

## A Quantum Pointer Signal Processing Research

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### **Abstract**

*In quantum gray-scale image processing, the storage in quantum states is the color information and the position information. According to the advantage of small range of the gray scale in a gray-scale image, a novel storage expression of quantum gray-scale image is proposed and demonstrated in this study. Besides, a new concept of "quantum pointer" is put forward based on the expression. Quantum pointer is the vinculum between the information of gray-scale and position of each pixel in quantum gray-scale images. The feasibility is verified for the proposed quantum pointer, and the properties of bi-direction and sub-block are used, the storing and other operations of quantum gray-scale image are simpler and more convenient.*

**Keywords:** Quantum image processing, quantum gray-scale image, quantum pointer, quantum gray-scale image storing

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### **1. Introduction**

Combining quantum mechanics and computer originated that Benioff used quantum mechanics to describe computer [1]. Feynman proposed quantum computer in 1982 [2], quantum mechanics disciplines with the depth and breadth both was fully integrated with the computer, and then there are a fundamental breakthrough in the development of quantum computing. Similar to the classic computer, quantum computer system is made to be more perfect, there must be the independent algorithms which adapted to the characteristics of the system to complete a variety of complex arithmetic operations. In order to effectively utilize quantum parallelism and quantum states' superposition, entangled nature, collapse, etc., Deutsch proposed Deutsch quantum algorithm [3], the algorithm not only demonstrated the characteristics of quantum parallelism, and it also reflected the speed and efficiency of a quantum computer if it was compared to a classical computer. Shor presented a quantum quality factoring algorithm in 1994 [4]. Tarsus factorization problem belongs to a class of typical NP (non-deterministic polynomial) complete problem, but this problem which Shor algorithm solved is unsolvable in the classical algorithm. Grover quantum search algorithm appeared in 1995 [5], the algorithm' largest contribution was that the complexity of the search will reduce to  $\sqrt{N}$  from the classic calculation  $N$ . The improved algorithm of Grover also followed [6,7,8]. The emergence of quantum algorithm can solve the problem which a lot of classic computing can not solve, because of these advantages precisely, there are more comprehensive research area with quantum combination [7], the quantum image processing is one of them.

Quantum image processing development so far, its direction can be broadly divided into three categories. The first category is that the image information is stored in the quantum state storage expression of the quantum states; the second is that the various digital conversion of the classic image is extending the quantum realm; the third category focuses on quantum image geometric transformation. Qubit Lattice [8, 9], Real Ket [10] and Flexible Representation of Quantum Image (FRQI) [11] expression belongs to the first category. Qubit Lattice and Real Ket methods are based on quantum entanglement system, the difference is that the former is aimed at all aspects of image processing, the latter major role is in image compression. FRQI proposed method is universal significance; it is able to have a good play in the image storage, compression and geometric transformations [12, 13]. Quantum gray image based on image expression in this study also belong to the first category. The main direction of extension in Classical Quantum Digital image converter are quantum wavelet transform [14], quantum

Fourier transform [15] and quantum discrete cosine transform [16, 17]. These transformations have full use of the parallel, superimposed quantum entanglement and other characteristics, various forms of quantum transform improve more obviously in the role and efficiency and other aspects than the classic form. Many types of unitary transformation operator with quantum geometric transformation were given [18, 19], and quantum circuit was designed through these transformations [20, 21], the applicability of geometric transformations are theoretically proved and can be implemented.

## 2. Materials and Methods

### 2.1. Quantum Expression of Grayscale Image

In classic gray image, the image is without color information, color saturation is zero, each pixel is constituted with its gray scale information and location information, and its gray scale is divided into 256 levels of gray, it is from 0-255. Similarly, in the quantum grayscale image, the image pixels are represented by the grayscale and position, they are expressed as follows:

$$|Q\rangle = \frac{1}{2^n} \sum_{j=0}^{2^n-1} |M\rangle \otimes |j\rangle \quad (1)$$

Where in:  $|Q\rangle$  is the quantum state representation of storing the entire gray scale image;  $|M\rangle$  is gray information for encoding image, M used expression is quantum-bit binary string. Because the image to be represented is a grayscale image, its gray values range from 0 to 255, there is a lesser extent, and there is no representation that the image becomes very complicated. But note that, M is a gray value binary string representation, gray may be the same in different locations, but the gray scale information must be unique on the same location. In other words, M expression of a position must be only one, but M itself can vary between 0-255 gray scale values, it is a string of variable expression;  $|j\rangle$  coding is the position of the image information; representation j is still binary strings.  $j = 0, 1, \dots, 2^n - 1$ ; n is the quantum bit number which is required to encode, while  $|j\rangle = |0\rangle, |1\rangle, \dots, |2^n-1\rangle$  is 2n quantum ground states which the involved grayscale image quantum state represents;  $\otimes$  is a quantum computation operator, it is called the tensor product. In classical digital image representation, each pixel can be represented by the coordinates, according to the horizontal ordinate representation and the pixel expression, location state of quantum grayscale image may represent a further split, namely:

$$|j\rangle = |h\rangle |v\rangle \quad (2)$$

Where:  $|h\rangle$  represents the x-directional information;  $|v\rangle$  is the coded image information in the y direction.

Thus, based on the above expression, Figure 1 is the 2 x 2 image, its expansion of the expression is as follow:

$$|Q\rangle = \frac{1}{2} (|M_0\rangle \otimes |00\rangle + |M_1\rangle \otimes |01\rangle + |M_2\rangle \otimes |10\rangle + |M_3\rangle \otimes |11\rangle) \quad (3)$$

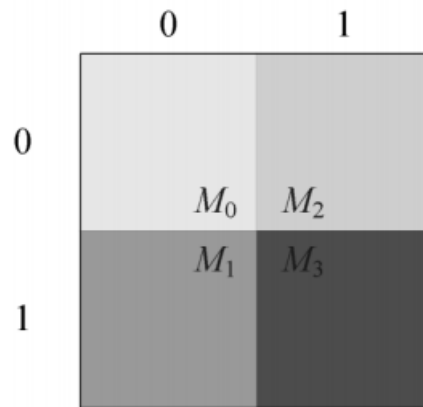


Figure 1. 2x2 gray image

It should be noted that the expression of  $M_0, M_1, M_2, M_3$  are represented by a binary string, and since each pixel gray are different,  $M_0, M_1, M_2, M_3$  is not equal to each other, and each position has a unique gray scale value.

**Proof :** From a quantum algorithm, the establishment of the expression is prepared from the initial quantum state, an expression of the storage state is converted by quantum gate change.

**Step 1 :** Preparation of the initial quantum state is  $|S\rangle = |0\rangle^{\otimes 2n+1}$ , and the initial state  $|0\rangle^{\otimes 2n+1}$  is divided into  $|S\rangle = |0\rangle^{\otimes 2n} |0\rangle$ .

**Step 2 :** Hadamard Transform  $|H\rangle^{\otimes 2n}$  is used, The  $2n$  H doors act on the  $2n$  initial qubit of the quantum state at the same time. After the effect of Hadamard door, intermediate quantum state can be obtained:

$$|G\rangle = H^{\otimes 2n} (|0\rangle^{\otimes 2n} |0\rangle) = \frac{1}{2^n} |0\rangle \otimes \sum_{j=0}^{2^n-1} |j\rangle \tag{4}$$

**Step 3 :** Swap operation is used, the state  $|0\rangle$  is converted any  $|M\rangle$  state, the final state is obtained:

$$|Q\rangle = \frac{1}{2^n} \sum_{j=0}^{2^n-1} |M\rangle \otimes |j\rangle \tag{5}$$

The proving shows that the quantum expression  $|Q\rangle = \frac{1}{2^n} \sum_{j=0}^{2^n-1} |M\rangle \otimes |j\rangle$  of the presented grayscale images here can be converted from a series of quantum unitary transformation and from the initial quantum state  $|0\rangle^{\otimes 2n+1}$ , the basis is provided for the next application of the expression.

**Quantum pointer:** In classical computer, the pointer is used to indicate the address of the memory unit stores, the memory unit of its address can be found through a pointer, which means that the address of a variable is the variable pointer. In quantum expressions of grayscale images, a gray scale represents 8 binary string, position indication also is a binary string, to diagram is whether each pixel location or gray information are binary strings in their forms of expression. For quantum grayscale images, each pixel is composed of the gray value and its location representation, a combination of both is exactly similar to the classic computer pointer, there is the idea of quantum pointer.

## 2.2. Quantum Pointer Bidirectionality

Quantum pointer is different from the classic pointer. In classical pointer, the address is a pointer, and this representation is fixed. However, the pointer is not fixed in quantum pointer, it is with two-way directivity. In both gray value and the position value, the gray value may be represented as a pointer, the position value may also be expressed as a pointer, and this choice is determined based on the quantum specific image processing operation. The quantum pointer bidirectionality is expressed specifically as follows:

- 1) When the gray value of quantum gray image is used to represent quantum pointer, the position values of the image is the content which the pointer points to ;
- 2) When the position values of quantum gray image represent quantum pointer, the gray values of the image is the content which the pointer points to.

In different image processing, both gray value pointer or position value pointer is selected as a pointer on the basis of which the process is relatively simple, it is easily implemented. For example, when the same gray-scale pixel gray value is adjust in the image, gray can be as a pointer to find a pointer of the gray, then gray value is directly changed, the gray which is corresponding to the pointer position is changed along with it. If the location is as a quantum pointer, which convenient effects will be produced? Here comes to the next nature of quantum pointer, which is sub-blocks.

**Quantum pointer sub-block properties:** Quantum pointer sub-block refers to the block division and combination of pointers. Because the pointer expression is a binary string, which consists of 0 and 1, according to the partition of 0 and 1, and quantum pointer can be divided on different types. When the position value is used as quantum pointer and is faced with more pixels, the pointer can be divided into blocks (or split). Similarly, when the gray scale value is used as quantum pointer, it can represent a binary string of which is representing gray can be divided, the positions different gray scale will be divided into a large sub-blocks.

## 2.3. Quantum Grayscale Image Storage

### 1) Storage Based on Quantum Gray Pointer

In the proposed expression,  $|M\rangle$  is used to encode the gray scale information, it is assumed that the gradation information is as quantum pointer. In a fixed grayscale image, each pixel has gray scale information, and some pixel grayscale of these pixels is the same. Gray pointer with the same gray value points to pixel location with the same gray-scale information, the building relationship is no longer the one to one relationship in the classic image, but it is the one to many relationship, the one to many relationship makes the pixel be without a single change, after the pixel sub-blocks can be formed, large scale transformation is made. Because the gray value changes between 0-255, the position of the same gray value is for unified storage, 256 units of quantum states most only be needed for storage.

Let  $|P_i\rangle = \{|p_{i1}\rangle, |p_{i2}\rangle, \dots, |p_{im}\rangle, \dots\}$ , here,  $|P_i\rangle$  collection is used to represent all the position states of which grayscale value  $M_i$  corresponding to, and  $|p_{im}\rangle$  represents various locations of gray value  $M_i$ , the corresponding position of each gray value is assumed to be  $m$ ,  $0 \leq m \leq 2^{2n} - 1$ , quantum stored expression of gray pixel with the corresponding grayscale value  $M_i$  is in equation (6).

$$|Q_i\rangle = \frac{1}{\sqrt{m}} \sum |M_i\rangle \otimes |p_{im}\rangle \quad (6)$$

$M_i$  must correspond  $P_i$ , and  $|Q_i\rangle$  is sub-expression of the expression

$$|Q\rangle = \frac{1}{2^n} \sum_{j=0}^{2^{2n}-1} |M\rangle \otimes |j\rangle, \text{ it also corresponds to a component with a gray value.}$$

### 2) Storage Based on Quantum Position Pointer

Quantum position pointer storage relies mainly on the sub-blocks of quantum pointer, the sub-block of location information bits is divided to achieve the purpose of the block storing image pixel information.

Pixel position is represented as  $|j\rangle = |P\rangle = |h_1 h_2 \dots h_n\rangle |v_1 v_2 \dots v_n\rangle$ , where  $|h_1 h_2 \dots h_n\rangle$  and  $|v_1 v_2 \dots v_n\rangle$  are respectively Qubit expression of the position bits in the x direction and y direction. Quantum position pointer is determined by the position string expression, which the biggest match quantum bits is determined in  $|h_1 h_2 \dots h_n\rangle, |v_1 v_2 \dots v_n\rangle$ . The maximum matching quantum bit string is that the entire image pixels are divided into a number of different chunks. In chunks, chunk is divided into different pieces by looking for matching string, it implements the sub-blocks of quantum pointer, the grayscale image storage is completed based on quantum location pointer.

**3) Storage Based on Quantum Position Pointer**

Storage based on quantum mixed pointer: From the description of two quantum pointer, there is a possibility of both combinations. In quantum gray pointers, for the same gray, the location of their remains can be divided into sub-blocks; thereby the child pointer of quantum position is generated. According to the same reason, the pixel block is processed, if the gray species are not particularly much in the block, or they can be combined based on gray. This can also serve as a future research direction in practical application.

From the point of view of two quantum pointer stored representation, processing operations are made for different quantum image, both have advantages, to choose the most appropriate pointers in various operations is the key to quantum pointer applications.

**3. Test and Discussion**

**3.1. Gradation Transformation**

The gray-scale transformation purpose is to change a gray value in the image, and all gray pixels of gray values must change, all the gray values have changed accordingly in the gray scale corresponding position.

First, the quantum image storage is completed based on gray pointer, then the gray qubits of the image is processed. Because the range of gray values is from 0 to 255, the need is showing up to an 8-bit binary string. Changes in gray value are changes of eight characters 0 and 1. The original gray values are compared with the target gray value, different bits are found between the two, and the swap door is used to achieve conversion between 0 and 1.

**Position conversion:** Position conversion is based on the quantum position pointer transformation, it is belong to the quantum image geometric transformation, the position mobile is achieved and the associated transformation is changed. A 8×8 grayscale image is used to illustrate the changing positions.

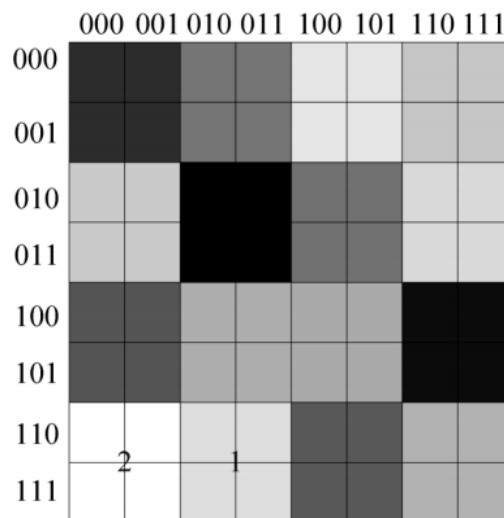


Figure 2. 8×8 original grey image

Figure 2 is a 8×8 grayscale image, each pixel identifies on the vertical and horizontal coordinates is shown in Figure 2. For a block 1 in FIGURE, the position is  $|P_1\rangle = \{|010110\rangle, |011110\rangle, |010111\rangle, |011111\rangle\}$ , and the position of the block 2 is  $|P_2\rangle = \{|000110\rangle, |001110\rangle, |000111\rangle, |001111\rangle\}$ . The stored qubits of the corresponding position are compared in position 1 and position 2,  $|010110\rangle$  and  $|000110\rangle$ ,  $|011110\rangle$  and  $|001110\rangle$ ,  $|010111\rangle$  and  $|000111\rangle$ ,  $|011111\rangle$  and  $|001111\rangle$  is contrasted respectively, Only the 2-th position is different in two bits, and for each qubit pair, only the 2-th original qubit transforms from 1 to 0, transformation can be completed from the original position to the target position.

**Application Test:** The described operation in Figure 3 is the above stored and transformation process.

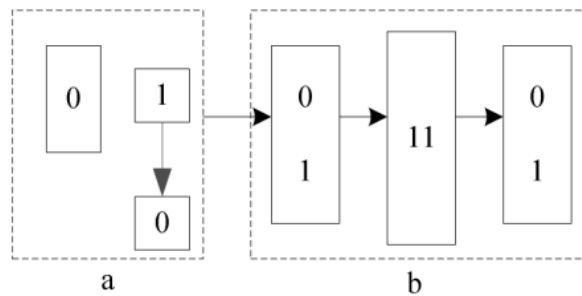


Figure 3. Quantum bit swap

First, position  $|P_1\rangle$  is stored. Lateral view exchange is shown in Figure 3, a dotted box is the first child block of the quantum pointer, which is the first layer of a pointer; second sub-block is a selection block, the provided and selected qubit is 0 and 1; the third sub-block is composed of "11" sub-blocks; the 4-th sub-block is also a selection block, it is composed of 0 and 1. From the division of the sub-blocks, the need transform is a first sub-block, arrow in the dashed box is a 1-0 conversion, and the 2-th, 3-th, 4-th sub-blocks constitute a perfectly matched combination of a memory, as is shown in the dashed box b. The qubit string block storage allows us to quickly find the blocks that need to change, the transition is implemented between bits, and different sub-blocks are divided into different images, which can also change the efficiency of the image processing. After the completion of the image conversion, it is shown in Figure 4.

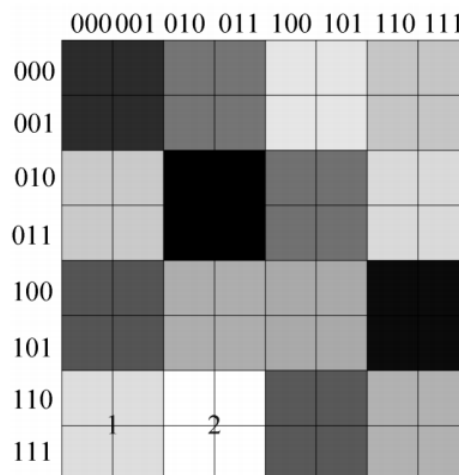


Figure 4. 8×8 target grey image

Figure 5 is a group of mobile photos, the content image blocks continuously exchange with the blank blocks to reach the mobile mass, the require image VI is obtained from the original image I. Each image is a 16×16 gray-scale image, while the entire image is divided into 16 blocks of equal sizes, and then the entire image is completed by moving the blocks. Description block moves is the first step, block 2 exchanges with blank block 16.

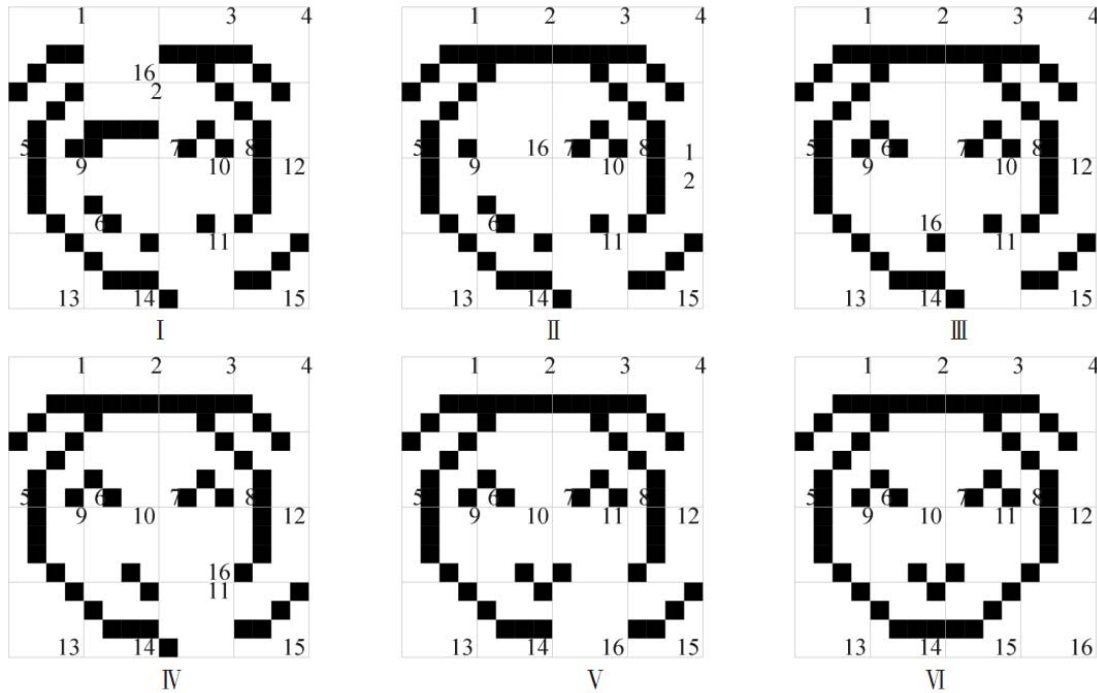


Figure 5. Original image, target image and middle image got by the image transformation

Pixel blocks 2 and 16 are made of 4×4 pixels. Based on the expression (2), the position of each pixel is represented by the eight quantum bits. The position set of the block 16 and block 2 are as follows (quantum bit string is too long and is not marked in Figure, Reference can be seen in Figure 2)

$$|P_{16}\rangle = \left\{ \begin{array}{l} |01000000\rangle, |01010000\rangle, |01100000\rangle, |01110000\rangle, \\ |01000001\rangle, |01010001\rangle, |01100001\rangle, |01110001\rangle \\ |01000010\rangle, |01010010\rangle, |01100010\rangle, |01110010\rangle, \\ |01000011\rangle, |01010011\rangle, |01100011\rangle, |01110011\rangle \end{array} \right\} \quad (7)$$

$$|P_2\rangle = \left\{ \begin{array}{l} |01000100\rangle, |01010100\rangle, |01100100\rangle, |01110100\rangle, \\ |01000101\rangle, |01010101\rangle, |01100101\rangle, |01110101\rangle \\ |01000110\rangle, |01010110\rangle, |01100110\rangle, |01110110\rangle, \\ |01000111\rangle, |01010111\rangle, |01100111\rangle, |01110111\rangle \end{array} \right\} \quad (8)$$

The blocks 2 and 16 need to be changed, the storage of block 2 and block 16 are only listed. Figure 6 shows the storage of two transform blocks, and the need quantum bit conversion is achieved.

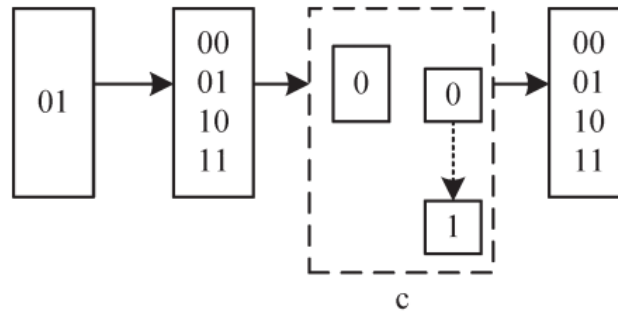


Figure 6. First transformation: exchange between 16 and 2

Location listed set P16 and P2 are compared in corresponding position, only the dotted line arrow marked bits are different in the corresponding position of P2 set. According to the exchange of the dashed arrows in Figure 6, the exchange will be able to be completed between block 16 and block 2.

**Discussion:** Because quantum pointer is a directional, we only need to change the position of the pixel, and then the role of quantum position pointer can be to change the corresponding position of gray, which is a major advantage of quantum pointer.

I to II conversion can be completed in Figure 5, then the other transformations is similar. Total 5 conversion is required from the original image I to the target image VI. The first step transformation is completed in Figure 6; the converted exchanges of the other four stores are empathy.

Other quantum image geometric transformation is compared, the pixel movement and transformation are used first in the quantum pointer form, it is no longer to change each individual pixel, but pixel block sizes vary in units, this can reduce unnecessary operations redundancy and simplify procedures. Furthermore, the point role of quantum pointer can produce a series of chain reactions, time is shorten to lock the operating range, efficiency is improved for image processing operations..

#### 4. Conclusion

A quantum expression of gray image is presented according to the gray property and location property of gray image pixel, and a new concept of quantum pointer is put forward in accordance with the image expression, based on the quantum pointer property, grayscale conversion and position conversion be made from the quantum theory of grayscale image. The different type of quantum pointer is used in different image processing operations, the complexity of the operation is simplified. As can be seen from the example, based on the image grayscale or the location information change, the connection role of quantum pointer can complete the transformation operation of the entire image, the complexity is the half.

In this paper, a relatively simple representation is proposed for grayscale images quantum expression. In the classic color image, the pixel has the two properties of the color and position. In a grayscale image, each pixel gray value and its location is both represented, the range of gray values is the smaller, which simplifies the representation of the expression. The expression extends to the color image is a direction for future research. In addition, a new concept of the quantum pointer is proposed based on the proposed expression. The proposed quantum pointer here is somewhat similar with classical computer pointer, but there are very different, quantum images stored representation is more convenient, so that the image has a higher efficiency and and better results in the process of transformation.

#### Acknowledgements

This study is sponsored by the Scientific Research Project (NO. 14A084) of Hunan Provincial Education Department, China.



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