

# Optimal Power and Modulation Adaptation Policies with Receiver Diversity over Rayleigh Fading Channel

Muhammad Imran Tariq\*, Razvan Beuran, and Yoichi Shinoda

Japan Advanced Institute of Science and Technology  
1-1 Asahidai, Nomi city, Ishikawa, Japan

\*Corresponding author, e-mail: imran@jaist.ac.jp

## Abstract

Efficient bandwidth utilization is paramount in wireless communication systems, particularly in fading environments, since fading is one of the major constraints that impair communication in wireless systems. The bandwidth efficiency of a wireless communication system can be enhanced significantly by employing power and modulation adaptation policies with diversity combining gain. In this work, first we examine an analytically-derived solution for Maximum Combining Ratio (MRC) diversity technique for the capacity per unit bandwidth. Then, we design an adaptive transmission system to utilize the diversity combining gain while retaining the target BER by adapting power and constellation size using continuous power, channel inversion with fixed rate and continuous power and discrete-rate. By considering the effect of diversity combining gain, the designed system yields a reasonable spectral efficiency with respect to target BER that grows as the number of diversity levels increase. Furthermore, the presented results show continuous power and discrete-rate adaptation policy reduces probability of outage unlike its achieved spectral efficiency is close to other selected policies, which ratifies the optimized switching thresholds and makes it best candidate for imperfect channel conditions.

**Keywords:** Adaptive transmission, diversity combining, bit error rate, switching threshold, Rayleigh fading

Copyright © 2016 Institute of Advanced Engineering and Science

## 1. Introduction

With the advent of high data transmission, quadruple-play applications in particular and high quality contents rich streams in wireless communication, in general are highly demanded and will continue in the future. The demand for those applications expeditiously have been burgeoning. Consequently, the limited wireless resources are increasingly demanded. Therefore, the capacity is a major factor during designing wireless communication systems, since wireless links are greatly impaired due to fading. Thus, the current fading mitigation schemes are utilized in wireless communication systems such as Digital Video Broadcasting-Satellite Version 2 (DVB-S2) [1], WiMAX [2] and LTE [3]. Diversity combining and adaptive modulation are capacity improving techniques that can be used to overcome fading in wireless communication systems. Figure 1. shows such adaptive systems acclimating to fading by utilizing diversity combining gain, optimal power adaptation and different component channel codes having various constellations sizes.

Increasing the system capacity and compensating fading in wireless communication systems, is a thoroughly studied topic and investigated in the recent literature [4, 5, 6, 7, 8, 9, 10, 11].

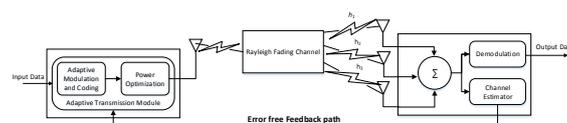


Figure 1. Overview of adaptive transmission system [4].

In [12, 13], Goldsmith and Varaiya presented some of the adaptive transmission policies, namely: optimal transmit power with rate adaptation - constant power with optimal rate adaptation - channel inversion with fixed rate. The idea of adapting optimal power combining with modulation and coding rate was to determine the *maximum average spectral efficiency* (MASE) over flat fading channels having perfect channel state information (CSI) with respect to both the transmitter and receiver. A practical approach in determining the spectral efficiency is adapted by [14], for discrete-rate multilevel quadrature amplitude modulation (MQAM) for fading channels. While in [8] using trellis coded modulation have overcome the observed gap between the achievable spectral efficiency and MASE. Furthermore, Hole in [9] extended [14, 8] to develop a general technique to evaluate average spectral efficiency of the coded modulation for  $2L$ -D trellis codes over Nakagami multipath fading (NMF).

One of the fading compensation techniques as well as boosting link performance is the diversity combining technique which accumulates signals received from several paths. In [4], the authors use diversity combining techniques to design a system for maximum spectral efficiency. They have evolved closed-form expressions for the general theory of adaptive transmission from [13] into three adaptive transmission and diversity combining techniques for Rayleigh fading channels. Although, diversity provides large capacity gain for each unit bandwidth over all the techniques, however in [15, 4] it has been proved, there must be a tradeoff between the conflicting targets; complexity and capacity of the adaptation methods. The evaluation of various power and rate adaptation policies were also considered in [6], where closed-form solutions were derived for maximum-combining ratio (MRC) of the generalized Rician fading channel. It is reported that truncated channel inversion adaptation policy has performance advantage over the different single antenna reception policies. Moreover, channel inversion with fixed rate policy is the preferable MRC policy and equal gain combining (EGC) diversity techniques [16].

Fading mitigation/compensation methods with diversity combining enable the adaptive transmission policies to achieve capacity close to shanon limits in flat fading channel [15]. Shanon capacity is considered idealistic limit for communication systems, and also a de-facto reference to compare adaptive schemes with respect to spectral efficiency. Unlike prior mentioned work [4]-[16], design adaptive transmission system employing fading compensation methods (e.g power and rate adaptation) that adequately consider the target BER in conjunction with MRC diversity. The design also optimized adaptive power and constellation size for continuous power and rate adaptation while handling truncated channel inversion with fixed rate and continuous power and discrete rate adaptation. Hence, the primary purpose of this adaptive transmission system is to balance the link budget in real-time through adaptive variation of the various important parameters such as symbol rate, transmit power, constellation size, coding rate, or any combination of these parameters [14, 4]. Moreover, a suitable model of the diversity combining adaptive process was also outlined to extend the obtained results to different adaptive transmission policies with respect to those considered.

The proposed mechanism takes the influence of channel in terms of attenuation caused by multipath fading into account, and also examines the diminishing effect of BER in conjunction with MRC diversity on selected adaptive transmission schemes. These considerations formulate adaptive constellation size for transmitting optimal symbols to improve the spectral efficiency when considering a certain fading distribution. The main contributions of this paper are as follows:

- The target BER and diversity combining gain at Rayleigh fading channel are assumed while deriving closed-form expressions using constrained spectral efficiency maximization of adaptive transmission policies;
- The constellation switching thresholds and its adaptive power for discrete-rate policy are optimized to maintain the target BER in conjunction with diversity combining.
- A series of numerical results that validate the operations of proposed system, show the introduction of power adaptation with respect to diversity combining and target BER in the selected policies have good agreement in terms of spectral efficiency increase and reducing the possibility of no transmission, even when the number constellations size are finite.

The rest of this paper is structured as follows. In section 2, the system model is presented by focusing the problem that is investigated. Section 3 discusses and derives the adaptive transmission policies with respect to the target BER and MRC diversity. In section 4, obtained numerical results are presented with detail disucssion. Finally, in section 5 the conclusions are drawn.

## 2. System Model and Problem Formulation

### 2.1. System Model

In this section, a single-link wireless communication model will be considered as shown in Figure 1, similar to the one described in [4, 5]. In a given wireless communication model, the discrete-time channel oscillates slower degree than the data rate. In this analysis, it is assumed to be slow varying and frequency-flate Rayleigh fading channel. By the probity of these assumption the distribution of the received Signal-to-Noise Ratio (SNR)  $\gamma$  is represented by an exponential distribution [17].

$$f_{\gamma}(\gamma) = \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}}, \quad (1)$$

Where  $\bar{\gamma}$  is the average received SNR and  $\gamma$  represents the instantaneous received SNR.

In this analysis, MRC combining technique is considered where the amplitudes and phases of the received signals are assumed known by the receiver perfectly. By virtue of this assumption we can term it *perfect combining*. It is proven in [17, 15, 4], that MRC yields maximum spectral efficiency improvement relative to all combining techniques as a result of *perfect combining*. It requires the received signals to be weighted by their own SNR (weighting parameter) independent from each diversity branch to compose the output decision variable. For a Rayleigh fading channel, the output of a linearly combined MRC combiner with  $L$ -branch is given in [17] as a distribution of the instantaneous received SNR.

$$f_{\gamma_{mrc}}(\gamma) = \frac{1}{(L-1)!} \frac{\gamma^{L-1}}{\bar{\gamma}^L} e^{-\gamma/\bar{\gamma}}, \quad (2)$$

We simply consider throughout in our study, as in [14, 13], the channel state is perfectly tracked by the transmitter via error-free feedback channel. Accordingly, the proposed model coordinattes with power adaptation scheme  $P(\cdot)$  to adapt transmit power. Let  $N$  denotes the quantization levels of available constellations  $M_n$  that represent the instantaneous received SNR  $\gamma$ . The  $\gamma$  range is partitioned into  $N+1$  nonoverlapping successive fading regions, with boundry points denoted by the switching thresholds  $\{\gamma_T^n\}_{n=0}^{N+1}$ . Specifically, constellation  $n$  having spectral efficiency  $SE$ , is chosen when  $\gamma \in [\gamma_T^n, \gamma_T^{n+1})$  and within this region, the transmission rate does not change; yet the adaptive policy might change the transmit power in order to compensate for the fading. To overcome strong channel fadings, data would be buffered for output SNR of fading regions  $\gamma_T^0 \leq \gamma < \gamma_T^1$ . For convience, we suppose  $\gamma_T^0 = 0$  and  $\gamma_T^N = \infty$ .

### 2.2. Problem Formulation

This section shows capacity of flat fadding channel having a perfect channel state information (CSI) with respect to the transmitter side and the receiver as well. Which is widely represented as  $C(\gamma) = B \log_2(1 + (P(\gamma)/\bar{P})\gamma)$  data bits/s. This capacity is considered as a yardstick to evaluate adaptive transmission schemes regarding its spectral efficiency [14]. By taking this effective diversity combining based approach, there exists constellation size that can achieve spectral efficiencies that can achieve  $C(\gamma)$  bits/s, while maintaining certain target BER. The presence of such constellation/modulation is assured according to the well known Shannon's theorem (channel coding). Our aim of this paper is to optimize a set of capacities with transmission modulation rates and switching threshold to guarantee the desired BER and power adaptation shcemes. This should maximize the corresponding spectral efficiency in the given fading distribution.

The outage probability  $P_{out}$  of adaptive policy with target BER can only appear when the set of channel states is below the optimal cutoff SNR  $\gamma_0$ , which normally appears within the

first interval of fading regions. Only at that time data is buffered for the corresponding period. The associated coding rate of other fading regions accommodate fluctuations. Thus, for adaptive systems the resulting spectral efficiency can be expressed as

$$SE = \sum_{n=1}^N M_n P_n \quad (3)$$

Where  $M_n = \log_2(M_n)$  represents the constellation size or modulation code, and  $P_n$  is defined as the probability of selecting code  $n$ :

$$P_n = \int_{\gamma_T^n}^{\gamma_T^{n+1}} f_{\gamma_{mrc}}(\gamma) d\gamma \quad (4)$$

### 3. Adaptive Transmission Policies with MRC diversity

The center of attention in this paper is on the performance characterization of an adaptive system with MRC diversity. The former studies emphasize on maximizing spectral efficiency of adaptive transmission under constant power constraints while maintaining target BER [18] and different power levels [19] unlike those studies, this research is interested in evaluating the adaptive system with MRC diversity while sustaining a certain BER, in selected adaptation policies. Accordingly, it is first argued in [20, 21] that the approximate BER of a rectangular MQAM uncoded modulation in AWGN can be expressed by the form

$$P_b \approx a \cdot \exp\left(-\frac{b\gamma}{(M-1)}\right) \quad (5)$$

Where  $a$  and  $b$  are positive fixed constants used to approximate the bounds of the expression and  $M$  is the size of a constellation. The desired bound within 1 dB it is achieved with  $a = 0.2$  and  $b = 1.5$  for  $M \geq 4$  and received SNR  $0 \leq \gamma \leq 30$  dB [12]. Though, evaluations of  $P_b$  e.g. lose or tighter bounds described in literature clearly argue that using curve fitting techniques we optimize the value of  $a$  and  $b$  such that the expression yields good accuracy even for low  $\gamma$ . The error probability for MRC diversity combining with  $L$ -branches, in independent and identically distributed (i.i.d) Rayleigh-fading channels can be derived by substituting (2) the pdf of  $\gamma$  into (5). That way for the AWGN channel with uncoded modulation, the error rates would be averaged out. It is possible to obtain closed-form expression by simplifying as in the following

$$\begin{aligned} P_b^{mrc} &= \int_0^{\infty} P_b \cdot f_{\gamma_{mrc}}(\gamma) d\gamma \\ &\approx a \cdot \exp\left(-\frac{b\bar{\gamma}}{(M-1)} + 1\right)^{-L} \end{aligned} \quad (6)$$

Accordingly, we use this expression when needed, since it is easy to invert. Hence, for a given target  $P_b$ , adaptive transmission policy and  $L$ -branches or diversity combining levels, the required  $M$  constellation level can be determined numerically.

By selecting the adaptive transmission policies, now we are ready to determine the rate and power which fluctuate according to time-variations of the channel. The transmitter compensates/reciprocates to channel fluctuations by adjusting the constellation size  $M(\gamma)$  and the transmission power  $P(\gamma)$  relative to diversity combining and  $BER_T$ . At the receiver, the received SNR becomes  $\gamma \frac{P(\gamma)}{\bar{P}}$ , thus the approximation of instantaneous  $P_b$  can be approximated by (6) for each value of  $\gamma$  as:

$$P_b^{mrc}(\gamma) \approx a \cdot \exp\left(-\frac{b}{(M-1)} \frac{\gamma P(\gamma)}{\bar{P}}\right)^{-L} \quad (7)$$

By re-arranging the above in terms of  $M$ , we obtain an expression for maximum constellation size as a function of target BER  $BER_T$ , diversity level  $L$  and instantaneous received SNR  $\gamma$ :

$$M(\gamma) = 1 + K_n \frac{\gamma P(\gamma)}{\bar{P}} \quad (8)$$

Where  $K_n \triangleq b.L\sqrt{a.BER_T}$  represents MRC diversity combining and  $a = 5.0$  and  $b = 1.5$ , to power diminishing parameter inversely proportion to  $BER_T$ .

### 3.1. Continuous power and rate adaptation policy

In the following, we maximizing spectral efficiency of the MQAM scheme for a specified average transmission power with the target BER  $BER_T$  in the first adaptation policy. The spectral efficiency of a fading channel and received SNR distribution  $f_{\gamma_{mrc}}(\gamma)$  with respect to diversity combining, is maximized by maximizing (8) as follows [12]:

$$E[\log_2 M(\gamma)] = \int \log_2 \left( 1 + \frac{K_n \gamma P(\gamma)}{\bar{P}} \right) f_{\gamma_{mrc}}(\gamma) d\gamma \quad (9)$$

Introducing a maximum spectral efficiency scheme (9), which subject to average power constraint  $\bar{P}(\gamma)$ , then constrict the average power (using an equal) rather than just boundng (using inequality)  $\frac{\partial J}{\partial P(\gamma)} = 0$  Lagrangian optimization solution. Optimal power adaptation for MQAM mentioned in [12] can be expressed as follows:

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} \frac{1}{\gamma_0^*} - \frac{1}{\gamma K_n} & \gamma \geq \gamma_0^*/K_n \\ 0 & \gamma < \gamma_0^*/K_n \end{cases} \quad (10)$$

Where  $\gamma_0^*/K_n$  is the optimal level SNR at which at the cutoff fade. Beyond that level transmission would be deferred. This optimal cutoff and MRC combining  $\gamma_0^*$  follow:

$$\int_{\gamma_0^*}^{+\infty} \left( \frac{1}{\gamma_0^*} - \frac{1}{\gamma K_n} \right) f_{\gamma_{mrc}}(\gamma) d\gamma = 1 \quad (11)$$

For the above equation of optimal cutoff SNR while maintaining BER performance, a closed-form expression adopts the numerical root finding techniques and substitutes (2) in (11) to find  $\gamma_0^*$ :

$$\frac{\Gamma(L, \frac{\gamma_0^*}{\bar{\gamma}})}{\frac{\gamma_0^*}{\bar{\gamma}}} - \frac{1}{K_n} \Gamma(L-1, \frac{\gamma_0^*}{\bar{\gamma}}) = (L-1)! \bar{\gamma} \quad (12)$$

Where  $\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1} e^{-t} dt$  is a complementary incomplete gamma function [15],[22]. Let  $x = \frac{\gamma_0^*}{\bar{\gamma}}$  and define:

$$f_{K_n}^{mrc}(x) = \frac{\Gamma(L, x)}{x} - \frac{1}{K_n} \Gamma(L-1, x) - (L-1)! \bar{\gamma} \quad (13)$$

Note that  $\frac{\partial f_{K_n}^{mrc}(x)}{\partial \gamma_0^*} < 0 \forall x > 0$  and  $L \geq 2$ . By virtue of  $\lim_{x \rightarrow 0^+} f_{K_n}^{mrc}(x) = +\infty$  and  $\lim_{x \rightarrow +\infty} f_{K_n}^{mrc}(x) = -(L-1)! \bar{\gamma}$ , hence it is assumed that there is a unique positive  $\gamma_0^*$  such that  $f_{K_n}^{mrc}(\gamma_0^*) = 0$  and restricted by (12). Gnu Scientific Library (GSL) optimization routines [23] are used to obtain numerical results. The results show  $\gamma_0^*$  with MRC diversity and BER performance always lies in the interval  $[0, 1]$ .

Based on the result of (13), we can devise the closed-form expression of (9) by substituting (2) in (9), for the channel capacity  $\langle C \rangle_{\text{CPRA}}$  as:

$$\langle C \rangle_{\text{CPRA}}^{\text{mrc}} = \mathcal{J}_L \left( \frac{\gamma_{K_n}^*}{\bar{\gamma}} \right) \frac{\gamma_{K_n}^{*L}}{\bar{\gamma} \log_2(e)(L-1)!} \quad (14)$$

Where  $\gamma_{K_n}^* = \gamma_0^*/K_n$  is a cutoff SNR threshold. Beyond that threshold deferring of the data transmission occurs.  $B$  represents to channel bandwidth (in Hertz). Letting  $\alpha = \frac{\gamma_{K_n}^*}{\bar{\gamma}} > 0$ , using the evaluation of  $\mathcal{J}_L(\alpha)$  which is given in Appendix A [22], and rearranging the (14). For MRC diversity, we obtain the SE [bits/s/Hz] and BER performance by using continuous power and rate adaptation policy as:

$$\frac{\langle C \rangle_{\text{CPRA}}^{\text{mrc}}}{B} = \log_2(e) \left( E1(\alpha) + \sum_{k=1}^{L-1} \frac{\mathcal{P}_k(\alpha)}{k} \right) \quad (15)$$

Where  $\mathcal{P}_k(\cdot)$  is Poisson distribution defined as  $\mathcal{P}_k(\alpha) = e^{-\alpha} \sum_{j=0}^{k-1} \frac{\alpha^j}{j!}$  and  $E1(\alpha) = \int_1^\infty \frac{e^{-\beta t}}{t} dt$  is an exponential integral function of first-order [22]. To achieve the capacity (15), instantaneous received SNR  $\gamma$  should not fall below  $\gamma_{K_n}^*$ . Since, no data transmission occurs when  $\gamma < \gamma_{K_n}^*$ , the policy is vulnerable to outage. Outage probability under  $BER_T$  performance for the MRC diversity case is expressed below:

$$f_{\text{out}_{\text{mrc}}} = \int_0^{\gamma_{K_n}^*} f_{\gamma_{\text{mrc}}}(\gamma) d\gamma = 1 - \int_{\gamma_{K_n}^*}^{\infty} f_{\gamma_{\text{mrc}}}(\gamma) d\gamma \quad (16)$$

Substituting (2) in (16), for the probability of outage, we obtain the following closed form expression:

$$f_{\text{out}_{\text{mrc}}} = 1 - \mathcal{P}_L(\alpha) \quad (17)$$

### 3.2. Truncated channel inversion policy with fixed rate

Another adaptation policy where the transmitter inverts the channel higher than the fade depth  $\gamma_0^*$  to maintain the SNR received. This power adaptation policy is termed Truncated Channel Inversion with Fixed Rate (TCIFR) that is employed in inner-loop power control mechanism [24], which adapts the transmitting power to achieve a desired SNR at the receiver. Then, adaptive system transmits fixed-rate MQAM constellation that maintains the  $BER_T$  with diversity combining. Hence, introducing  $P(\gamma)/\bar{P} = \sigma/\gamma$  in (8) and multiplying by the probability that  $\gamma > \gamma_0^*$ , we obtain a new expression for channel capacity expressed by:

$$\langle C \rangle_{\text{tcifr}}^{\text{mrc}} = B \log_2 \left( 1 + K_n \frac{1}{\int_{\gamma_{K_n}^*}^{\infty} \frac{f_{\gamma_{\text{mrc}}}(\gamma)}{\gamma} d\gamma} \right) (1 - f_{\text{out}_{\text{mrc}}}) \quad (18)$$

Where  $f_{\text{out}_{\text{mrc}}}$  is calculated as in (16) to maximize (18). In order to find the closed-form expression, we substitute (2) into (16) and rearrange to obtain the SE [bits/s/Hz] under TCIFR policy and  $BER_T$  performance for MRC diversity as:

$$\frac{\langle C \rangle_{\text{tcifr}}^{\text{mrc}}}{B} = \log_2 \left( 1 + \frac{(M-1)! \bar{\gamma}}{\Gamma(M-1, \gamma_0^*/\bar{\gamma})} \right) \frac{\Gamma(M, \gamma_0^*/\bar{\gamma})}{(M-1)! \bar{\gamma}} \quad (19)$$

Using properties of  $\Gamma(\cdot, \cdot)$  of complementary incomplete gamma function [15][22], (19) further can be simplified as follows

$$\frac{\langle C \rangle_{\text{tcifr}}^{\text{mrc}}}{B} = \log_2 \left( 1 + \frac{(M-1) \bar{\gamma}}{\mathcal{P}_{M-1}(\gamma_0^*/\bar{\gamma})} \right) \mathcal{P}_M(\gamma_0^*/\bar{\gamma}) \quad (20)$$

### 3.3. Continuous power and discrete-rate adaptation policy

We now proceed to extend the design of adaptive system for continuous power and discrete-rate adaptation policy, by restricting adaptive MQAM to  $N$  fading regions, whose constellations size  $M_n = 2^{2(n-1)}$  and whose BER can be approximated by (7) for  $n = 2, \dots, N-1$

under diversity combining. Recall that instantaneous SNR  $\gamma$  is partitioned into  $N$  fading regions [12], [9], the modulation size  $M_n$  is used whenever  $\gamma \in [\gamma_T^n, \gamma_T^{n+1})$ . Thus, the adaptive system is lower bounded by  $\gamma_T^n$  to maintain the  $BER_T$  with diversity combining, after transmission power adaptation.

Following (8), this policy also requires the boundaries of switching threshold be determined. Therefore, by substituting (10) into (8) yields the constellation size for a given  $\gamma$  as:

$$M(\gamma) = \frac{\gamma}{\gamma_\beta^*} \quad (21)$$

Where  $\gamma_\beta^*$  is a optimized parameter found by numerical methods to optimize the switching thresholds and maximize spectral efficiency. Former, the fading region  $n$  of switching threshold and associated constellation size  $M_n$  are determined, then we obtain the power adaptation policy based on (8) to maintain the fixed  $BER_T$  and satisfies the average power constraint  $\int P(\gamma) f_{\gamma_{mrc}}(\gamma) \leq \bar{\gamma}$  as follows:

$$\frac{P_n(\gamma)}{P} = \begin{cases} \frac{(M_n-1)}{\gamma K_n} & M_n < \frac{\gamma}{\gamma_\beta^*} \leq M_{n+1} \\ 0 & M_n = 0 \end{cases} \quad (22)$$

Where  $P_n(\gamma)$  represents post adaptation power, and when  $\gamma < \frac{\gamma}{\gamma_\beta^*}$  no transmission occurs and data is buffered. The maximized spectral efficiency SE of CPDA policy for a given diversity level and  $BER_T$  with distribution  $f_{\gamma_{mrc}}(\gamma)$ , is defined as the sum of spectral efficiencies associated with each of the fading regions:

$$\frac{\langle C \rangle_{CPDA}^{mrc}}{B} = \sum_{n=1}^{N-1} \log_2(M_n) f_{\gamma_{mrc}}(M_n \leq \frac{\gamma}{\gamma_\beta^*} < M_{n+1}) \quad (23)$$

Since  $M_n$  is a function of  $\gamma_\beta^*$ , by maximizing (23) with respect to  $\gamma_\beta^*$ , we arrive at the following optimization problem where (23) is maximized subject to power constraint must be satisfied:

$$\sum_{n=1}^{N-1} \int_{\gamma_\beta^* M_n}^{\gamma_\beta^* M_{n+1}} \frac{P_n(\gamma)}{\bar{\gamma}} f_{\gamma_{mrc}}(\gamma) d\gamma = 1 \quad (24)$$

If we assume equation (2) Rayleigh fading channel with diversity combining the closed-form expression of (24) can be written as follows:

$$f_{mrc}(\gamma_\beta^*) = \sum_{n=1}^{N-1} \frac{M_n - 1}{\bar{\gamma} K_n (L-1)!} \left( \Gamma(L-1, \frac{M_{n+1} \gamma_\beta^*}{\bar{\gamma}}) - \Gamma(L-1, \frac{M_n \gamma_\beta^*}{\bar{\gamma}}) \right) \quad (25)$$

With the increase of diversity gain, relative to others polices, the discrete-rate adaptive policy will have reasonable agreement in term spectral efficiency but significantly has less probability of outage as we show in results section.

#### 4. Numerical Results and Discussion

In this section, we obtain the numerically evaluated results for *continuous power and rate adaptation* (CPRA), *truncated channel inversion with fixed-rate adaptation* (TCIFR) and *continuous power and discrete-rate adaptation* (CPDA) at target  $BER_T$ . For the following, a Rayleigh fading distribution and MRC diversity combining technique have been assumed.

Using described techniques in [15] and [4], we have designed an adaptive MQAM system which takes the target  $BER_T$  into account and utilizes the diversity combining gain, to maximize

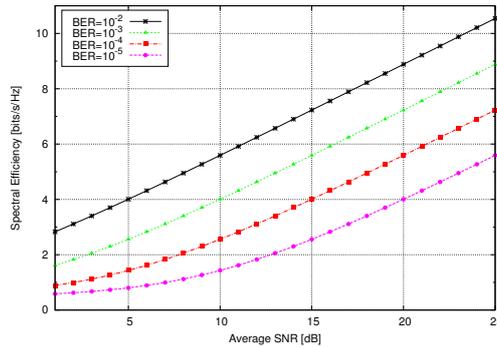


Figure 2. Spectral efficiency of continuous power and rate adaptation for various target  $BER_T$  with diversity level  $L=4$  , versus average received SNR  $\bar{\gamma}$

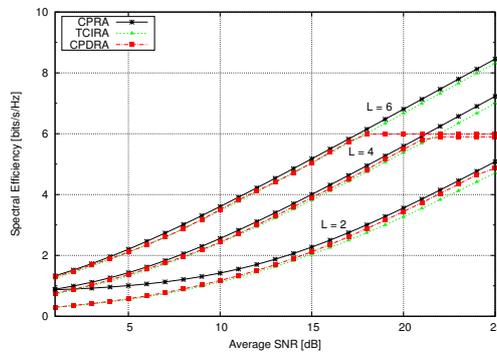


Figure 3. Spectral efficiency of various adaptive policies with various diversity levels and  $BER_T = 10^{-4}$  , versus average received SNR  $\bar{\gamma}$ .

the spectral efficiency. As can be seen in Figure 2, the gap between the spectral efficiencies is induced by a effective power loss parameter  $K_n$ , since it is a function of  $BER_T$  and number of  $L$  diversity levels. Moreover, we also observe that  $K_n$  diminishes the performance within 4 dB with constant  $L$  as  $BER_T$  advances. Hence, we fix the target  $BER_T$  in the following, and evaluate its effect on spectral efficiency and power adaptation for proposed adaptive system with diversity combining techniques.

Figure 3 presents the resulting channel capacity for each bandwidth unit or spectral efficiency acquired using closed-form expression (15), (19) and (23) . The capacity per unit bandwidth increases with the number of diversity levels. We noted that the spectral efficiency curve for CPRA policy has increasing trend at low received SNR (0 dB to 10 dB) in the beginning, afterwards gradually comes close to other policies keeping slightly larger spectral efficiency(when SNR is greater than 10 dB). The cause of this effect is the CPRA policy of leveraging the water-filling nature at lower received SNR and it only holds this instict for diversity level( $L = 1$  or  $L = 2$ ).

In Figure 3, the discrete-rate policy has similar behaviour even in diversity combining [12], by restricting it to the fading regions and it yields low spectral efficiency. That resulting efficiency is 1 dB compared to that produced by CPRA. However, the spectral efficiency curves are very close to each other in almost all diversity levels.

The optimal switching thresholds  $\{\gamma_T^n\}_{n=1}^N$  for CPDA policy which is suitable for adaptive system, are shown in Figure 4, at target  $BER_T = 10^{-4}$  and for  $0 \text{ dB} < \bar{\gamma} < 25 \text{ dB}$  under various diversity levels. Obtain the thresholds using expression (21) with 4 fading regions and optimized power adaptation (22), and  $\gamma_\beta^*$  found through numerical methods, where the threshold  $\gamma_T^n$  equals

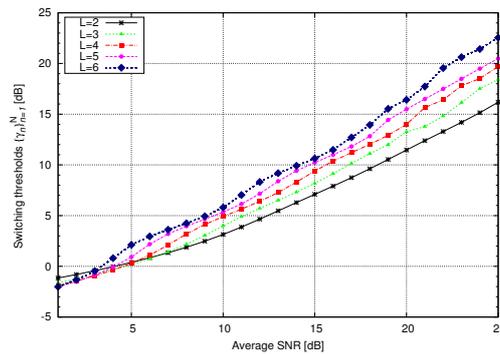


Figure 4. Switching thresholds  $\{\gamma_T^n\}_{n=1}^N$  under continuous power adaptation policy with target  $BER_T = 10^{-4}$  as a function of various diversity levels.

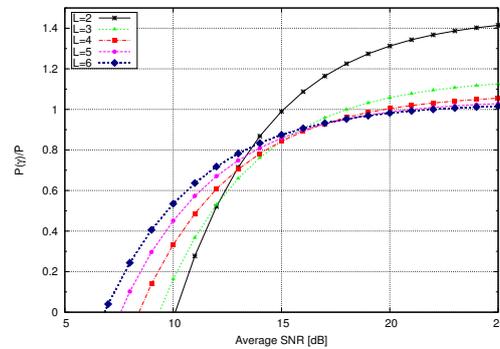


Figure 5. Power adaptation scheme  $P(\gamma)/\bar{P}$  with various diversity levels with respect to average received SNR, plotted for an average SNR  $\bar{\gamma} = 15$  dB.

the minimum target SNR to obtain the desired  $BER_T$ .

As, it is obvious in Figure 4  $\gamma_T^n$  in each diversity level exploits the diversity gain after 3 dB and it monotonically widens with the increment of diversity level. By taking the leverage of diversity gain, the adaptive system switches the modulation code when  $M_{n+1} > M_n$  and maximizes the spectral efficiency by the probability  $\int_{\gamma_T^n}^{\gamma_T^{n+1}} f_{\gamma_{mrc}}(\gamma)d\gamma$  that the instantaneous received SNR falls in region  $n$ .

For each  $\bar{\gamma}$  of interest, designed the constellation size above for spectral efficiency. The optimized policy for power adaptation is used and presented in (10). Figure 5, highlights the continuous power adaptation policy used in designing the adaptive system, for  $\bar{\gamma} = 15$  dB. We noticed the range of the optimized transmission power  $P(\gamma)$  is 0 to  $1.4\bar{P}$ . Following the water-filling nature, it can be seen that most of the power is allocated to the best SNR channels regardless diversity gain. In addition, it is interesting to point out that with the upsurge of diversity gains the optimized power becomes uniform especially when  $\bar{\gamma} \geq 15$  dB. It is pertinent to mention that due to low diversity combining gain, 35% more transmit power is allocated when diversity level is  $L = 2$ . In this analysis, it is also noticed that by virtue of various diversity gain,  $P(\gamma)$  does not take rigorous peak values.

Figure 6 depicts the normalized spectral efficiency with diversity level  $L$  of MRC diversity relation to cutoff SNR  $\gamma_0^*$ . The optimal cutoff SNR (the channel is not used below that) is numerically found using (20) for a certain value of  $\gamma_0^*$  to maximize spectral efficiency. It was found that spectral efficiency ameliorates by increasing the diversity levels. Furthermore, it can be observed the gradual fall of the curves in Figure 6 as a result of higher probability of outage.

As, it was stated in [12], [4], truncated channel inversion policy yields more spectral ef-

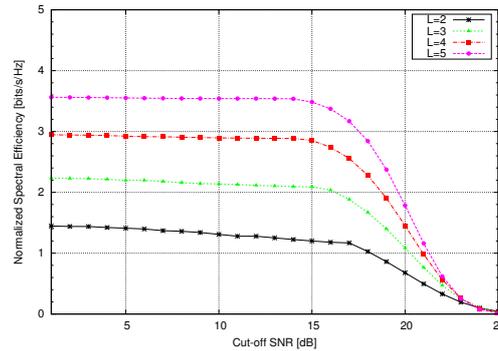


Figure 6. Normalized spectral efficiency of TCIFA policy with various diversity levels and  $\bar{\gamma} = 12$  dB with *cutoff* SNR  $\gamma_0^*$

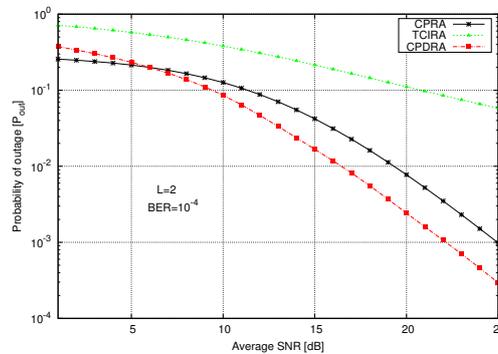


Figure 7. Probability of outage of selected adaptatio policies with diversity level L=2 and  $BER_T = 10^{-4}$ , as a function of average received SNR  $\bar{\gamma}$

efficiency sacrificing the probability of outage, as explained in Figure 7. The corresponding probability of outage  $P_{out}$  for CPRA, TCIFA and CPDA policies are calculated according to (17) with respect to target  $BER_T$  in Figure 7. The given adaptive policies then are rectified to achieve the maximum spectral efficiency under the given constraint of probability of outage. Formerly, we explained TCIFA policy has higher probability of outage compared with CPRA and CPDA policies, because it inverts the fading channel and compensates the fading to maximize the spectral efficiency.

## 5. Conclusion

In this article, power and rate adaptation policies have been devised to maximize the spectral efficiency of MQAM system in Rayleigh fading channel with MRC diversity combining technique. In particular, closed-form expressions of selected adaptive policies are obtained, in order to maintain target BER and maximize spectral efficiencies in conjunction with diversity combining.

The results were compared for a continuous power and rata adaptation policy under the constraint of BER and its effect on spectral efficiency under diversity gain. We concluded that the performance of the proposed adaptive system significantly influenced by  $K_n$  value. On the other hand, we observed that the difference in each step of BER alone extends/reduces the system spectral efficiency almost 4 dB. Additionally, we optimized the switching threshold for discrete-rate adaptation policy and power adaptation policies through numerical techniques. It is interesting to note that the continuous power and rate adaptation policy has only a little better spectral efficiency compared to discrete-rate and truncated channel inversion policy, despite the

increment in diversity branches and average SNR retaining it.

We have also seen, with diversity combining gain the deep channel fade almost gradually falls, consequently little transmitting power is required to make up for channel fading. Particularly, the truncated channel inversion policy inverts fading beyond  $\gamma_0^*$  to obtain the maximum spectrum efficiency, by sacrificing the outage probability. Finally, we note that the adaptive policies rendered in this article have identical aspects which endorse the correct operations of devised closed-form expressions and also of switching thresholds. Analyzing these expressions and switching thresholds with multiple power levels for each modulation and coding rate, in real-time system it can be a potential topic for future research.

## References

- [1] ETSI, "Digital video broadcasting-satellite version 2," <http://www.dvb.org>.
- [2] I. . working group, *802.16e-2006: IEEE Standard for Local and Metropolitan Area Networks Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems*. IEEE Standard, Feb.2006.
- [3] T. S. G. R. A. N. Std., *Physical Layer Procedures (FDD)*. Third Generation Partnership Project, September 2005, vol. Rev. TS25.214 (Release 6).
- [4] A. G. M-S Alouini, "Capacity of rayleigh fading channels under different adaptive transmission and diversity-combining techniques," *IEEE Transaction on Vehicular Technology*, vol. 48, no. 4, pp. 1165–1181, 1999.
- [5] G. Ø. A. GjendemsjØ, Yang. Hong-Chuan and M.-S. Alouini, "Joint adaptive modulation and diversity combining with downlink power control," *IEEE Transactions on Vehicular Technology*, vol. 57, no. 4, pp. 2145–2152, July 2008.
- [6] V. Bhaskar, "Spectral efficiency evaluation for mrc diversity schemes over generalized rician fading channels," *International Journal of Wireless Information Networks*, vol. 14, no. 3, p. 209223, September 2007.
- [7] *Optimal power control for discrete-rate link adaptation schemes with capacity-approaching coding*. St. Louis, MO,: IEEE, Nov-Dec 2005.
- [8] A. Goldsmith and S. G. Chua, "Adaptive coded modulation for fading channels," *IEEE Transactions on Communications*, vol. 46, no. 5, pp. 595–602, May 1998.
- [9] H. H. K.J. Hole and G. E. Øien, "Adaptive multidimensional coded modulation over flat fading channels," *IEEE Jouranl on Selected Areas in Communication*, vol. 18, no. 7, pp. 1153–1158, July 2000.
- [10] Q. L. Xin Wang and G. G.B, "Analyzing and optimizing adaptive modulation coding jointly with arq for qos-guaranteed traffic," *IEEE Transactions on Vehicular Technology*, vol. 56, no. 2, pp. 710–720, March 2007.
- [11] D. T. R. Fantacci, D. Marabissi and I. Habib, "Adaptive modulation and coding techniques for ofdma systems," *IEEE Transactions on Wireless Communication*, vol. 8, no. 9, pp. 4876–4883, September 2009.
- [12] A. Goldsmith, *Wireless Communications*. Cambridge University Press New York, NY, USA, August 2005.
- [13] V. P. Goldsmith A.J, "Capacity of fading channels with channel side information," *IEEE Transactions on Information Theory*, vol. 43, no. 6, pp. 219–233, November 1997.
- [14] A. Goldsmith and S. G. Chua, "Variable-rate variable-power mqam for fading channels," *IEEE Transactions on Communications*, vol. 45, no. 10, pp. 1218–1230, Oct 1997.
- [15] M.-S. A. Marvin K. Simon, *Digital Communication over Fading Channels*. Wiley-IEEE Press, November 2004.
- [16] V. Bhaskar, "Capacity evaluation for equal gain diversity schemes over rayleigh fading channels," *International Journal of Electronics and Communications*, vol. 63, no. 4, p. 235240, 2009.
- [17] W. C. Jakes, *Microwave Mobile Communications*, 2nd ed. Wiley-IEEE Press, May 1994.
- [18] D.-T. N. Nguyen Quoc-Tuan and L. S. Cong, "A 10-state model for an amc scheme with repetition coding in mobile wireless networks," *EURASIP Journal on Wireless Communications*

- and Networking*, vol. 2013, no. 1, pp. 1–15, September 2013.
- [19] J. Paris, "Optimum discrete-power adaptive qam scheme for rayleigh fading channels," *IEEE Communications Letters*, vol. 5, no. 7, pp. 281–283, July 2001.
- [20] A. G. M-S Alouini, "Adaptive modulation over nakagami fading channels," *Wireless Personal Communications*, vol. 13, no. 1, pp. 119–143, May 2000.
- [21] G.J.Foschini and J.Salz, "Digital communications over fading radio channels," *Bell Systems Technical Journal*, vol. 62, no. 2, pp. 429–456, February 1983.
- [22] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series and Products*, 5th ed. Academic Press, San Diego, CA, 1994.
- [23] M. Galassi, J. Davies, J. Theiler, B. Gough, G. Jungman, M. Booth, and F. Rossi, *GNU Scientific Library Reference Manual*, 3rd ed. Network Theory Ltd, January 2009.
- [24] *Joint SNR Target and Rate Adaptation Based on Bit Error Rate Estimation*. Siena, Italy: IEEE, September 2005.