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A Blind Identification and Equalization for MC-CDMA Transmission Channel Using A New Adaptive Filter Algorithm

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Abstract

A review of literature shows that there is a variety of adaptive filters. In this research study, we propose a new type of an adaptive filter that increases the diversification used to compensate the chan-nel distortion effect in the MC-CDMA transmission. First, we show the expressions of the filter's impulse responses in the case of a perfect channel. The adaptive filter has been simulated and experienced by blind equalization for different cases of Gaussian white noise in the case of an MC-CDMA transmission with orthogonal frequency baseband for a mobile radio downlink channel Bran A. The simulation results show the performance of the proposed identification and blind equalization algorithm for MC-CDMA transmission chain using IFFT.

Keywords: *Blind equalization; Blind identification; MC-CDMA; OFDM; Adaptive Filter*

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1. Introduction

The Multi-Carrier Code Division Multiple Access (MC-CDMA) transmission is a technique that combines the CDMA technique and Orthogonal Frequency Division Multiplexing (OFDM) method [1]. It is used in several application areas, such as television transmission, transmission of broadband data and digital mobile communication, because of its robustness to minimize inter-ference (ISI) due to multipath propagation [2]. The adaptive equalization is an important step in digital transmission which is primarily based on the adaptive filter that allows a self-recovery sig-nal [3] to offset the distortions of the channel by eliminating interference (ISI). Thus, it minimizes the effect of noise, which leads to a possibility of increase in channel flow. Adaptive equalization was introduced by Sato [3] developed by Godard [4], and updated by Shalvi and Weinstein [5], [6]. It is of great importance in many application areas such as astrophysics, telecommunications and imaging. There is a variety of adaptive filters according to the pattern transmission. There are those based on minimization of the peak distortion and others based on the minimization of the mean quadratic error [7], [8].

However, these filters have some limitations as in the case of Z-forcing; the effect of noise is ampli-fied when one of the channel impulse responses is too small [7]. In this paper, we propose in the blind equalization a new adaptive filter that increases the diversification, in order to compensate the channel distortion effect. The new adaptive equalizer is based on a recursive adaptive filter, with impulse responses do not depend on the inverses of the channel impulse responses (for instance the Z-forcing), this permits to alleviate the divergence due to the noise. These impulse responses are obtained from a blind identification of the channel. For a noisy channel with Gaus-sian noise, we use cumulants-based algorithms [9-14]. Now, we present the principle of the proposed model, we demonstrate the expression of the impulse responses in the case of a perfect channel, we apply the blind channel identification by using AtAlg algorithm [14], we equalize the signal by the proposed filter and we calculate the bit error rate based on SNR, with a sample size in the order of 100. To validate our approach, we simulate for various values

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of signal to noise ratio (SNR), a downlink radio mobile channel, Bran A, noisy by Gaussian noise.

2. The Proposed Model
2.1 Hypotheses

We often model a multipath channel with a Single Input Single Output (SISO) by a linear FIR digital filter of an assumed known *q* order. Let *H* = [*h*(0)*;* *h*(1)*; :::h*(*q*)] the impulse responses of the filter. *h*(*i*) is a constant for a stationary channel time-invariant [15]. The model for medium finite differences adjusted channel (moving average: MA) without noise, is represented by the following relationship [14], [16], [17]:

$$y(n) = \sum_{j=0}^{q} h(j).x(n-j), \quad h(0) = 1 \ \text{(out noiseless)} \tag{1}$$

In the case of a noisy channel the output at time *n* is *z*(*n*) such as

$$z(n) = y(n) + b(n) \tag{2}$$

Where *x*(*n*) is the initial information at time *n*, with a non-Gaussian finite energy. *b*(*n*) the additive white Gaussian noise at time *n*. The *b*(*n*) are independent and identically distributed (iid) with a zero-mean. *z*(*n*) the output of noisy channel at time *n*. To eliminate interference symbol (IES) we offset the distortion of the channel by an adaptive filter according to the following pattern:

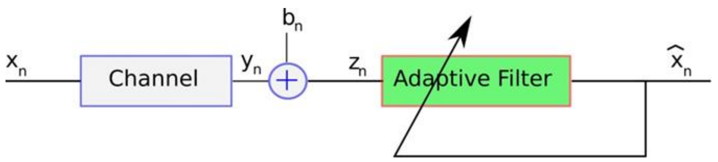

Figure 1. FIR model channel with adaptive filter

2.2 The Theoretical Expressions of the Impulse Responses in the Proposed Adaptive Filter

The role of the adaptive filter is to offset the distortion of a multipath channel. It's assumed as a FIR with a *q* order as a channel. We denote by *G* = [*g*(0)*;* *g*(1)*; : : : ;* *g*(*q*)] the impulse responses of the adaptive filter. We denote by *x*^(*n*) the output signal of the equalizer, it represents the reported estimated *x*(*n*).

For a linear invariant time of order *q* channel, information received at time *n* is written as:

$$y(n) = \sum_{i=0}^{q} h(i)x(n-i) \tag{3}$$

Where *x*(*n*) is the information issued at time *n* and *h*(*i*) the channel impulse response.

The use of the adaptive filter as a FIR is the same as the channel to estimate the transmitted information *x*^(*n*) by the following relationship:

$$\hat{x}(n) = \sum_{j=0}^{q} g(j)y(n-j) \tag{4}$$

Where *g*(*j*) the impulse response of the adaptive filter and *y*(*n j*) the the signal received at time *n j,* such that:

$$y(n-j) = \sum_{i=0}^{q} h(i)x(n-j-i) \tag{5}$$

So,

$$\hat{x}(n) = \sum_{j=0}^{q} g(j) \sum_{i=0}^{q} h(i)x(n-j-i) = \sum_{j=0}^{q}\sum_{i=0}^{q} g(j)h(i)x(n-j-i). \tag{6}$$

We put $k = i + j$ so

$$\begin{array}{ll} & 0 \leq k \leq 2q \\ and & 0 \leq j = k - i \leq q \\ so & k - q \leq i \leq k \end{array} \tag{7}$$

From where:

$$\hat{x}(n) = \sum_{i=0}^{q}\sum_{j=0}^{q} h(i)g(j)x(n-i-j) = \sum_{k=0}^{2q} x(n-k)\left(\sum_{i=k-q}^{k} h(i)g(k-i)\right) \tag{8}$$

It is well known that if $i < 0$ or $i > q$

$$h(i) = 0 \tag{9}$$

We conclude that under the following assumption:

$$\sum_{i=k-q}^{k} h(i)g(k-i) = 0, \quad \text{for all } k \neq 0, \tag{10}$$

We get the identity:

$$\hat{x}(n) = x(n) \tag{11}$$

We deduce from (8) that for $k = 0$:

$$\begin{array}{lll} k = 0 & \Longrightarrow & g(0) = \frac{1}{h(0)} \\ k = 2q & \Longrightarrow & g(q) = 0 \end{array} \tag{12}$$

$$g(k) = \frac{1}{h(0)}\left(-\sum_{i=1}^{k} h(i)g(k-i)\right) \tag{13}$$

Therefore, the proposed filter is a recursive filter, we also note that the impulse responses of order q of the adaptive filter are equal to zero.

3. The Blind Identification Algorithm

In the blind identification of Gaussian channel, the algorithm based on cumulant shows more robustness. In the first paragraph we present the fundamental relations of the identification cumulant algorithms, then in the second paragraph we review the AtAlg algorithm because it shows its effectiveness in blind identification [14].

3.1 The Fundamental Relations of the Identification Algorithm

The common point of all conventional methods of identifying adjusted average (MA) mod-els is the use of Brillinger and Rosenblatt formula [18], which, under the above assumptions is:

$$\begin{aligned} C_{m,Z}(\tau_1, ..., \tau_{m-1}) &= C_{m,Y}(\tau_1, ...\tau_{m-1}) \\ &= \gamma_{m,x} \sum_{i=0}^{q} b(i)b(i+\tau_1)...b(i+\tau_{m-1}) \end{aligned} \tag{14}$$

For $m = 2$, the autocorrelation is:

$$C_{2,Z}(\tau) = C_{2,Y}(\tau) + C_{2,N}(\tau) \qquad (15)$$

Where $C_{2;N}$ ( ) is the autocorrelation of the noise skewing results and $C_{2;Y}$ ( ) is the autocorrelation of the non-noisy signal expressed by:

$$C_{2,Y}(\tau) = \gamma_{2,x} \sum_{i=0}^{q} b(i)b(i+\tau), \quad (\gamma_{2,x} = \sigma_x^2) \qquad (16)$$

According to (14), one can easily demonstrate that the order cummulants *m* and *n*, with ($m > n$), verifies the following relationship:

$$\sum_{i=0}^{q} h(i) C_{m,Y}(i+\tau_1, ..i+\tau_{n-1}, \tau_n, ..., \tau_{m-1}) = \varepsilon_{m,n} \sum_{i=0}^{q} h(i) \left[\prod_{j=n}^{m-1} h(i+\tau_j)\right] C_{n,Y}(i+\tau_1, ..i+\tau_{n-1}) \qquad (17)$$

Algorithme 'C(q,k)' of Giannakis showed that the coefficients (FIR) can be expressed by the following formula [14].

$$b(\tau) = \frac{C_{m,Y}(q, \tau, 0, ..., 0)}{C_{m,Y}(q, 0, ..., 0)} \qquad (18)$$

With = 0,...,q and the cumulant of order m of excitation is:

$$\gamma_{m,x} = \frac{C_{m,Y}^2(q, 0, ..., 0)}{C_{m,Y}(q, q, ..., 0)} \qquad (19)$$

3.2 The AtAlg Algorithm

In this section, the impulse response $H = [h(0); h(1); :::; h(q)]$ is proposed to estimate a q order RIF channel using an algorithm that combines cumulants of order 3 and 4, as a previously proposed hypothesis. It also explains the method that improves the proposed algorithm [14].

Equation (17) is transformed into an equation which links *m* and *n* such that $m = n + 1$ as following:

$$\sum_{i=0}^{q} h(i) C_{m,Y}(i+\tau_1, ..i+\tau_{n-1}, \tau_n) = \varepsilon_{m,n} \sum_{i=0}^{q} h(i) h(i+\tau_n) C_{n,Y}(i+\tau_1, ..i+\tau_{n-1}) \qquad (20)$$

Especially $m = 4$ and $n = 3$, Equation (20) becomes:

$$\sum_{i=0}^{q} h(i) C_{4,Y}(i+\tau_1, i+\tau_2, \tau_3) = \varepsilon_{4,3} \sum_{i=0}^{q} h(i) h(i+\tau_3) C_{3,Y}(i+\tau_1, i+\tau_2) \qquad (21)$$

We take $\tau_1 = \tau_2 = q$ and $\tau_3 = \tau$, equation (21) becomes:

$$\sum_{i=0}^{q} h(i) C_{4,Y}(i+q, i+q, \tau) = \varepsilon_{4,3} \sum_{i=0}^{q} h(i) h(i+\tau) C_{3,Y}(i+q, i+q) \qquad (22)$$

Given that $C_{4,Y}(\tau_1, \tau_2, \tau_3) = C_{3,Y}(\tau_1, \tau_2) = 0$, si $\tau_i > q$; equation (22) becomes:

$$h(0)C_{4,Y}(q,q,\tau) = \varepsilon_{4,3}h(0)b(\tau)C_{3,Y}(q,q) \quad (23)$$

We deduce:

$$h(\tau) = \frac{C_{4,Y}(q,q,\tau)}{\varepsilon_{4,3}C_{3,Y}(q,q)} \quad (24)$$

With:

$$\varepsilon_{4,3} = \frac{\gamma_{4,x}}{\gamma_{3,x}} \quad (25)$$

According to equation (19), we deduce:

$$\varepsilon_{4,3} = \frac{C_{4,Y}^2(q,0,0)}{C_{4,Y}(q,q,0)}\frac{C_{3,Y}(q,q)}{C_{3,Y}^2(q,0)} \quad (26)$$

Than,

$$b(\tau) = \frac{C_{4,Y}(q,q,0)}{C_{4,Y}^2(q,0,0)}\frac{C_{3,Y}^2(q,0)}{C_{3,Y}(q,q)}\frac{C_{4,Y}(q,q,\tau)}{C_{3,Y}(q,q)} \quad (27)$$

The reduction of numerical calculations and the performance of the used statistical estimator can be a source of some divergence of values compared to the true value. To minimize these error differences signs we will also propose a selective choice of estimated values of im-pulse responses from the previous algorithms in the following format:

Since each calculated value is accompanied by an error, it is therefore considered as a fuzzy number [19] defined by an interval in the set R by the following Figure 2:

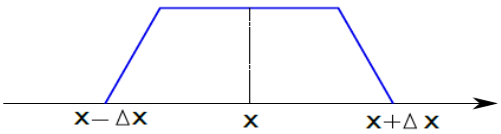

Figure 2. Fuzzy number representation

Figure 3 represents fuzzy values obtained by iterative simulation. Fuzzy values may be intersecting or not. We removed fuzzy extreme values having a zero intersection with the other fuzzy values. Indeed, these fuzzy values are far from the true value. Note that the number of fuzzy values, remaining after removal of the extrem ones must be greater than at least half of the iterations.

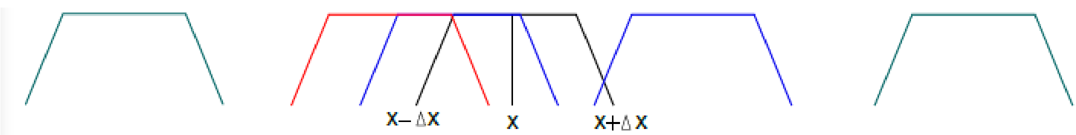

Figure 3. Representation of a fuzzy number of estimated series

Remark: AtAlg is the method of selection applied on our previous algorithm, given that the fuzzy variable is selected by:

$$H = \sum_{i=0}^{q} h(i) \quad (28)$$

where 2xΔH is the size fuzzy interval. The sum is fed to remove the divergence due to the undesired occurrence of the minus sign in one of the components of the impulse response.

4. A MC-CDMA Transmission Chain

The multicarrier modulation MC-CDMA is a robust technique especially for the attenuation of the signal due to multipath propagation. This method is used in radio communication for very high flow rates and because of the simplicity of implementation by IFFT.

The MC-CDMA transmission makes use of time and frequency diversity by spreading, on multi subcarriers orthogonal frequency, a pulse on a lot of chips, to ensure multiple users access of channel with a minimum (IES). In this paragraph we present the structure of the transmitter and the MC-CDMA receiver.

Figure 4 shows the steps of an MC-CDMA transmission baseband [1], [20], [21]. It notes a_i informative signal at time n from user i. $c_{i;j}$ with 1 j N_p, representing Walsh-Hadamard codes associated with the user i make it possible to ensure orthogonality between the information of different users. It attributes the chips by multiplixage of a_i on N_p code followed by the multi-pliants $c_{i;j}$. *IF F T* block is the inverse Fourier transform, ensures a multi-carrier modulation with orthogonality among frequencies. The *P=S* block is a dimultiplixing block by a summation signal for serial transmission on the same channel, are the x_i. The z_i are the receiver inputs signal in the equalizer. The $x\^{}_i$ is the estimated output of the equalizer. The $a\^{}_i$ are the estimated values of a_i after *F F T* between the series-parallel conversion and parallel-series.

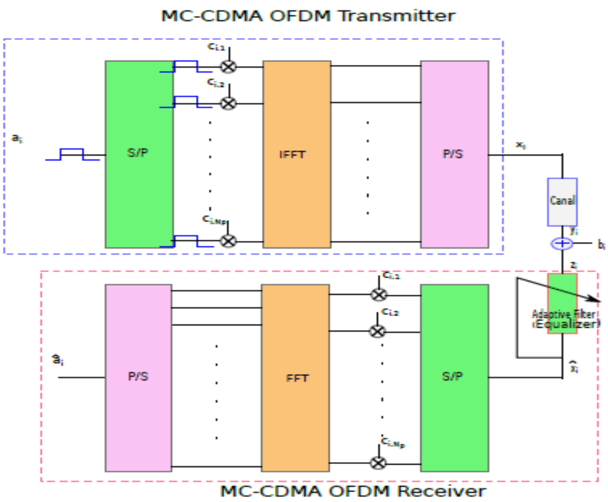

Figure 4. The proposed MC-CDMA transmision channel

4.1 The Structure of the MC-CDMA Transmitter

Data a_i from user i at the time nT are spread on N_p chips generated by the Walsh-Hadamard code. These are modulated by the subcarriers of the channel. Let $x(n)$ transmitter output, such as:

$$x(n) = \sum_{k=0}^{N_p-1} a_i c_{i,k} exp(j2\pi f_k nT); \quad (29)$$

N_p the number of subcarriers; $c_{i;k}$ codes Walsh-Hadamard associated with the user i. T duration inversely proportional to the width symbol of bandwidth channel B.

$$f_k = f_c + k\frac{B}{N_p} = f_c + k f_d \quad (30)$$

With $f_d = \frac{B}{N_p}$ width bands under f_c the center frequency of the bandwidth and $0 \leq k \leq N_{p-1}$. The f_k are the orthogonal frequencies. The previous expression (29) becomes:

$$x(n) = exp(j2\pi f_c n) \sum_{k=0}^{N_p-1} a_i c_{i,k} exp(j2\pi k \frac{B}{N_p} nT); \quad (31)$$

Where: $B = \frac{1}{T}$. We find the expression of the output baseband signal:

$$x(n) = \sum_{k=0}^{N_p-1} a_i c_{i,k} exp\left(j2\pi \frac{kn}{N_p}\right); \quad (32)$$

The orthogonality between the frequency (OFDM) is ensured by the inverse Fourier transform *IF F T* of $a_i c_{i;k}$ [1].

4.2 The Structure of the MC-CDMA Receiver

In the receiver, *z*(*n*) received at time *nT* are deconvoluted to remove signal distortion due to channel then multiplixed after they are multiplied by the user code generated by Walsh-Hadamard. The *F F T* operation is then finally applied to the dimultiplixed generated estimated values $a\hat{}_i$.

5. Simulations and Results

We take a sample size in the order of 100. The selected channel is the mobile radio downlink to a Bran A around a center frequency of 5*;* 24 GHz. The number of subcarriers is 64 bandwidth of the order of 1*;* 5 MHz. The duration of T_s symbol is equal to $\frac{1}{1.5} \mu s$ the same value as the life of a T_d chips, phase identification using 100 iterations. To evaluate the performance of the model we trace the signal to noise ratio (SNR) based on the bit error rate (BER) for a digital transmission BPSK. The SNR range from 0*;* 5 dB up to 25*;* 5 dB.

The following simulation results are obtained by using the Scilab mathematical software. The following figures show the transformations undergone by the signal on the transmission chan-nel for different values of SNR.

5.1 A Blind Identification and Equalization Channel for SNR = 0.5 dB

The impulse responses are determined following a blind channel identification, Figure (5). In the case of a very noisy channel, the impulse responses are far from those of the perfect channel. However, this difference plays an important role in the channel noise compensation in impulse responses estimation phase of the adaptive filter.

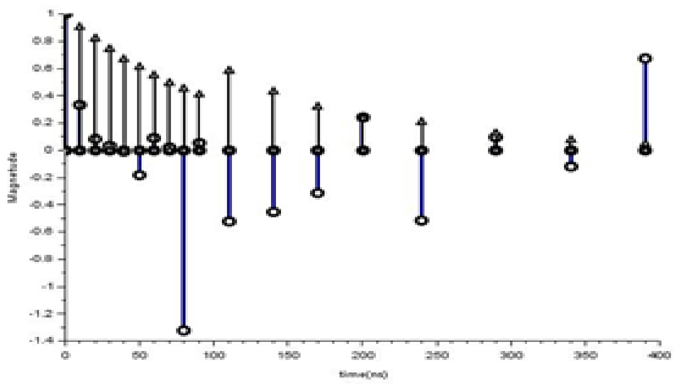

Figure 5. The impulse responses of the ideal channel behind and after blind identification based on time for SNR = 0,5 dB

The signal to the transmitter input is the BPSK signal which takes the values 1, Figure (6-a). The curve in Figure (6-b) represents the signal at the output that crosses the channel, it is noisy by Gaussian white noise of the order 0; 5 dB Figure (6-c). The output signal of the transmitter is completely hidden by the channel noise, then is received by the receiver after the signal processing is obtained, figure (6-d). However, after signal recovery, the number of errors is minimal. These errors can be reduced or deleted by channel coding.

Figure (7) summarizes a series of processing steps of the data signal at the input of the receiver, Figure (7-a) through the noisy channel (7-b) and restitution Figure (7-c). Note that the recovered signal is almost identical to the transmitted signal. This shows the performance of our blind identification algorithm, equalization adaptive filter, and the structure of the transmitter and the receiver of the Gaussian channel.

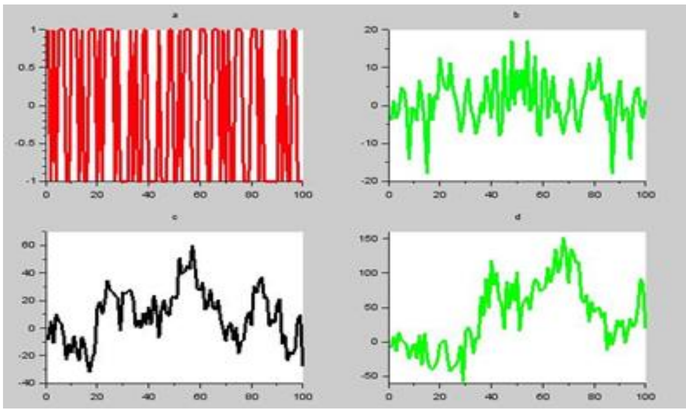

Figure 6. The different steps of transformation undergone by the signal from the transmitter and the channel for SNR = 0,5 dB

5.2 A blind Identification and equalization channel for SNR = 10,5 dB

The simulation is done for several values of SNR. In this pragraph, we take the case of SNR=10; 5 dB after a blind identification of the channel impulse responses are obtained. Figure (8) shows that the impulse responses reach the case of the ideal channel. This example shows that the deviation from the perfect event plays an important role in the channel noise compensation in the impulse responses estimation phase.

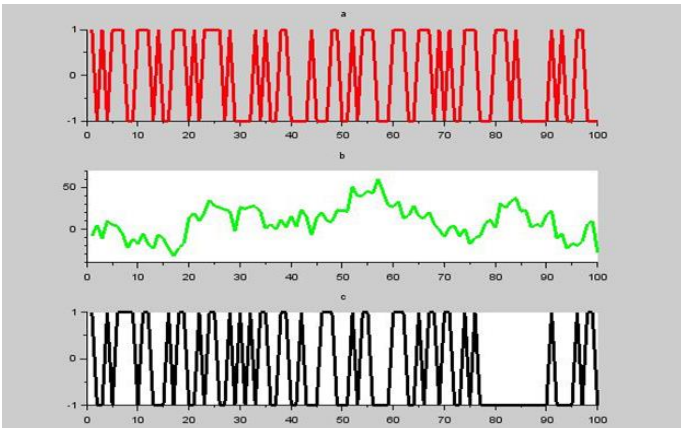

Figure 7. The different steps of processing of the signal from transmission through deconvolution baseband for SNR = 0,5 dB

The input signal to the transmitter is a BPSK signal that takes the values 1 , figure (9-a). The curve in figure (9-b) represents the signal at the output of transmitter. The signal passes through the channel. It is noisy by Gaussian white noise of around 10*;* 5 dB figure (6-c). The signal from the output of the transmitter is totally noisy, then it is received by the receiver after treatment yields the signal of figure (9-d) that perfectly resembles the transmitter output signal, which shows the effectiveness of the proposed adaptive filter.

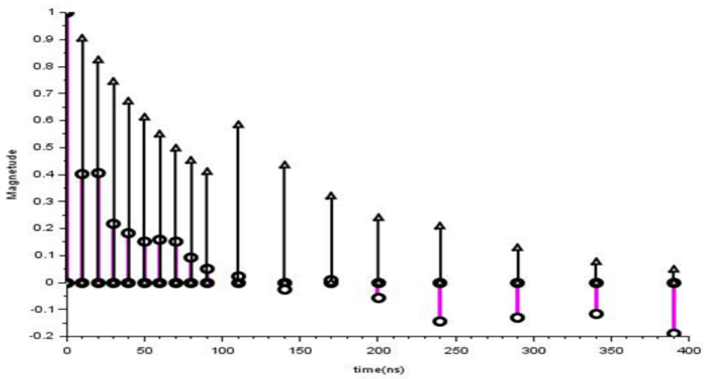

Figure 8. The impulse responses of the ideal channel and estimated blind identification based on time for SNR = 10,5 dB

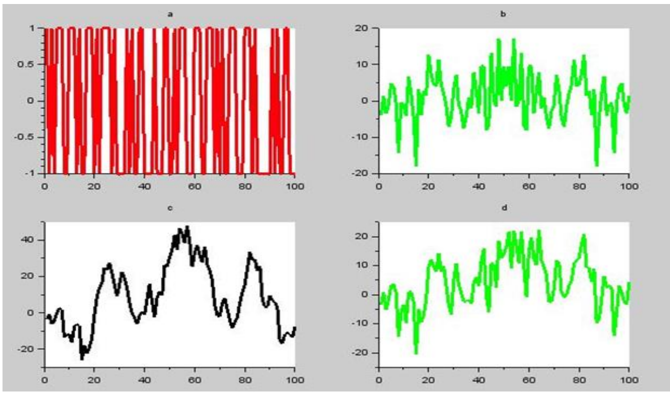

Figure 9. The different steps of transformation undergone by the signal from the transmitter and the channel for SNR = 10,5 dB

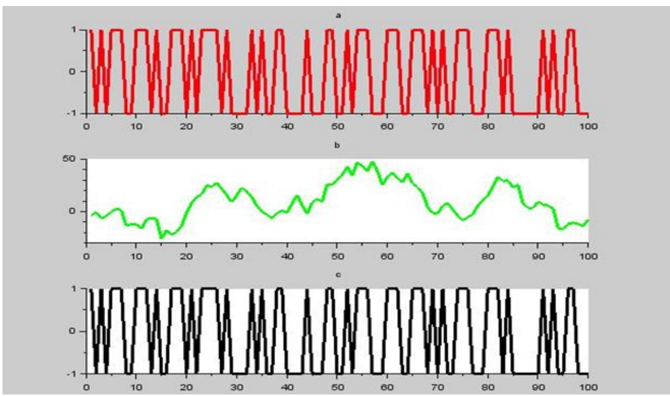

Figure 10. The different steps of processing of the signal from transmission through deconvolution baseband for SNR = 10,5 dB

Figure (10) shows the information signal (figure 10-a) and after the recovery signal (figure 10-c) and the noisy signal (figure 10-b). Notice that the recovered and the transmitted signals are identical. This shows the performance of the blind identification, equalization algorithm and the modulation mode selected for these algorithms.

5.3 Binairy Error Rate (BER) according to noise signal (SNR)

The simulation is made for different SNR values ranging from 0*;* 5 *dB* up to 25*dB* . Figure (11) represents the variation of the BER according to the SNR. From an SNR = 10 dB the proposed model gives no error. The proposed model of the adaptive filter therefore shows its effectiveness in equalization and transmission channel including the selection of the identification algorithm.

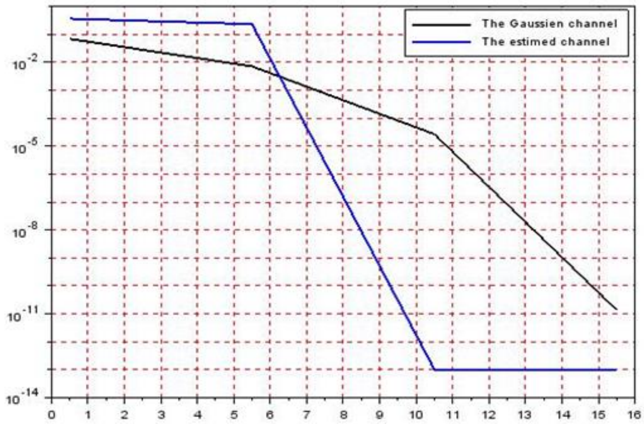

Figure 11. Representation of the binary error rate (BER) versus signal to noise ratio (SNR)

6. General Conclusion

The proposed recursive adaptive filter has been tested for different sample sizes less or greater than 100 for an MC-CDMA transmission system. The implementation uses the IFFT algorithm in the transmitter and the FFT algorithm in the receiver. The used blind identification algorithm is based on cumulants given the Gaussian nature of the noise of the channel. The approach shows its effectiveness for an intense noise. Indeed, from a noise whose SNR is greater than or equal to 10 dB the bit error rate tends to 0. The performance of the proposed algorithms of equalization, blind identification and the structure of the MC-CDMA transmission with IFFT and FFT are efficient and more satisfactory when they are combined together.

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