

An Improved Criterion for Induced l_∞ Stability of Fixed-Point Digital Filters with Saturation Arithmetic

Priyanka Kokil*, Xavier Arockiaraj S

Department of Electronics Engineering,
Indian Institute of Information Technology, Design and Manufacturing,
Kancheepuram, Chennai-600 127, India

*Corresponding author, e-mail: kokilnit@gmail.com

Abstract

This paper establishes a criterion for the induced l_∞ stability of fixed-point state-space digital filters with saturation nonlinearities and external interference. The criterion is established in a linear matrix inequality (LMI) setting, and therefore, computationally tractable. The criterion turns out to be an improvement over a previously reported criterion. A comparison of the presented criterion with existing criterion is made. Numerical examples are given to demonstrate the usefulness of the proposed approach.

Keywords: External interference, l_∞ criterion, Finite wordlength effect, Linear matrix inequality, Digital filter

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1. Introduction

When a recursive digital filter is implemented using fixed-point digital signal processors, one commonly come across with accumulator overflow and quantization of a product. Saturation, zeroing, two's complement and triangular are the common types of overflow nonlinearities [1, 2]. The possible occurrence of zero-input limit cycles in digital filters owing to the presence of such nonlinearities (e.g., quantization and overflow) corresponds to an unstable behavior and is undesirable [1-4]. In order to design a digital filter, an important need is to select the filter coefficients so that the designed filter is limit cycle-free. The global asymptotic stability of the zero solution of the system ensures nonexistence of zero input limit cycles. Therefore, the study of stability properties of digital filters has attracted the attention of many researchers [3-19].

The implementation of high-order digital filters is usually achieved by splitting high-order filters into several low-order digital filters. External interferences among these low-order digital filters are unavoidable while implementation. Such external interferences or disturbances may lead to malfunction and destruction phenomenon in the realized filter [20, 21]. In recent years, many results have been appeared for the stability of digital filters by considering the effect of external disturbance [11-15]. In [11, 12], conditions for H_∞ stability of digital filter under the effect of finite wordlength nonlinearities and external disturbance have been developed. The stability properties of fixed-point state-space digital filters with external disturbance by using l_2-l_∞ approach [13] and input-output state stability approach [14, 15] has been studied. The induced l_∞ norm which is also known as peak-to-peak gain is carried by magnitude bounded signal. This concept of induced l_∞ norm was given by [16] and has received significant attention in recent years [17-19]. Refs. [18, 19] deal with the stability analysis of digital filters employing saturation nonlinearities and external disturbance by using induced l_∞ stability approach. However, there still remains scope to reduce the conservatism of the stability criterion available.

Motivated by the preceding discussion, we revisited the problem of induced l_∞ stability of state-space digital filters using saturation nonlinearities and external disturbances. It is desirable to reduce the conservatism of the existing stability criterion for the state-space digital filters as much as possible. Therefore, we focused on developing a criterion which may lead to less conservative results than [18]. The organization of this paper is as follows. Section 2

provides the notation used and introduces the system under consideration. In Section 3, a criterion for the induced L_∞ stability of fixed-point digital filters with saturation nonlinearities and external interference is established. Numerical examples are given in Section 4 to show the usefulness of the proposed result. Finally, Section 5 provides concluding remarks.

2. System Description

Notations: \mathbf{R}^p and $\mathbf{R}^{p \times q}$ indicate set of $p \times 1$ real vectors and $p \times q$ real matrices, respectively. \mathbf{B}^T denotes the transpose of the real matrix \mathbf{B} . For a given real matrix \mathbf{B} , $\mathbf{B} > \mathbf{0}$ ($\mathbf{B} < \mathbf{0}$) stands for the matrix \mathbf{B} which is symmetric positive (negative) definite. $\lambda_{\min}(\mathbf{B})$ is the minimum eigenvalue of the matrix \mathbf{B} ; $\lambda_{\max}(\mathbf{B})$ represents the maximum eigenvalue of matrix the \mathbf{B} . The null matrix or null vector of appropriate dimension is denoted as $\mathbf{0}$. The notation used to represent identity matrix of appropriate dimension is \mathbf{I} . $\sup\{\cdot\}$ stands for supremum or least upper bound of a set. Any vector or matrix norm is indicated by $\|\cdot\|$.

The system under consideration is given by:

$$\begin{aligned} \mathbf{x}(r+1) &= \mathbf{f}(\mathbf{y}(r)) + \mathbf{w}(r) \\ &= [f_1(y_1(r)) \quad f_2(y_2(r)) \quad \cdots \quad f_n(y_n(r))]^T \\ &\quad + [w_1(r) \quad w_2(r) \quad \cdots \quad w_n(r)]^T, \end{aligned} \quad (1)$$

$$\mathbf{y}(r) = [y_1(r) \quad y_2(r) \quad \cdots \quad y_n(r)]^T = \mathbf{A} \mathbf{x}(r), \quad (2)$$

$$\mathbf{z}(r) = [z_1(r) \quad z_2(r) \quad \cdots \quad z_p(r)]^T = \mathbf{H} \mathbf{x}(r), \quad (3)$$

Where $\mathbf{x}(r) \in \mathbf{R}^n$ denotes the state vector, $\mathbf{z}(r) \in \mathbf{R}^p$ represents the linear combination of the states, $\mathbf{w}(r) \in \mathbf{R}^n$ is the external interference, $\mathbf{A} \in \mathbf{R}^{n \times n}$ the known coefficient matrix, and $\mathbf{H} \in \mathbf{R}^{p \times n}$ is a known constant matrix. The function $f_i(y_i(r))$ represents the saturation nonlinearities given by:

$$f_i(y_i(r)) = \begin{cases} 1, & y_i(r) > 1 \\ y_i(r), & -1 \leq y_i(r) \leq 1, \\ -1, & y_i(r) < -1 \end{cases} \quad i = 1, 2, \dots, n, \quad (4)$$

Are under consideration.

Note that the saturation nonlinearities are restricted to sector $[0, 1]$, i.e.,

$$f_i(0) = 0, \quad 0 \leq \frac{f_i(y_i(r))}{y_i(r)} \leq 1, \quad i = 1, 2, \dots, n. \quad (5)$$

For a given $\gamma > 0$, the paper intends to establish an induced L_∞ based stability criterion such that the system (1)-(3) with $\mathbf{w}(r) = \mathbf{0}$ is exponentially stable and under zero-initial conditions for all nonzero $\mathbf{w}(r)$. The parameter γ is known as induced L_∞ norm bound or the interference attenuation level. In this case, the digital filter (1)-(3) is known to be exponentially stable with induced L_∞ performance γ .

$$\sup_{r \geq 0} \{\mathbf{z}^T(r) \mathbf{z}(r)\} < \gamma^2 \sup_{r \geq 0} \{\mathbf{w}^T(r) \mathbf{w}(r)\}, \quad (6)$$

The following lemma is needed in the proof of main result.

Lemma 1 (see [9]) Let the matrix $D = [d_{ij}] \in \mathbf{R}^{n \times n}$ is characterized by the following form:

$$d_{ii} = g_i + \sum_{j=1, j \neq i}^n (\zeta_{ij} + \eta_{ij}), \quad i = 1, 2, \dots, n, \quad (7a)$$

$$d_{ij} = d_{ji} = (\zeta_{ij} - \eta_{ij}), \quad i, j = 1, 2, \dots, n, (i \neq j), \quad (7b)$$

$$\zeta_{ij} = \zeta_{ji} > 0, \quad \eta_{ij} = \eta_{ji} > 0, \quad i, j = 1, 2, \dots, n, (i \neq j), \quad (7c)$$

$$g_i > 0, \quad i = 1, 2, \dots, n. \quad (7d)$$

It is implicit that for $n = 1$, D corresponds to a positive scalar. Then:

$$(\mathbf{y}(r))^T \mathbf{D}(\mathbf{y}(r)) - \mathbf{f}^T(\mathbf{Ax}(r)) \mathbf{D} \mathbf{f}(\mathbf{Ax}(r)) \geq 0, \quad (8)$$

Where $\mathbf{f}(\mathbf{y}(r))$ is given by (4).

3. Improved Criterion

We now prove the following result.

Theorem 1. For a given $\gamma > 0$, the system (1)-(3) is exponentially stable with induced L_∞ performance γ , if there exist $n \times n$ matrices $\mathbf{P} = \mathbf{P}^T > \mathbf{0}$, $\mathbf{D} = \mathbf{D}^T = [d_{ij}] > \mathbf{0}$, $n \times n$ positive definite diagonal matrix \mathbf{M} , $n \times n$ matrices \mathbf{Q}_1 , \mathbf{Q}_2 and positive scalars μ and $0 < \lambda < 1$ such that

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{A}^T \mathbf{D} \mathbf{A} + \lambda \mathbf{P} - \mathbf{P} & \mathbf{A}^T \mathbf{M} + \mathbf{Q}_1 & -\mathbf{Q}_1 & \mathbf{Q}_1 \\ \mathbf{M} \mathbf{A} + \mathbf{Q}_1^T & \mathbf{P} - \mathbf{D} - 2\mathbf{M} & \mathbf{Q}_2^T & \mathbf{P} \\ -\mathbf{Q}_1^T & \mathbf{Q}_2 & -\mathbf{Q}_2 - \mathbf{Q}_2^T & \mathbf{Q}_2 \\ \mathbf{Q}_1^T & \mathbf{P} & \mathbf{Q}_2^T & \mathbf{P} - \mu \mathbf{I} \end{bmatrix} < \mathbf{0}, \quad (9)$$

$$\begin{bmatrix} \lambda \mathbf{P} & \mathbf{0} & \mathbf{H}^T \\ \mathbf{0} & (\gamma - \mu) \mathbf{I} & \mathbf{0} \\ \mathbf{H} & \mathbf{0} & \gamma \mathbf{I} \end{bmatrix} > \mathbf{0}, \quad (10)$$

Where,

$$d_{ii} \geq \sum_{j=1, j \neq i}^n |d_{ij}|, \quad i = 1, 2, \dots, n. \quad (11)$$

Proof. Consider a Lyapunov function as:

$$\mathbf{V}(\mathbf{x}(r)) = \mathbf{x}^T(r) \mathbf{P} \mathbf{x}(r). \quad (12)$$

Calculating the difference of $\mathbf{V}(\mathbf{x}(r))$ along the trajectory of (1) gives:

$$\begin{aligned} \Delta \mathbf{V}(\mathbf{x}(r)) &= \mathbf{V}(\mathbf{x}(r+1)) - \mathbf{V}(\mathbf{x}(r)) \\ &= [\mathbf{f}(\mathbf{Ax}(r)) + \mathbf{w}(r)]^T \mathbf{P} [\mathbf{f}(\mathbf{Ax}(r)) + \mathbf{w}(r)] - \mathbf{x}^T(r) \mathbf{P} \mathbf{x}(r) \\ &= \mathbf{f}^T(\mathbf{Ax}(r)) \mathbf{P} \mathbf{f}(\mathbf{Ax}(r)) + \mathbf{f}^T(\mathbf{Ax}(r)) \mathbf{P} \mathbf{w}(r) \end{aligned}$$

$$\begin{aligned}
& +\mathbf{w}^T(r)\mathbf{P}\mathbf{f}(\mathbf{Ax}(r)) + \mathbf{w}^T(r)\mathbf{P}\mathbf{w}(r) - \mathbf{x}^T(r)\mathbf{P}\mathbf{x}(r) \\
& = \mathbf{f}^T(\mathbf{Ax}(r))\mathbf{P}\mathbf{f}(\mathbf{Ax}(r)) + \mathbf{f}^T(\mathbf{Ax}(r))\mathbf{P}\mathbf{w}(r) \\
& + \mathbf{w}^T(r)\mathbf{P}\mathbf{f}(\mathbf{Ax}(r)) + \mathbf{w}^T(r)\mathbf{P}\mathbf{w}(r) - \mathbf{x}^T(r)\mathbf{P}\mathbf{x}(r) \\
& + \hat{\Phi}(r) - \hat{\Phi}(r).
\end{aligned} \tag{13}$$

Where,

$$\hat{\Phi}(r) = -2\mathbf{f}^T(\mathbf{y}(r))\mathbf{M}[\mathbf{y}(r) - \mathbf{f}(\mathbf{y}(r))], \tag{14}$$

By adding (8) in (13), $\Delta V(\mathbf{x}(r))$ can be rewritten as:

$$\begin{aligned}
\Delta V(\mathbf{x}(r)) & \leq \mathbf{f}^T(\mathbf{Ax}(r))\mathbf{P}\mathbf{f}(\mathbf{Ax}(r)) + \mathbf{f}^T(\mathbf{Ax}(r))\mathbf{P}\mathbf{w}(r) \\
& + \mathbf{w}^T(r)\mathbf{P}\mathbf{f}(\mathbf{Ax}(r)) + \mathbf{w}^T(r)\mathbf{P}\mathbf{w}(r) - \mathbf{x}^T(r)\mathbf{P}\mathbf{x}(r) \\
& + 2\mathbf{f}^T(\mathbf{Ax}(r))\mathbf{M}[\mathbf{Ax}(r) - \mathbf{f}(\mathbf{Ax}(r))] \\
& - 2\mathbf{f}^T(\mathbf{y}(r))\mathbf{M}[\mathbf{y}(r) - \mathbf{f}(\mathbf{y}(r))] \\
& + [(\mathbf{Ax}(r))^T\mathbf{D}(\mathbf{Ax}(r)) - \mathbf{f}^T(\mathbf{Ax}(r))\mathbf{D}\mathbf{f}(\mathbf{Ax}(r))].
\end{aligned} \tag{15}$$

Furthermore, consider:

$$2[\mathbf{x}^T(r)\mathbf{Q}_1 + \mathbf{x}^T(r+1)\mathbf{Q}_2][\mathbf{f}(\mathbf{Ax}(r)) + \mathbf{w}(r) - \mathbf{x}(r+1)] = \mathbf{0}. \tag{16}$$

Together with (16), we obtain:

$$\begin{aligned}
\Delta V(\mathbf{x}(r)) & \leq \mathbf{f}^T(\mathbf{Ax}(r))\mathbf{P}\mathbf{f}(\mathbf{Ax}(r)) + \mathbf{f}^T(\mathbf{Ax}(r))\mathbf{P}\mathbf{w}(r) \\
& + \mathbf{w}^T(r)\mathbf{P}\mathbf{f}(\mathbf{Ax}(r)) + \mathbf{w}^T(r)\mathbf{P}\mathbf{w}(r) - \mathbf{x}^T(r)\mathbf{P}\mathbf{x}(r) \\
& + 2\mathbf{f}^T(\mathbf{Ax}(r))\mathbf{M}[\mathbf{Ax}(r) - \mathbf{f}(\mathbf{Ax}(r))] \\
& - 2\mathbf{f}^T(\mathbf{y}(r))\mathbf{M}[\mathbf{y}(r) - \mathbf{f}(\mathbf{y}(r))] \\
& + [(\mathbf{Ax}(r))^T\mathbf{D}(\mathbf{Ax}(r)) - \mathbf{f}^T(\mathbf{Ax}(r))\mathbf{D}\mathbf{f}(\mathbf{Ax}(r))] \\
& + 2[\mathbf{x}^T(r)\mathbf{Q}_1 + \mathbf{x}^T(r+1)\mathbf{Q}_2][\mathbf{f}(\mathbf{Ax}(r)) + \mathbf{w}(r) - \mathbf{x}(r+1)]
\end{aligned} \tag{17}$$

$$= \mathbf{\Omega}^T(r)\mathbf{\Gamma}\mathbf{\Omega}(r) - \lambda\mathbf{x}^T(r)\mathbf{P}\mathbf{x}(r) + \mu\mathbf{w}^T(r)\mathbf{w}(r) + \hat{\Phi}(r), \tag{18}$$

$$\mathbf{\Omega}(r) = [\mathbf{x}^T(r) \quad \mathbf{f}^T(\mathbf{Ax}(r)) \quad \mathbf{x}^T(r+1) \quad \mathbf{w}^T(r)]^T. \tag{19}$$

If $\mathbf{\Gamma} < \mathbf{0}$, we get:

$$\begin{aligned}
\Delta V(\mathbf{x}(r)) & < -\lambda\mathbf{x}^T(r)\mathbf{P}\mathbf{x}(r) + \mu\mathbf{w}^T(r)\mathbf{w}(r) \\
& = -\lambda V(\mathbf{x}(r)) + \mu\mathbf{w}^T(r)\mathbf{w}(r). \tag{20}
\end{aligned}$$

Accordingly, $\Delta V(\mathbf{x}(r)) < 0$ will be true, if $\lambda V(\mathbf{x}(r)) \geq \mu\mathbf{w}^T(r)\mathbf{w}(r)$. Since $V(\mathbf{x}(0)) = 0$ under the zero-initial condition, this implies that $V(\mathbf{x}(r))$ cannot exceed $\frac{1}{\lambda}[\mu\mathbf{w}^T(r)\mathbf{w}(r)]$.

$$\mathbf{x}^T(r)(\lambda\mathbf{P})\mathbf{x}(r) = \lambda V(\mathbf{x}(r)) < \mu\mathbf{w}^T(r)\mathbf{w}(r), \tag{21}$$

For $r \geq 0$. From (21), it gives:

$$\begin{aligned}
& \frac{1}{\gamma} \mathbf{x}^T(r) \mathbf{H}^T \mathbf{H} \mathbf{x}(r) - \gamma \mathbf{w}^T(r) \mathbf{w}(r) \\
&= \frac{1}{\gamma} \mathbf{x}^T(r) \mathbf{H}^T \mathbf{H} \mathbf{x}(r) - (\gamma - \mu) \mathbf{w}^T(r) \mathbf{w}(r) - \mu \mathbf{w}^T(r) \mathbf{w}(r) \\
&< \frac{1}{\gamma} \mathbf{x}^T(r) \mathbf{H}^T \mathbf{H} \mathbf{x}(r) - (\gamma - \mu) \mathbf{w}^T(r) \mathbf{w}(r) - \lambda \mathbf{x}^T(r) \mathbf{P} \mathbf{x}(r). \tag{22}
\end{aligned}$$

From (10), one gets:

$$\frac{1}{\gamma} \begin{bmatrix} \mathbf{H}^T \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{H} & \mathbf{0} \end{bmatrix} < \begin{bmatrix} \lambda \mathbf{P} & \mathbf{0} \\ \mathbf{0} & (\gamma - \mu) \mathbf{I} \end{bmatrix}. \tag{23}$$

Pre- and post-multiplying (23) by $[\mathbf{x}^T(r) \quad \mathbf{w}^T(r)]$ and $[\mathbf{x}^T(r) \quad \mathbf{w}^T(r)]^T$, respectively, gives:

$$\frac{1}{\gamma} \mathbf{x}^T(r) \mathbf{H}^T \mathbf{H} \mathbf{x}(r) - (\gamma - \mu) \mathbf{w}^T(r) \mathbf{w}(r) - \lambda \mathbf{x}^T(r) \mathbf{P} \mathbf{x}(r) < 0, \tag{24}$$

Which ensures:

$$\frac{1}{\gamma} \mathbf{x}^T(r) \mathbf{H}^T \mathbf{H} \mathbf{x}(r) - \gamma \mathbf{w}^T(r) \mathbf{w}(r) < 0. \tag{25}$$

Therefore, it is straightforward to see that:

$$\begin{aligned}
\mathbf{z}^T(r) \mathbf{z}(r) &= \mathbf{x}^T(r) \mathbf{H}^T \mathbf{H} \mathbf{x}(r) \\
&< \gamma^2 \mathbf{w}^T(r) \mathbf{w}(r), \tag{26}
\end{aligned}$$

And taking the supremum of (26) over $r \geq 0$ yields (6).

Note that $\mathbf{V}(\mathbf{x}(r))$ satisfies the following Rayleigh inequality [22]:

$$\lambda_{\min}(\mathbf{P}) \|\mathbf{x}(r)\|^2 \leq \mathbf{V}(\mathbf{x}(r)) \leq \lambda_{\max}(\mathbf{P}) \|\mathbf{x}(r)\|^2. \tag{27}$$

When $\mathbf{w}(r) = \mathbf{0}$, it follows from (20) that:

$$\Delta \mathbf{V}(\mathbf{x}(r)) < -\lambda \mathbf{x}^T(r) \mathbf{P} \mathbf{x}(r) \leq -\lambda_{\min}(\lambda \mathbf{P}) \|\mathbf{x}(r)\|^2. \tag{28}$$

Therefore, (27) and (28) ensure the exponential stability of the system under consideration. This completes the proof of Theorem 1.

Corollary 1. If $\mathbf{w}(r)$ is bounded as $\mathbf{w}^T(r) \mathbf{w}(r) < \chi$, the conditions (9) and (10) confirm that $\mathbf{x}(r)$ is bounded as:

$$\|\mathbf{x}(r)\| < \sqrt{\frac{\mu \chi}{\lambda_{\min}(\lambda \mathbf{P})}}, \quad r \geq 0. \tag{29}$$

Proof. From (21), gives:

$$\lambda_{\min}(\lambda \mathbf{P}) \|\mathbf{x}(r)\|^2 \leq \mathbf{x}^T(r) (\lambda \mathbf{P}) \mathbf{x}(r) < \mu \chi. \tag{30}$$

Thus, one can conclude that $x(r)$ is bounded. This completes the proof of Corollary 1.

Remark 1. It is interesting to note that by substituting $D = \delta I$, $Q_1 = \mathbf{0}$ and $Q_2 = \mathbf{0}$ in Theorem 1, it reduces to a Theorem 2 in [19]. Therefore, Theorem 2 in [19] may be considered as a special case of Theorem 1.

Remark 2. Theorem 1 is given in terms of LMIs for a fixed scalar $0 < \lambda < 1$. Thus, Theorem 1 can be easily tested using MATLAB LMI toolbox [23, 24].

Remark 3. The induced l_∞ norm which is also known as peak-to-peak gain is appropriate for the worst-case peak value of the state vector for all bounded peak values of the external disturbance signals.

Remark 4. The induced l_∞ norm [25, 26] is defined as:

$$\|T_{zw}\|_{l_\infty} = \frac{\sqrt{\sup_{r \geq 0} \{z^T(r) z(r)\}}}{\sqrt{\sup_{r \geq 0} \{w^T(r) w(r)\}}}, \quad (31)$$

Where T_{zw} is a transfer function matrix from $w(r)$ to $z(r)$. For a given level $\gamma > 0$, $\|T_{zw}\|_{l_\infty} < \gamma$ can be given in the equivalent form (6). Consider:

$$L(r) = \frac{\sup_{0 \leq k \leq r} \{z^T(k) z(k)\}}{\sup_{0 \leq k \leq r} \{w^T(k) w(k)\}}, \quad (32)$$

The relation (6) can be characterized by $L(\infty) < \gamma^2$, which is supported by the plot of $L(r)$ (Please refer Figure 1).

4. Numerical Results

To illustrate the usefulness of Theorem 1, we now consider the following examples.

Example 1. Consider a second-order system (1)-(3) with:

$$A = \begin{bmatrix} 1.05 & 0.6 \\ -0.5 & 0.01 \end{bmatrix}, \quad H = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, \quad w(r) = 0.5 \begin{bmatrix} \cos(2r) \\ 2 \sin(r) \end{bmatrix}. \quad (33)$$

For the design objective (6), assume l_∞ performance be $\gamma = 0.75$ and $\lambda = 0.5$. By using MATLAB LMI toolbox [23, 24] one can easily check the present example falls outside the application scope of Theorem 2 in [19]. However, it is found that (9) and (10) are feasible for the following values of unknown parameters:

$$P = \begin{bmatrix} 0.0828 & 0.0781 \\ 0.0781 & 0.0909 \end{bmatrix}, \quad M = \begin{bmatrix} 0.0015 & 0 \\ 0 & 0.0079 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} -0.0019 & 0.0036 \\ -0.0008 & 0.0001 \end{bmatrix}, \\ Q_2 = \begin{bmatrix} 0.0039 & 0.0004 \\ 0.0006 & 0.0145 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1107 & 0.1075 \\ 0.1075 & 0.1181 \end{bmatrix} \\ (g_1 = 0.0016, g_2 = 0.0090, \zeta_{12} = 0.1083, \eta_{12} = 7.8831 \times 10^{-4}), \quad \mu = 0.6745. \quad (34)$$

Thus, Theorem 1 succeeds to determine the exponential stability of the present system with induced l_∞ performance $\gamma = 0.75$.

Figure 1 shows the plot of $L(r)$ for the present example. From Figure 1, one can see that $L(\infty) < \gamma^2 = 0.5625$, which shows that the induced l_∞ norm from $w(r)$ to $z(r)$ is reduced within the induced l_∞ norm bound γ . The state trajectories for the present example with initial conditions $(x_1(0), x_2(0)) = (20, -15.8)$ are shown in Figure 1.

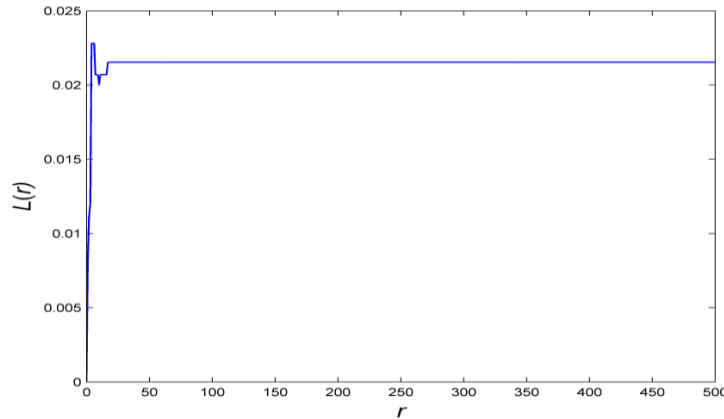


Figure 1. The plot of $L(r)$

The criteria reported in [3-10] cannot be utilized to determine the stability of the system under consideration for the present example since they do not study the effect of external interference.

Example 2. Consider the system described by (1)-(3) with:

$$\mathbf{A} = \begin{bmatrix} 0.5 & 0.1 \\ -0.1 & 0 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} 0.1 & -2 \\ 0.12 & 0.1 \end{bmatrix}, \quad \mathbf{w}(r) = 0.5 \begin{bmatrix} \cos(2r) \\ 2 \sin(r) \end{bmatrix}, \quad \lambda = 0.025. \quad (35)$$

The minimum lower bound of γ for present example achieved by using Theorem 1 is 13.6502. In contrast, Theorem 2 in [19] gives the minimum lower bound of γ as 12.8651. Since the minimum lower bound of γ found by using Theorem 1 is smaller than that obtained by Theorem 2 in [19], therefore, it is clear that Theorem 1 provides the less conservative results than Theorem 2 in [19] for the present example.

5. Conclusion

An LMI based criterion for the induced l_∞ stability of fixed-point digital filters employing saturation nonlinearities and external interference is established. The criterion turns out to be less conservative than an existing criterion. Two numerical examples to support the usefulness of the proposed approach are arranged.

The promising extension of the proposed idea to the problems of optimal non-static error compensation controller design [27], state delayed systems [28, 29], uncertain non stationary continuous systems [30] and to other such situations appears to be interesting for future investigation.

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