

## Adaptive-Fuzzy Back Stepping Control of Cable Robot Vessels

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### Abstract

Importance of sea in development, promotion and safety of the related countries make access to data and recognizing the environment become very important. Surface vessels help access to sea easier regarding technological advancement. Controlling surface vessels is considered as an active field of research regarding offshore, military and research uses. The main problem of controlling surface vessels is that these vehicles are often underactuated. Underactuation means that the system has lower number of independent actuators than degrees of freedom. In this study, designing an output feedback controller in the presence of parametric and non-parametric uncertainties independent of speed signal measurement and regarding actuators dynamic for path tracking of underactuated surface vessels, was studied. In designing controller, dynamic surface control method is used in order to reduce complexity of back stepping control. Moreover, adaptive robust techniques were used to control uncertainties. Based on Liapanov theory, stability of the proposed controlling is studied analytically and in the results tracking errors are uniformly bounded. Efficiency of the proposed controller is compared with an adaptive back stepping controller by numerical simulations and the reference tracking ability of the proposed controller is studied. It has been observed that the proposed controller compared to adaptive back stepping controller shows better performance in the presence of parametric and non-parametric uncertainties. Moreover, the controller shows shorter transient responses.

**Keywords:** underactuated surface vessel, tracking control, back stepping control, dynamic surface control, parametric and non-parametric uncertainties

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### 1. Introduction

Long before, human civilization has always been searching for outlying places to provide food and in order to reach the goal passing seas and rivers was important. Temptation of passing through seas and rivers made human to build vehicles for this purpose. In fact, vessels have developed beside humans and they were the key to huge discoveries and technological advancements in history.

Prominence of the first ocean vehicle dates back to Neolithic period approximately 10 thousand years ago but these vehicles are not considered as ships. The first vessels used wood to build the body, wool strip to join wood pieces and animal skin to build sails. Until Renaissance, maritime technology was remained primitive and initial advancements in building small, large and larger ships of higher facilities had a slow development. In the late 14th century, turrets are put on carrack ships' chests which caused reduction in stability of the ship and they were replaced by the decks. In the era, technology advanced very fast in Europe. Lots of new commercial routes were established and sea trade has developed using transportation system advances. Moreover, this causes advancement in high-tech military and commercial ships.

Ship design has remained intact till the late 19th century. Industrial revolution opened new horizons in science and experience was replaced by idea. Innovating modern technologies and tools speeded up and human needs increased as well.

Modern mechanical methods of propulsion and building ships from metal made a huge development in ship design and ships were improved for new uses like saving and survey. Today, human has been successful in designing safe, fast and maneuverable vessels using modern technologies.

Autonomous surface robots are considered as autonomous cable robots regarding their structural similarity. These robots can do the specific job without human interference on the surface of the water. These robots are in small, medium, large and very large dimensions with different uses. However, small sizes often used in places that are dangerous for humans like military sea operations, electronic war and special operation. These vessels are often considered as navy robotic systems and they have been developing very fast to become smaller, cheaper and more durable. During the last two decades, advancements were made in the field of navy robotic technology. However, more studies are needed to solve some issues such as autonomous control, mapping, navigation and sensor, which are all considered as active fields for the study.

Controlling underactuated cable robots is an active research field regarding important uses like oil and gas, cabling, environmental survey and travel. underactuated cable robots have two actuators in the longitudinal and rotation axis. These actuators are usually two propellers. In other words; the main problem of underactuated cable robots is that they don't have independent actuator in the latitude axis [1-7].

Design of a tracking controller for underactuated robots are presented using direct Liapanov and back stepping technique, assuming that all the parameters are known. In this paper, adaptive controller was developed to improve performance of the controller in the presence of uncertainties. Liapanov stability analysis shows that tracking errors are asymptotically converging [8-10].

## 2. Research Method

### 2.1. Kinematic and Dynamic Description of Cable Robot on the Surface

In surface vessels, inertia and damping matrixes depend on wave frequency. Assuming these matrixes known, might cause careless movements of the surface vessel [11]. On the other hand, adaptive controller makes the movement more accurate in the presence of uncertainty. In fact, the purpose of developing adaptive control in the paper, is to make the surface vessel track the reference path asymptotically in the presence of uncertainty [12]. Therefore, assuming that inertia matrix and damping matrixes are unknown, an adaptive controller is developed to estimate the uncertainties. Designing an adaptive controller has been divided into 3 parts:

1. Defining a control law involving uncertainties.
2. Defining an adaptive controller to estimate uncertainties
3. Stability analysis of the controller [13-14].

Mathematical model of a surface cable robot evolving kinematics and dynamics equations is written as follows:

$$\dot{\eta} = J(\eta)v \quad (1)$$

$$M\dot{v} = -C(v)v - Dv + \tau + \tau_E \quad (2)$$

The model of (1) can be rewritten as follows:

$$\dot{\eta} = S(\Psi)v + \delta(v, \Psi) \quad (3)$$

$\eta = [x, y, \Psi]^T$  is the position and direction vectors on Earth's reference frame,  $v \in \mathcal{R}$  is a vector of linear and angular speed in longitude and direction axis respectively and  $\delta(v, \Psi) \in \mathcal{R}^3$  is the unmatched disturbances vector as follows [10]:

$$S(\Psi) = \begin{bmatrix} \cos \Psi & 0 \\ \sin \Psi & 0 \\ 0 & 1 \end{bmatrix}, \delta(v, \Psi) = \begin{bmatrix} -v \sin \Psi \\ v \cos \Psi \\ 0 \end{bmatrix}, v = [u, r]^T \quad (4)$$

Dynamic equations of (2) can be rewritten as follows:

$$M_1\dot{v} + C_1(v)v + D_1v = \tau_v + \tau_{w1}(u, v, t) \quad (5)$$

In equation (5),  $M_1$  is the inertia matrix,  $C_1(v)$  is the Coriolis matrix,  $D_1$  is the damping matrix,  $\tau_{w1}(u, v, t)$  is the force and momentum vector caused by external disturbances and  $\tau_a$  is the vector of controlling input.

$$M_1 = \begin{bmatrix} M_{1i} & 0 \\ 0 & M_{33} \end{bmatrix}, C_1(v) = \begin{bmatrix} 0 & -m_{22}v \\ (m_{22} - m_{11})v & 0 \end{bmatrix}, D_1 = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{33} \end{bmatrix}, \tau_{w1}(t) = \begin{bmatrix} \tau_{wu}(t) \\ \tau_{wr}(t) \end{bmatrix} \quad (6)$$

Equation (5) has the following features:

- $M_1$  is a symmetric and positive matrix. In other words;  $M_1 = M_1^T > 0$   
In which upper and lower bound are as:  $\lambda_{m1} \|x\|^2 \leq x^T M_1 x \leq \lambda_{M1} \|x\|^2 \forall x \in \mathcal{R}^2$  and  $0 < \lambda_{m1} \leq \lambda_{M1} < \infty$  such that  $\lambda_{m1} := \min \lambda_{min}(M_1)$  and  $\lambda_{M1} := \max \lambda_{max}(M_1)$ .
- Following bounds existed for dynamic model (5) and kinematic model (3):

$$\|s(\Psi)\| \leq s_1, \|C_1(v)\| \leq \lambda_{C1} \|v\|, \|D_1\| \leq \lambda_{D1}, \|\tau_{w1}(t)\| \leq \lambda_{w1} \quad (7)$$

In equation (7)  $s_1, \lambda_{C1}, \lambda_{D1}$  and  $\lambda_{w1}$  are positive constants.

## 2.2. Dynamic model of actuators

To consider dynamics of actuators, it has been assumed that cable robot is stimulated by DC motors connected to gearbox. Drive system is shown in figure 1. Electrical equation of  $i$ th DC motor is written as follows [11,13]:

$$\begin{aligned} u_{ai} &= l_{ai} \dot{I}_{ai} + r_{ai} I_{ai} + k_{bi} \dot{\theta}_{mi} + u_{di} \\ \tau_{mi} &= k_{\tau i} I_{ai} \end{aligned} \quad (8)$$

In equation (8),  $k_{bi}$  is the recursive driving force,  $r_{ai}$  and  $l_{ai}$  are resistance and inductance of armature respectively,  $u_{ai}$  is the input voltage,  $u_{di}$  is the non-parametric uncertainty,  $\tau_{mi}$  is the actuator torque. Looking at the relation between torque and speed,  $\dot{\theta}_{mi} = n_i \dot{\theta}_i$  and  $\tau_i = n_i \tau_{mi}$  in which  $n_i$  is the gear ratio. Dynamics of actuator is written as follows:

$$L_a \dot{I}_a + R_a I_a + NK_b X v + u_d = u_a \quad (9)$$

$$\tau = NK_{\tau} I_a \quad (10)$$

In the (10):

$$\begin{aligned} L_a &= \text{diag}[l_{a1}, l_{a2}], R_a = \text{diag}[r_{a1}, r_{a2}] \\ N &= \text{diag}[n_1, n_2], k_b = \text{diag}[k_{b1}, k_{b2}] \\ k_{\tau} &= \text{diag}[k_{\tau 1}, k_{\tau 2}] \end{aligned}$$

$\tau, u_a, I_a \in \mathcal{R}^2$  Shows torque vectors, voltage and current of armature and  $X$  is the transfer matrix which transfers angular velocity of the propellers into linear velocity into longitude direction and angular velocity into rotational direction.

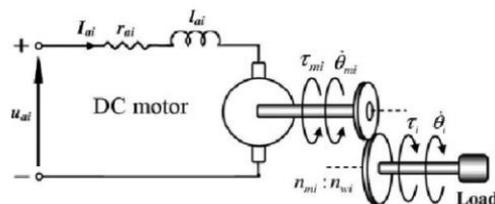


Figure 1. Drive System of Underactuated Surface Cable Robot [15]

In order to design controller, output equations are defined as follows:

$$z = h(\Psi) = [x + L \cos(\Psi), y + L \sin(\Psi)]^T \quad (11)$$

In equation (11),  $z \in \mathcal{R}^2$  is a variable of position and  $L$  is the look-ahead distance shown in figure (2). In the figure, a surface cable robot with two propellers is shown.  $O_b$  is the origin of coordinate system and  $P_l$  is the virtual origin on the  $X$  axis in the body coordinate with the distance of  $L$  from  $O_b$ . Derivate equation (11) and replacing it in equation (3) makes:

$$\dot{z} = J(\Psi)v + J_\delta(v, \Psi) \quad (12)$$

in equation (12):

$$\begin{aligned} J(\Psi) &= J_h(\Psi)S(\Psi) \in \mathcal{R}^4 \\ J_\delta(v, \Psi) &= J_h(\Psi)\delta(v, \Psi) \end{aligned} \quad (13)$$

In which  $J_h(\Psi) := \partial(\eta)/\partial\eta$  is as:

$$J(\Psi) = \begin{bmatrix} \cos(\Psi) & -L \sin(\Psi) \\ \sin(\Psi) & L \cos(\Psi) \end{bmatrix}, J_\delta(v, \Psi) = \begin{bmatrix} -v \sin(\Psi) \\ v \cos(\Psi) \end{bmatrix} \quad (14)$$

Since  $\det(J(\Psi)) \neq 0$ . Therefore;  $J(\Psi)$  is reversible and its reverse is as follows:

$$J^{-1}(\Psi) = \begin{bmatrix} \cos(\Psi) & \sin(\Psi) \\ -\sin(\Psi)/L & \cos(\Psi)/L \end{bmatrix} \quad (15)$$

Using equation (12) we have [16]:

$$v = J^{-1}(\Psi)(\dot{z} - J_\delta(v, \Psi)) \quad (16)$$

$$\begin{aligned} \dot{v} &= J^{-1}(\Psi) \left[ \ddot{z} - \dot{J}_\delta(v, \Psi) \right] - j(\Psi)J^{-1}(\Psi)(\dot{z} - J_\delta(v, \Psi)) \\ &= J^{-1}(\Psi) \left[ \ddot{z} - j(\Psi)J^{-1}(\Psi)\dot{z} \right] + \rho(v, \Psi) \end{aligned} \quad (17)$$

In equation (17):

$$\rho(v, \Psi) = -J^{-1}(\Psi)[\dot{J}_\delta(v, \Psi) - j(\Psi)J^{-1}(\Psi)J_\delta(v, \Psi)] \quad (18)$$

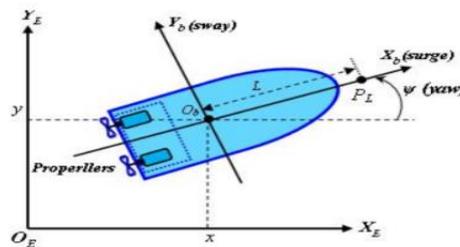


Figure 2. A Surface Cable Robot Moving Surface in Horizontal Plane [17]

Replacing equation (16) and (17) in the equation (5):

$$\begin{aligned} M_1 J^{-1}(\Psi) \ddot{z} + C_1(v) J^{-1}(\Psi) \dot{z} + D_1 J^{-1}(\Psi) \dot{z} - M_1 J^{-1}(\Psi) j(\Psi) J^{-1}(\Psi) \dot{z} + M_1 \rho(v, \Psi) - \\ C_1(v) J^{-1}(\Psi) J_\delta(v, \Psi) - D_1 J^{-1}(\Psi) J_\delta(v, \Psi) - \tau_{w1}(t) = \tau_a \end{aligned} \quad (19)$$

Regarding equation (10) and multiplying equation (19) by  $J^{-T}(\Psi)$ , following relation is defined in Earth's coordinate:

$$M_2(\Psi)\ddot{z} + C_2(\Psi, \dot{z})\dot{z} + D_2(v, \Psi)\dot{z} + \tau_{w2}(v, \Psi, t) = J^{-T}(\eta)I_a \quad (20)$$

In equation (20):

$$\begin{aligned} M_2(\Psi) &= (NK_T)^{-1}J^{-T}(\Psi)M_1J^{-1}(\Psi) \\ C_2(\Psi, \dot{z}) &= -(NK_T)^{-1}J^{-T}(\Psi)M_1J^{-1}(\Psi)j(\Psi)J^{-1}(\Psi) \\ D_2(v, \Psi) &= (NK_T)^{-1}J^{-T}(\Psi)(D_1 + C_1(v))J^{-1}(\Psi) \\ \tau_{w2}(v, \Psi, t) &= (NK_T)^{-1}J^{-T}(\Psi)[M_1\rho(v, \Psi) - \\ &\quad C_1(v)^{-1}J_\delta(v, \Psi) - D_1J^{-1}(\Psi)J_\delta(v, \Psi) - \tau_{w1}(t)] \end{aligned} \quad (21)$$

Equation (20) has below features:

1.  $M_2(\Psi)$  is a symmetrical and definite positive matrix in which the upper and lower bound are as follows [18]:

$$\lambda_{m2}\|x\|^2 \leq x^T M_2 x \leq \lambda_{M2}\|x\|^2 \quad \forall x \in \mathcal{R}^2, 0 < \lambda_{m2} \leq \lambda_{M2} < \infty$$

Such that  $\lambda_{M2} := \max_{\Psi \in \mathcal{R}} \lambda_{max}(M_2(\Psi))$ ,  $\lambda_{m2} := \min_{\Psi \in \mathcal{R}} \lambda_{min}(M_2(\Psi))$

2. Matrix  $\dot{M}_2 - 2C_2$  is anti-symmetric. In other words;

$$x^T(\dot{M}_2 - 2C_2)x = 0, \forall x \in \mathcal{R}^2 \quad (22)$$

3. Positive constants  $\lambda_{M2}$ ,  $\lambda_{C2}$ ,  $\lambda_{D2}$ ,  $\lambda_J$ ,  $\lambda_{J\delta}$  and  $\lambda_{w2}$  exist as follows:[19]

$$\begin{aligned} \|M_2(\Psi)\| &\leq \lambda_{M2}, \|C_2(\Psi, \dot{z})\| \leq \lambda_{C2}\|\dot{z}\|, \|D_2(v, \Psi)\| \leq \lambda_{D2} \\ \|\tau_{w2}(v, \Psi, t)\| &\leq \lambda_{w2}, \|J(\Psi)\| \leq \lambda_J, \|J_\delta(v, \Psi)\| \leq \lambda_{J\delta} \end{aligned} \quad (23)$$

### 2.3. Controlling Purposes and Hypothesis

Definition 1: consider the bounded reference path  $z_d(t) = h(\eta_d(t))$ , designing a feedback control law for system 20 considering actuators dynamics without measuring velocity signals in such that error tracking  $z_e(t) = z(t) - z_d(t)$  is uniformly bounded in the presence of parameter and non-parameter uncertainties are the purpose of this study.

Method 1: latitudinal dynamics of a surface cable robot can be stated as  $\dot{v} = m_{22}^{-1}[-m_{23}\dot{r} - c_{23}r - d_{22}v - d_{23}r]$  such that  $\partial c_{23}/\partial v = 0$  and  $d_{22} > 0$ . For bounded signals  $u$  and  $r$ , if  $|d_{23}/d_{22}| < \infty, \forall v$  then latitudinal velocity is always bounded. To study more details and proving above method, refer to reference [20].

To reach the desired controlling purpose, considering following hypothesis is necessary:

1. Measurements are accessible for the output vector  $z \in \mathcal{R}^2$  in any moment.
2. Reference path  $z_d(t)$  is chosen in such that signals  $z_d(t)$ ,  $\dot{z}_d(t)$ ,  $\ddot{z}_d(t)$  and  $\ddot{z}_d$  have bounds meaning  $\sup_{t \geq 0} \|z_d\| < B_{dp}$ ,  $\sup_{t \geq 0} \|\dot{z}_d\| < B_{dv}$ ,  $\sup_{t \geq 0} \|\ddot{z}_d\| < B_{da}$  and  $\sup_{t \geq 0} \|\ddot{z}_d\| < B_{dj}$  in which  $B_{dp}$ ,  $B_{dv}$ ,  $B_{da}$  and  $B_{dj}$  are bounded constants.
3. Velocity is bounded in the longitudinal direction [21-22].

### 3. Designing the Controller

In this part, an output feedback controller using controlling dynamic surface method for tracking path of a surface cable robot is designed. Then, stability analysis regarding Liapanov theory is performed uniformly bounded tracking error and state estimation is gained [23]

### 3.1. Designing the Controller and the Observer

First step: consider following definitions:

$$\dot{z}_r := \dot{z}_d - \Lambda(\hat{z} - z_d) = \dot{z}_d - \Lambda z_e + \Lambda z_z \quad (24)$$

$$\dot{z}_0 := \dot{\hat{z}} - \Lambda z_z \quad (25)$$

$$r := \dot{z} - \dot{z}_0 = \dot{z}_z + \Lambda z_z \quad (26)$$

Here,  $z_z := z - \hat{z}$  is definite error vector,  $\Lambda \in \mathcal{R}^4$  is a diagonal gain matrix and it is definite positive. Based on the definitions, the first dynamic level is defined as follows:

$$S_1 := \dot{z} - \dot{z}_r = \dot{z}_e + \Lambda z_e - \Lambda z_z \quad (27)$$

Based on equations of (20) and (27):

$$M_2(\Psi)\dot{S}_1 = -C_2(\Psi, \dot{z})S_1 - C_2(\Psi, \dot{z}_r)S_1 - D_2(v, \Psi)S_1 + \xi + J^{-T}(\Psi)I_a \quad (28)$$

In equation (28),  $\xi = -M_2(\Psi)\ddot{z}_r - C_2(\Psi, \dot{z}_r)\dot{z}_r - D_2(v, \Psi)\dot{z}_r + \tau_{w2}(v, \Psi, t)$  contains non-linear uncertainty which is bounded regarding equation (23), in the other words,  $\|\xi\| \leq f(\dot{z}_r, \ddot{z}_r)$ . Virtual control of  $\bar{I}_a$  is defined as:

$$\bar{I}_a = J^T(\Psi)(-K_1(\dot{z}_0 - \dot{z}_r) - k_2(z_e + z_z) - f_c(\hat{f}(\dot{z}_r, \ddot{z}_r)sgn(S_1 + r))) \quad (29)$$

In equation (29),  $K_1, K_2 \in \mathcal{R}^4$  are definite positive and diagonal matrixes.  $f_c$  is the continuous time approximation of sign function which satisfies the following conditions [21,24].

$$\begin{aligned} (S_1 + r)^T f_c(\hat{f}sgn(S_1 + r)) &\geq 0 \\ \hat{f}\|S_1 + r\| - (S_1 + r)^T f_c(\hat{f}sgn(S_1 + r)) &\leq \delta_1(t) + \delta_2(t) \end{aligned} \quad (30)$$

In which  $\delta_i(t), i = 1, 2$  are bounded positive time-variable scalar. In equation (29),  $\hat{f}(\dot{z}_r, \ddot{z}_r) = F(\dot{z}_r, \ddot{z}_r)\hat{a}$  is the upper bound prediction of  $f(\dot{z}_r, \ddot{z}_r)$  in which  $\hat{a}$  and  $F(\dot{z}_r, \ddot{z}_r) = [1 \quad \|\dot{z}_r\|^2 \quad \|\ddot{z}_r\|]$  is updated using following relation:

$$\hat{a} = \Gamma_1 F^T(\dot{z}_r, \ddot{z}_r)\|S_1 + 1\| - \Gamma_1 \sigma_1(\hat{a} - \alpha_0) \quad (31)$$

In equation (31),  $\Gamma_1 = \gamma_1 I_4$  is adaption gain,  $\sigma_1$  is the small positive constant and  $\alpha_0 \in \mathcal{R}^4$  is the initial estimation of the variables. Moreover, the following linear observer is used to estimate speed vectors [22,25]:

$$\dot{\hat{z}} = \dot{\hat{z}}_0 + \Lambda z_z = k_0 z_z \quad (32)$$

$$\ddot{\hat{z}}_0 = \ddot{z}_r + k_0 \Lambda z_z \quad (33)$$

Fuzzy logic model is based on experience and depends on operator's experience, not technological understanding of the system. Fuzzy logic is able to imitate behavior with very high speed. Expert fuzzy system is a system which uses fuzzy logic instead of Boolean logic, in logical data. These systems lead to database which is flexible and very enriched and easily to understood by the experts. Law abiding Systems have three general advantage over other common classification of systems. First, a large training set is not needed. Second, expert fuzzy systems are parallel for act in a parallel way for all effective rules [25].

Parallel operations have many advantages for fuzzy systems. Programming becomes easier and more applicable and it becomes significantly faster than an equivalent numeric system and that is what we need in real time fatigue recognition system. Third, law based expert systems have the ability to make law and add data to the database. This feature causes the developed system to be adaptable for hidden situations and later developments. For the fuzzy

controller, two inputs and an output is considered. Limit and range for the output is shown below [26-28].

$179.6 \leq \text{speed} \leq 180$	<i>good</i>
$164 \leq \text{speed} \leq 180$	<i>medium small</i>
$140 \leq \text{speed} \leq 172$	<i>very small</i>
$180 \leq \text{speed} \leq 196$	<i>medium big</i>
$188 \leq \text{speed} \leq 220$	<i>very big</i>

#### 4. Results and Analysis

In this section, to evaluate performance and robustness of proposed controller for an underactuated surface cable robot in the presence of parametric and non-parametric uncertainties, using MATLAB software, two simulations on a small model with the length of 1.2 meters and mass of 17.5 kg and other parameters taken from the reference 29 is performed.

Table 1. Parameters of Surface Cable Robot

Parameter	Symbol	Value	Unit
Mass added with extra mass longitudinally	$m_{11}$	25.805	kg
Mass added with extra mass attitudinally	$m_{22}$	33.856	kg
Mass added with extra mass circularly	$m_{33}$	2.743	kg
Length attenuation	$d_{11}$	12.436	Kg/s
Latitudinal attenuation	$d_{22}$	17.992	Kg/s
Circular attenuation	$d_{33}$	0.564	Kg/s

It is assumed that system dynamics and actuators parameters are indefinite. Moreover, in order to simulate non-parametric uncertainties of systems, dynamics and actuators are defined as:

$$\begin{aligned} \tau_{w2} &= [5\sin(t/20) \ 5\sin(t/20)] \\ u_d &= [\sin(t/20), \sin(t/20)] \end{aligned} \quad (34)$$

Moreover, zero-mean random noise is added to measured output signal for simulation. In the first simulation, a circular reference path is considered for evaluating performance of the controller:

$$z_r = [x_f + R \cos(\omega_r t), y_f + R \sin(\omega_r t)] \quad (35)$$

Here,  $\omega_r = 0.05$  and  $R = 2m$ ,  $(x_f, y_f) = [2.5m, 5.5m]$  are the path reference parameters. The forward distance is considered  $L=0.15$ .

In simulations, controlling parameters are defined as follows to reach suitable tracking:

$$\Lambda = 0.3I_4, k_1 = 9I_4, k_2 = 10.5I_4, k_3 = 4I_4, k_0 = 20, \tau = 0.025 \quad (36)$$

However, big controller gains are not used to prevent actuator saturation. Adaptive gains are increased from zero to obtain suitable convergence rate.  $\Gamma_1 = 2 \text{diag}[1,1,0.25,1]$  و  $\Gamma_2 = 10^{-3} \text{diag}[1,1,1,1]$  were defined as gains for simulations. Other parameters for the controller are as follows:

$$\sigma_1 = 0.009, \sigma_2 = 0.0005, \sigma_0 = 0, \delta_0 = 0 \quad (37)$$

According to notation 1, Robust control law of equations (29) and (39) were chosen as:

$$f_c = (\hat{f} \text{sgn}(\hat{S}_1 + \hat{r})) = \frac{(\hat{S}_1 + \hat{r})\hat{f}^2}{\hat{f}\|\hat{S}_1 + \hat{r}\| + \delta(t)} \quad (39)$$

$$h_c = (\hat{h} \operatorname{sgn}(S_2)) = \frac{S_2 \hat{h}^2}{\hat{h} \|S_2\| + l(t)} \tag{40}$$

Such that conditions (29) and (39) are satisfied. Thickness of boundary layer in control laws of (39) and (40) is considered as follows:

$$\delta(t) = \begin{cases} 5000 & \text{if } 0 \leq t \leq 5 \\ 1 + 5000e^{-0.2(t-5)} & \text{if } t > 5 \end{cases} \tag{41}$$

Initial conditions of surface cable robot for the first simulation are:  $[x(0), y(0), \Psi(0)] = [6, 5, \frac{\pi}{7}]$ .

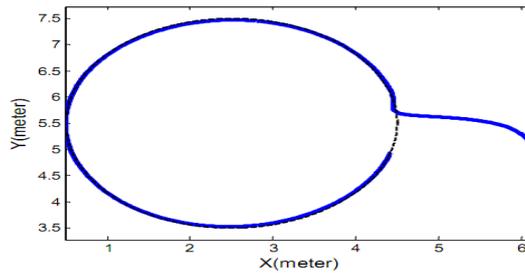


Figure 3. Reference Path Curve and Real Device Path Curve on the (x,y) Plane

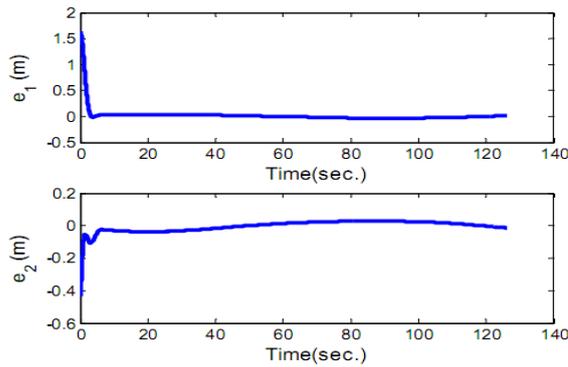


Figure 4. Output Tracking Errors

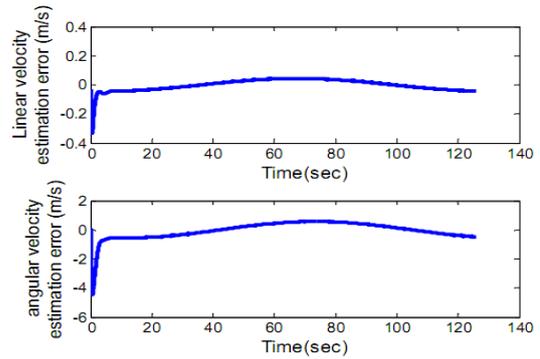


Figure 5. Estimation Error of Linear and Angular Velocity

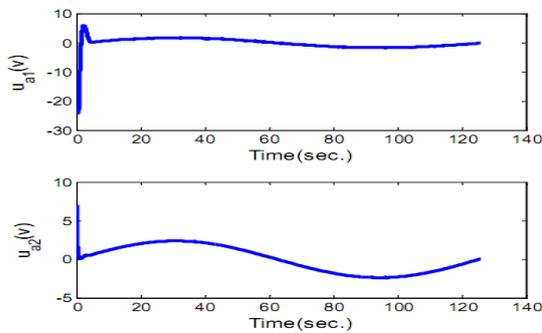


Figure 6. Control Inputs

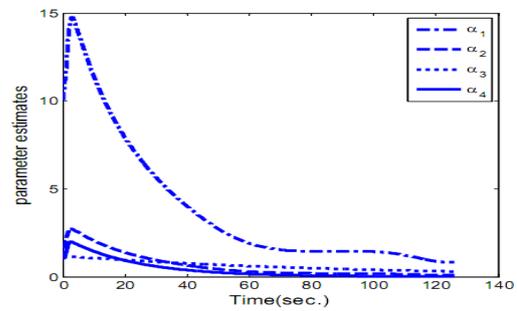


Figure 7. Estimated Parameters

Figure 3 shows reference path curve and real device path curve in the (x,y) plane. Output tracking errors were shown in Figure 4. As it is shown in these figures, regardless of parametric and non-parametric uncertainties, tracking and robustness of the proposed controller are completely satisfying. Figure 5 shows estimation error of linear and angular velocities. Figures 4 and 5 show that tracking and velocity estimation errors converge to zero. Figure 6 shows the control inputs and Figure 7 shows estimated parameters. As it is shown in Figure 7, estimated parameters are bounded.

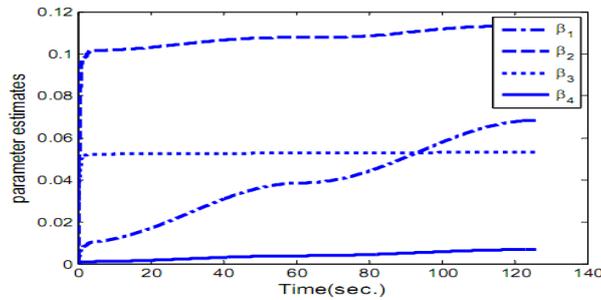


Figure 8. Stimated Parameters

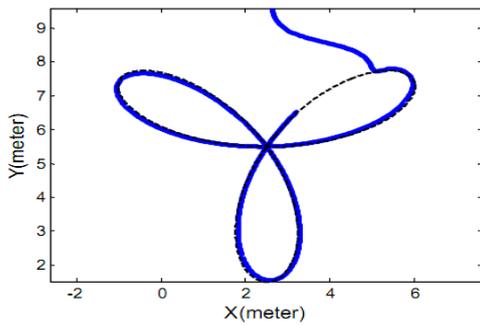


Figure 9. Reference Path Curve and Real Device Path Curve on the (x,y) Plane

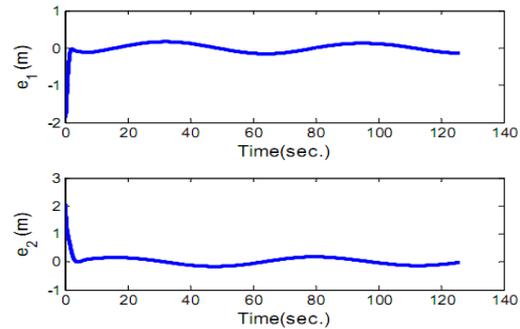


Figure 10. Output Tracking Errors

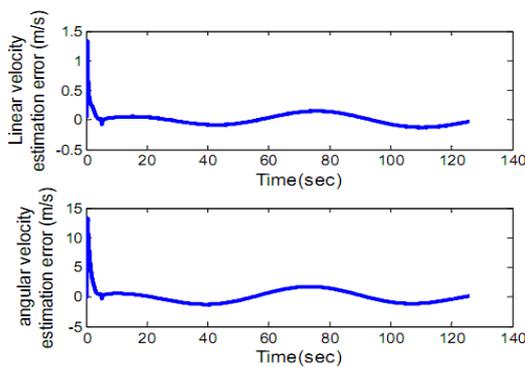


Figure 11. Error of Linear And Angular Velocity

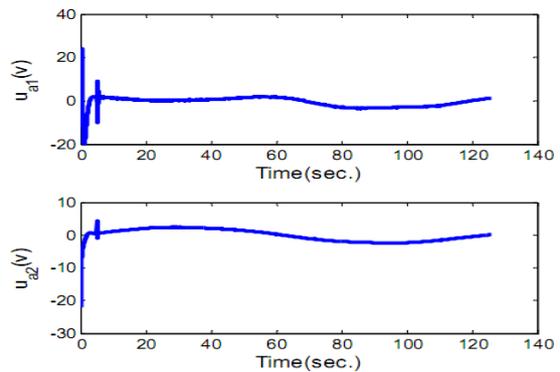


Figure 12. Control Inputs

In the second simulation, a more complicated path is considered for evaluating controller performance as follows:  
(80)

$$z_r = [x_f + R \sin(2\omega, t) + R \cos(\omega, t), y_f + R \sin(\omega, t) + R \cos(2\omega, t)]$$

In this simulation  $k_3 = 3I_4$ ,  $\Lambda = 1.2I_4$  and other controlling parameters and reference path parameters were chosen like the first simulation. Initial conditions of surface cable robot for above simulation equals to  $[x(0), y(0), \Psi(0)] = [2.5, 9.5, \pi/6]$ . Like the first simulation, figures (9) to (14) show reference path tracking results for the proposed controller. As seen in these figures, controller performance is suitable. Therefore, results of the simulations show the controller effectivity for tracking the path of surface cable robots without measuring velocity.

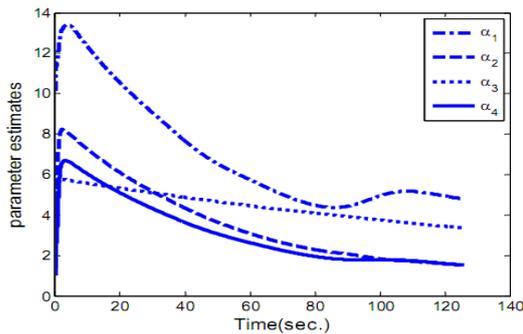


Figure 13. Estimated Parameters

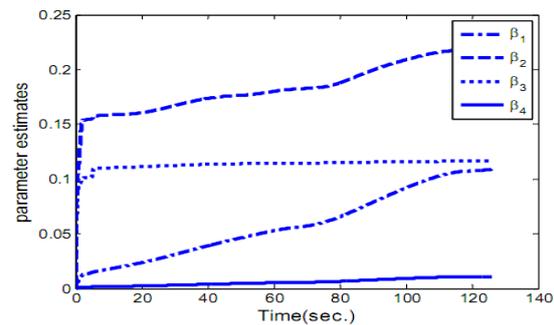


Figure 14. Estimated Parameters

#### 4.1. Fuzzy-Adaptive Back-Step Control in Comparison to Revised Fuzzy-Adaptive Back-Step Control

In this section, we deal with fuzzy-adaptive back-stepping control and revised fuzzy-adaptive back-stepping control. As seen in the previous section, long and time-consuming calculations were done to obtain regression matrix in developing fuzzy-adaptive controller. The problem in revised fuzzy-adaptive back-step control was solved filtering virtual control in a first order filter and a controller with the least complexity was obtained. Comparison between simulations of the two controllers shows the followings:

Tracking reference path by real cable robot in fuzzy-adaptive back-stepping controllers and revised fuzzy-adaptive back-stepping controllers in Figures 1, 3 and 9 are obvious. Figure 15 shows the suitable performance of fuzzy-adaptive back-stepping cable controller in the presence of parametric uncertainties. Regardless of parametric and non-parametric uncertainties in the fuzzy-adaptive back-stepping control system, figures 3 and 9 show the suitable performance of this controller in tracking reference path. Short transient response in fuzzy-adaptive back-stepping (around 200 seconds) was obvious in Figure 16 and it is shows that tracking errors converge to zero asymptotically. However, observing Figures 4, 5, 10 and 11 show the shorter transient response of revised fuzzy-adaptive back-stepping controller against fuzzy-adaptive back-stepping controller, tracking error convergence and estimation toward zero vicinity.

As seen before, revised fuzzy-adaptive back stepping control has supremacy over fuzzy-adaptive back-stepping control regarding less design complexity, easier applicability, shorter transient response, and robustness over parametric and non-parametric uncertainties and no need for velocity measurement.

In this section, it is explained the results of research and at the same time is given the comprehensive discussion. Results can be presented in figures, graphs, tables and others that make the reader understand easily [2], [5]. The discussion can be made in several sub-chapters.

## 5. Conclusion

In this article, in order to track the path of underactuated surface cable robots regarding actuators dynamics and in the presence of parametric and nonparametric uncertainties, a robust fuzzy-adaptive output feedback controller was designed. At the beginning, a virtual control was considered, then; cable controller was developed using dynamic surface control method needless of virtual control derivative. Tracking error uniform bound and state estimation error is obtained by Liapanov stability analysis. Finally, simulations confirmed controller effectiveness.

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