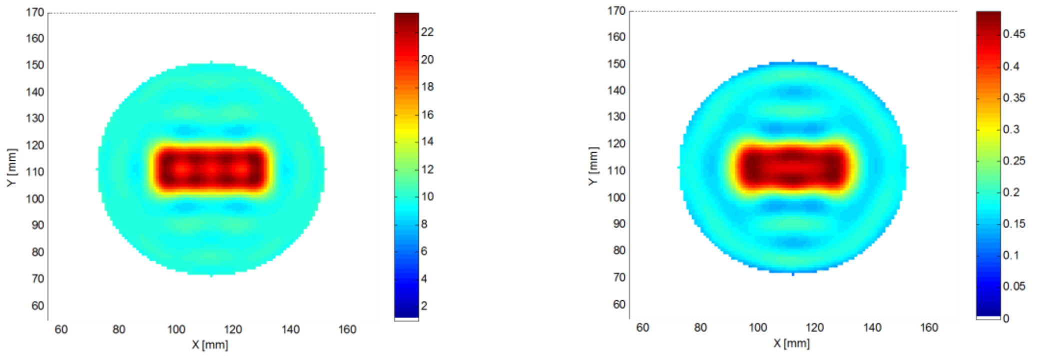
3. Results and Analysis

Several simulations of a simple object have been carried out to investigate the effectiveness and improvement of the reconstructed image as regularization method applied into the FBTS system. Figure 2 shows the actual profile of relative permittivity and conductivity of the simple object through accumulation of synthetic data. Figure 3 shows the reconstructed image of relative permittivity and conductivity of the simple object by FBTS without applying regularization method. Both of the relative permittivity and conductivity of the simple object have been well reconstructed through FBTS method. However, the reconstructed ROI and simple object have some vibrations, uneven surfaces and smoothed edges. TV regularization method is then applied to improve FBTS image reconstruction. Figure 4 shows the reconstructed image of relative permittivity and conductivity of the simple object by FBTS incorporating TV regularization method. Compared with FBTS simulation without regularization method, both of relative permittivity and conductivity reconstructions have showend an improvement. The simple object is reconstructed more firm and stable in shape which make the object detection more accurate. Uneven surfaces appeared more flat and smoothed. Figure 5 shows the cross sectional view of comparing image reconstructed by FBTS with and without TV regularization method. From cross sectional view, it is clear that reconstructed image of relative permittivity with TV regularization method is smoothed of irregular surface and more accurate, thus resembling the actual image.



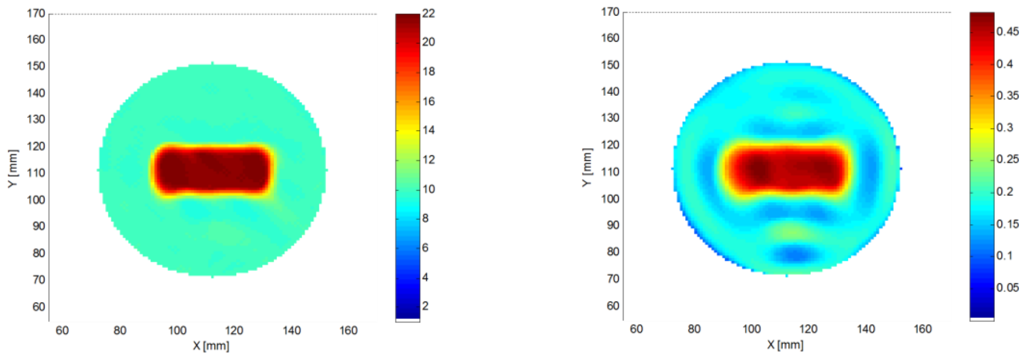
(a) Relative permittivity (b) Conductivity

Figure 2. Actual profile of relative permittivity and conductivity



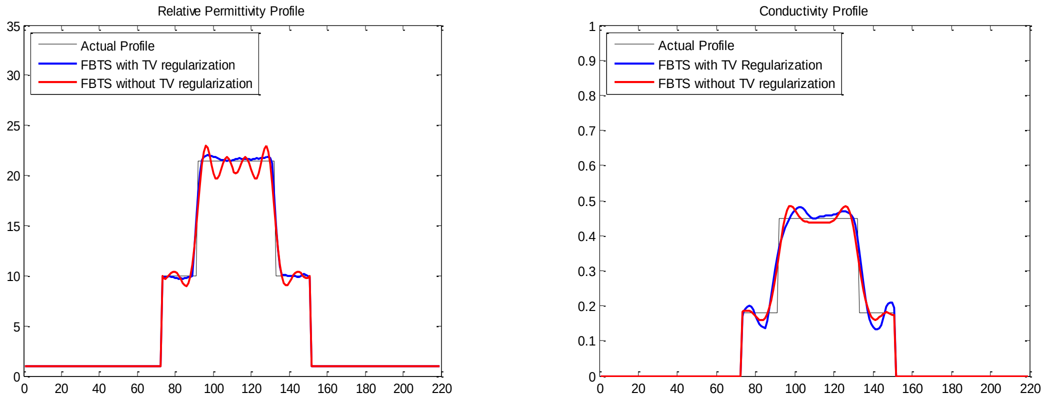
(a) FBTS reconstruction of relative permittivity (b) FBTS reconstruction of conductivity

Figure 3. Reconstruction of relative permittivity and conductivity using FBTS without TV regularization method



(a) FBTS-TV regularization reconstruction of relative permittivity (b) FBTS-TV regularization reconstruction of conductivity

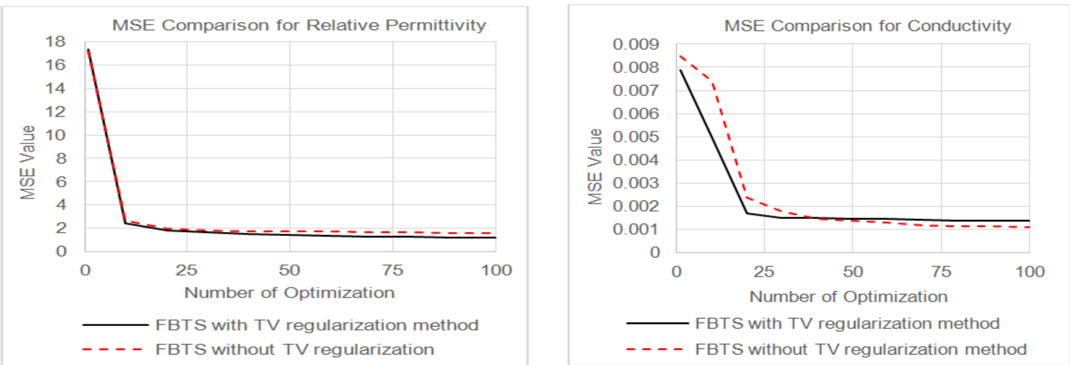
Figure 4. Reconstruction of relative permittivity and conductivity using FBTS with TV regularization method



(a) Cross sectional view of comparison of reconstruction of relative permittivity

(b) Cross sectional view of comparison of reconstruction of conductivity

Figure 5. Cross sectional view of comparison of reconstruction of relative permittivity and conductivity



(a) MSE value of reconstruction of relative permittivity

(b) MSE value of reconstruction of conductivity

Figure 6. Comparison of MSE calculation versus number of optimization for FBTS with and without TV regularization method

It is observed that the corner edges of the object to be smoothed and blurred. The image gradients are force to be sparse through TV regularization method and it tend to be penalized evenly causing low contrast structures of the reconstructed image might loss some information and over smoothed, this explained the round edges of the corner of the object. In contemplation to increase the performance of the reconstructed image especially at the low contrast profile region, the edge preserving technique could be added up to the existing TV regularization algorithm in such way to reduce the penalty weight of the regularization as in [30]. In the other hand, image reconstructed of relative permittivity is better than conductivity. This phenomena had been explained in [13]. The accuracy of the algorithm can be quantified by measuring the Mean Square Error (MSE). The MSE is calculated as follows"

$$MSE = \frac{1}{MN}\sum_{x=1}^{M}\sum_{y=1}^{N}[U(x,y) - U'(x,y)]^2 \qquad (6)$$

Where *U(x, y)* is the actual profile, $U'(x,y)$ is the reconstructed profile, and *M* × *N* is the range of the image. Figure of MSE comparison for FBTS with and without TV regularization method is as shown as Figure 6. The MSE values for FBTS with TV regularization method declined proportionally with optimization increment as expected. Reconstruction of relative permittivity utilizing FBTS with regularization method has a lower MSE value than the reconstruction without TV regularization method. In conductivity reconstruction, the MSE value for FBTS with TV regularization method showed a great performance for the first 40 iterations of optimization. Table 1 shows the MSE value of the performance of FBTS with and without TV regularization method. For a fair comparison, the smallest value of MSE of the performance throughout the optimization is taken.

Table 1. The MSE Value of FBTS With And Without TV Regularization Method

MSE Value	Relative Permittivity	Conductivity
Without TV Regularization	1.5488	0.0011
With TV Regularization	1.1745	0.0014

4. Conclusion

TV regularization method has a unique ability to enhance image reconstruction by smoothing irregular contours while preserved the edges. TV regularization method was applied in FBTS and through the numerical simulations, the proposed method has shown a good performance of preserving image sharpness and edges for the reconstructed images, and hence improve the FBTS algorithm stability and accuracy. The simple object has been successfully detected and its reconstruction is more solid and shape up like the actual object.

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