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Reducing Computational Complexity and Enhancing Performance of IKSD Algorithm for Uncoded MIMO Systems

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Abstract

The main challenge in MIMO systems is how to design the MIMO detection algorithms with lowest computational complexity and high performance that capable of accurately detecting the transmitted signals. In last valuable research results, it had been proved the Maximum Likelihood Detection (MLD) as the optimum one, but this algorithm has an exponential complexity especially with increasing of a number of transmit antennas and constellation size making it an impractical for implementation. However, there are alternative algorithms such as the K-best sphere detection (KSD) and Improved K-best sphere detection (IKSD) which can achieve a close to Maximum Likelihood (ML) performance and less computational complexity. In this paper, we have proposed an enhancing IKSD algorithm by adding the combining of column norm ordering (channel ordering) with Manhattan metric to enhance the performance and reduce the computational complexity. The simulation results show us that the channel ordering approach enhances the performance and reduces the complexity, and Manhattan metricalonecan reduce the complexity. Therefore, the combined channel ordering approach with Manhattan metric enhances the performance and much reduces the complexity more than if we used the channel ordering approach alone. So our proposed algorithm can be considered a feasible complexity reduction scheme and suitable for practical implementation.

***Keywords**: MIMO, KSD, MLD, channel ordering, and Manhattan metric*

1. Introduction

In recent years, two main research activities have dominated the design of power and bandwidth efficientwireless communication systems: First, multiple inputs, multiple outputs (MIMO) [1] that embody the meaning of communication through multiple antennas. MIMO technique permits simultaneous transmit of multiple symbols from multiple transmit antennas. This results in a linear increase in the channel capacity commensurate to the number of transmit antennas when there are a suitable number of receive antennas [2]. Second, the Iterative detection, it is a practical method to improve the symbol-error-rate (SER) performance for communication systems. So the study of combining the Iterative detection and MIMO techniques to approaching from the capacity of MIMO channels [3].

MIMO detection is a challenging and important topic for researchers and communication system designers, massive research efforts were done in the last years giving the birth to a variety of detection techniques that differ in strategy adapted, computational complexity, and performance. In order to solve the detection problem in MIMO systems, the researchers have been focused on suboptimal detection techniques which are efficient in terms of both performance and computational complexity, and powerful in terms of error performance and are practical for implementation purposes [4].

A novel and efficient MIMO detection algorithm for any wireless communication systems must include some important features such as low-complexity, near-optimal performance and robust scheme. The MLD [2] can present outstanding performance; but, it suffers from high computational complexity in practical implementation especially when increasing the number of transmit antennas to achieve a good transmission capacity in MIMO systems. Different near-optimal MIMO detection techniques have been proposed in previous literatures some of them

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based on zero-forcing (ZF) [3], minimum mean-squared-error (MMSE) [3], successive interference cancellation (SIC) [5], parallel interference cancellation (PIC) [6] and ordered SIC (OSIC) [7]. Unfortunately, all of them cannot achieve the performance of an MLD. Sphere detection /decoder (SD) [8-12] was investigated to achieve the ML performance by using reliable radius. The idea of SD was introduced in [13] and it has been furthermore debated in various researches [14,15]. The K-best sphere decoder (KSD) [12] for MIMO detection appears in the area of detection techniques because of its fixed throughput and parallel implementation. In the other side, the use of the depth-first tree search in conventional SD giving non-constant throughput, which limits the detection efficiency. So instead of using a depth-first to traverse the tree, the KSD executes a breadth-first search and keeps only K-best nodes in each layer. In KSD algorithm to achieve close- ML performance [12], the KSD especially needs for very large values of K, which in turn leads to a higher complexity than that who in the conventional SD. Nonetheless, due to advantages of the KSD algorithm, some variants have been proposed to improve its performance and/or reducing its complexity [16-19].

The computational complexity of an MIMO detection algorithms depends on the number of spatially multiplexed data streams (number of transmit antennas) and the symbol constellation size, but frequently on the instantaneous MIMO channel realization and the signal-to-noise ratio (SNR) [20]. The computational complexity of tree search algorithms is determined by two norms: Firstly, the number of nodes that have to be examined and Secondly, the operational cost per node. In SD, the number of visited nodes depends on the choice of initial sphere radius and on the decreasing of the radius constraints due to a radius update [21]. The complexity of K-best SD algorithms depends critically on the preprocessing stage (QR decomposition), the ordering (back-substitution) in which the components of information signals are considered, and the initial choice of the radius of the sphere.

In this work, we propose enhancing IKSD algorithm, this can be achieved by divided the K-best SD algorithm work into two parts. The first part is known as the "*preprocess par"t*, the preprocess can be achieved by execution the column norm ordering [22] (channel ordering) for channel matrix due to that the computation complexity is so sensitive to the order of the columns of the channel matrix. The second part is known as the "*search part"*, it is computed the ML solution of transmitted vector from the received vector, in this part we propose using the Manhattan norm to calculate the ML solution in order to reduce the complexity of this part.

The rest of this paper is organized as follows: Section 2 presents the model of the MIMO system and K-Best SD algorithm. The enhanced IKSD algorithm is present in Section 3. The column norm ordering (channel ordering approach) preprocessing is described in Sub-section 3.1. Manhattan metric propose to use in search part in Sub-section 3.2. Simulation results presented in Section 4. Finally, in Section 5, we present the conclusions.

2. System Model and K-Best SD Algorithm

We consider an uncoded M-QAM 4×4 MIMO system having $N_t$transmit and N_r receive antennas where $N_r \geq N_t$. Under the assumption of a flat-fading channel, the received vector can be expressed as

$$\bar{y} = \bar{H}\bar{s} + \bar{w} \quad (1)$$

where$\bar{s} = [\bar{s}_1, \bar{s}_2, \dots\dots, \bar{s}_{N_t}]^T$ denotes $(N_t \times 1)$ transmitted vector, and the entries of $\bar{s}$ are selected from a complex constellation, $\bar{y} = [\bar{y}_1, \bar{y}_2, \dots\dots, \bar{y}_{N_r}]^T$ denotes $(N_r \times 1)$ complex-valued received vector, $\bar{H}$ denotes the $(N_r \times N_t)$ complex-valued channel matrix with elements are assumed to be independent and identically distributed (i.i.d.) complex Gaussian variables with zero mean and unit variance, and $\bar{w}$is the complex-valued of additive white Gaussian noise (AWGN) with zero mean and σ^2variance.

For simplifying the system, the complex-valued received vector is transformed into an equivalent real-valued received vector by representing the real part and the imaginary part of $\bar{y}$ as

$$y = Hs + w \quad (2)$$

where the dimension is doubled such that $m = 2N_t$, $n = 2N_r$, i.e.,

$$\begin{bmatrix}\Re[\bar{y}]\\ \Im[\bar{y}]\end{bmatrix} = \begin{bmatrix}\Re[\bar{H}] & -\Im[\bar{H}]\\ \Im[\bar{H}] & \Re[\bar{H}]\end{bmatrix}\begin{bmatrix}\Re[\bar{s}]\\ \Im[\bar{s}]\end{bmatrix} + \begin{bmatrix}\Re[\bar{w}]\\ \Im[\bar{w}]\end{bmatrix} \tag{3}$$

where $\Re[\cdot]$and $\Im[\cdot]$ denote the real and imaginary parts of $[\cdot]$ respectively, $s = \begin{bmatrix}\Re[\bar{s}]\\ \Im[\bar{s}]\end{bmatrix}$ denotes $(m \times 1)$ real-valued transmitted vector, $y = \begin{bmatrix}\Re[\bar{y}]\\ \Im[\bar{y}]\end{bmatrix}$ denotes $(n \times 1)$ real-valued received vector, $w = \begin{bmatrix}\Re[\bar{w}]\\ \Im[\bar{w}]\end{bmatrix}$ denotes $(n \times 1)$ real-valued noise vector, and $H = \begin{bmatrix}\Re[\bar{H}] & -\Im[\bar{H}]\\ \Im[\bar{H}] & \Re[\bar{H}]\end{bmatrix}$ denotes $(n \times m)$ real-valued channel matrix [14], [23]. Assume that the receiver has a perfect channel knowledge, so the MLD problem can be formulated as

$$s_{ML} = \arg\min_{s \in A} \parallel y - Hs \parallel^2 \tag{4}$$

where $A = D^m$, *D* is real-valued signal constellation set, for 16-QAM*, D* = [-3,-1,1,3].
In SD algorithms the search includes only the lattice points (*Hs*) inside the hypersphere centered at the received vector(*y*) with radius (*d*) instead of comprehensive search for all lattice points as in MLD, and can be written as

$$s_{ML} = \arg\min_{s \in A} \parallel y - Hs \parallel^2 \leq d^2 \tag{5}$$

By decomposed the channel matrix H using the standard QR decomposition, we can get

$$\tilde{y} = Rs + \widetilde{w} \tag{6}$$

where $\tilde{y} = Q^{\mathrm{H}}y$, $\widetilde{w} = Q^{\mathrm{H}}w, (\cdot)^{\mathrm{H}}$denotes Hermitian matrix transposition, *R* is an $(m \times m)$ upper triangular matrix, and Q is an$(n \times n)$orthogonal matrix.Utilizing the triangular nature of R, the left-hand side of (6) can be rewritten as

$$s_{ML} = \sum_{i=1}^{m} \parallel \tilde{y}_i - \sum_{j=i}^{m} R_{i,j}s_j^{(i)} \parallel^2 \tag{7}$$

From (7) we can see the detection problem as a tree that has its root just above the m-th layer and leaves on the 1st layer, and each survived candidate of *i-th* layer is defined as $s^{(i)} = [s_i^{(i)}, s_{i+1}^{(i)} \dots \dots, s_m^{(i)}]$. The Euclidean distance in (7) can be computed iteratively by defining $P(s^{(i)}) = P_i$ with the partial Euclidean distances (PEDs)[24].

$$P_i = P_{i+1} + \parallel e_i \parallel^2 , \quad i = m, \dots\dots, 1 \tag{8}$$

The initialization $P_{m+1} = 0$, and the distance increments are

$$\parallel e_i \parallel^2 = \parallel \tilde{y}_i - \sum_{j=i}^{m} R_{i,j}s_j^{(i)} \parallel^2 \tag{9}$$

The PED,P_i, depend on the symbol vector (*s*) through the partial symbol vector $s^{(i)}$, the SD problem has been changed into a weighted tree-search problem. The SD algorithm with depth-first tree search suffers from non-constant throughput and non-efficiency decoding [25]. To overcome these problems, the K-best SD algorithm is used, with applying the breath-first tree search strategy. The K-best algorithm simplifies the complexity of SD algorithm by shortening the paths in each detection layer from the m-th layer to the 1st layer and only the smallest K nodes are kept in each layer (except the 1st layer), which will be extended into

$(K \times D)$ nodes in the next layer. We can describe the K-best algorithm through the following steps:

a) Apply QR decomposition according to H = QR.
b) Compute the PEDs P_i according to Eq. (6) to (9) (path extension).
c) Prune the paths that greater than the radius based on the PEDs of step (b).
d) Do sorting to K nodes and choose the smallest one.
e) Do path update by updating i by $i = i - 1$, If $i > 1$, go back to step (b).
f) If $i = 1$, go to step (b) and (c), then select the path with minimum PED as a decision.

If K is not large enough, the K-best SD algorithm not able to guarantee the same performance as SD and ML algorithm. Thus, there is a pressing need to find efficient K-best algorithms that can do the best trade-off between performance and complexity.

3. The Enhancing IKSD Algorithm

As noted earlier, we can enhance the performance of the IKSD algorithm by adding channel ordering approach to the preprocessing part, and also adding the Manhattan metric to the search part. Where the channel ordering approach is working to improve SER performance and reducing the complexity, the Manhattan metric is working to reduce the complexity more. As it will be clarified in the next two sub-sections. The enhancing IKSD algorithm is described in Algorithm-I.

Algorithm-I: The enhancing IKSD algorithm

Input:y, *H*, *K*, *A*, Δ, d
Output:$\hat{S}$
Initialization $P_i = 0$ (the branch metric) and S_o is the root node (level $i = m$);
$H_{ordering} = \Pi H$;
$[Q, R] = qr\_decomp(H_{ordering})$;
$\tilde{y} = Q^{\mathrm{H}} y$;
$P_i = 0$; and start from level $i = m$
while *i* ≥ *1* **do**
$\ell = 1$;
for *j=1***to** length (P_i) **do**
$\widetilde{\boldsymbol{P}}_{\boldsymbol{\ell}} = \boldsymbol{P}_{\boldsymbol{j}} + \left\|\widetilde{\boldsymbol{y}}_{\boldsymbol{i},\boldsymbol{j}} - \boldsymbol{r}_{\boldsymbol{i},\boldsymbol{i}}\boldsymbol{s}_{\boldsymbol{i}}\right\|, \forall \boldsymbol{s}_{\boldsymbol{i}} \in \boldsymbol{A}$;
$\ell = \ell + 1$;
end
sort all the components of P_i in an ascending order ;
if length $\widetilde{P} \leq K$**Then**
Keep all the candidates in tree;
else
Only keep the elements whose cost indexes satisfy $\widetilde{P} \leq P_i + \Delta$ in tree ;
end
Replace the $P_i \leftarrow \widetilde{P}$;
$i = i - 1$;
end
Return $\hat{S} \leftarrow$ the 1st element in the tree

3.1. The Channel Ordering Approach (Preprocess Part)

The computation complexity of K-best SD is quite sensitive to the order of the columns of the channel matrix, which rely on both the channel matrix and the received signal. So, the random detection order is not the best detection order, particularly for low SNR or high order modulation. Usually, re-arranging the columns of the matrix appropriately is to get a good detection process (low complexity).

The QR decomposition performance can be improved if the channel matrix is pre-processed before QR decomposition. So, we suggest using the preprocess of column norm ordering(channel ordering approach) [22] before QR decomposition. The columns of channel matrix can be reordered in accordance with the norm of each column, so the signals with higher signal-to-noise ratio (SNR) are detected first. This can be achieved by multiplying channel matrix H by a permutation matrixΠ, i.e., $\Pi\mathrm{H} = \mathrm{QR}$.

The column norm ordering method includes the following procedure, arranging the columns h_i of the channel matrix $H = [h_1, \ldots\ldots, h_{m-1}, h_m]$ in accordance with their Euclidean distance $\| h_i \|$ in an ascending manner. The arranging of $h_i$is processed by the permutation ρ so that

$$\| h_{\rho(i)} \| \leq \| h_{\rho(j)} \| \quad for\ \ i<j \tag{10}$$

Then, the arranging channel matrix is represented as

$$H_{ordering} = \Pi H \tag{11}$$

where Π is $(m \times m)$permutation matrix such as $\Pi = \left[e_{\rho(1)}, e_{\rho(2)}, \ldots\ldots, e_{\rho(m)}\right]$, where e_i is the column vector of which entries are one *i-th*position only and are zero in every other positions.
One advantage of channel ordering of the generator matrix *H* is that the column norm ordering does not distort or disturb the boundaries of the finite lattice can be easily determined and exploited. This can be understood as the column arranging of *H* simply leads to a re-arranging of the components of transmit signal vectors.

3.2. The Manhattan Metric (Search Part)

In this section, we propose to use Manhattan metric (MM) to calculate ML solution instead of using the Euclidean metric (EM), in order to reduce the complexity in search part. The purpose of using MM or EM is to calculate the weights of each candidate node [26]. In EM, the brute-force MLD can be converted into a full tree structure search by using EM such as

$$s_{ML(\mathrm{EM})} = \arg\min_{s\in A} ||\tilde{y}_i - R_i s_i|| = \arg\min_{s\in A} \sum_{i=1}^{m} \left| \tilde{y}_i - \sum_{j=i}^{m} R_{i,j} s_j^{(i)} \right|^2 \tag{12}$$

From (12) the MIMO-MLD searches a candidate $s^{(i)}$ that minimizes the squared EM between $\tilde{y}_i$ and $R^i s^{(i)}$ that is referred to as $s_{ML(\mathrm{EM})}$, and we can see that the operations performed depend on summation and multiplication due to square term.The hardware implementation is infeasible due to a logic resource limitation of the target device because there are $4N_r M^{N_t}$= 1,048,576 real multiplications (for 16-QAM) are required to compute all the EM. According to (12) this type of detection algorithm is practically impossible to implement in MIMO systems that utilize high order modulation such as (16-QAM, 64-QAM). So we adopted a practical metric like MM to avoid the use of arithmetic multiplications, the MM is computed by adding absolute values of $\tilde{y}_i$ and$R^i s^{(i)}$, as in (13).

$$s_{ML(\mathrm{MM})} = \arg\min_{s\in A} |\tilde{y}_i - R_i s_i| = \arg\min_{s\in A} \sum_{i=1}^{m} \left| \tilde{y}_i - \sum_{j=i}^{m} R_{i,j} s_j^{(i)} \right| \tag{13}$$

As shown in (13), the operations performed depend only on summation and didn't have a square termand therefore it does not need for arithmetic multiplicationsas in (12).

4. Simulation Results

In this section, we discuss and compare the performance (the symbol error rate, SER) and computational complexity (the number of nodes visited) for both traditional KSD and IKSD algorithms, with cases of no ordering, ordering, and combine ordering with MM. To make a fair comparison for all cases, suppose the initial radius for all cases is the same.

Firstly we discuss the effect of column norm ordering and MM on SER performance in both algorithms traditional KSD and IKSD. An uncoded 4x4 MIMO system with 16-QAM and 64-QAM are simulated over a flat Rayleigh fading channel. From figure (1) and figure (2), can show the same SER performance in the case of no ordering for 16-QAM and 64-QAM just the performance of the IKSD needs for (K=2), while the traditional KSD needs for (K=16), and we can see improved the SER performance when using ordering and combine ordering with MM.

From figure (1) we have observed that for 16-QAM and at an SER=10^{-2}, the performance gain about 2.5dB when using the channel ordering and combine channel ordering with MM compared to using of no ordering. From figure (2) can conclude that the performance enhances by 3.9dB at an SER=10^{-1}.

Secondly, we discuss the effect of column norm ordering and MM on computational complexity in both algorithms traditional KSD and IKSD. Figure (3) and figure (4) shows the comparison of the complexity (visited nodes) in traditional KSD and IKSD algorithms with no ordering, ordering, and combine ordering with MM. And it also shows that the complexity of the IKSD algorithm with combined ordering and MM are lower than that of IKSD with ordering and with no ordering, and the complexity of the IKSD algorithm with ordering is less than the complexity of the IKSD algorithm with no ordering. In other words, for example (case of Figure 3), while the IKSD algorithm with ordering visit on average 44 nodes to obtain a performance comparable to IKSD algorithm with no ordering, and the IKSD algorithm with combined ordering and MM is able to achieve the same performance visiting on average 30 nodes while IKSD algorithm with no ordering visits 61 nodes. From figure (3) we can compare the complexity between the curves of no ordering case and combine ordering with MMof IKSD algorithm at a minimum (SNR=0 dB) and maximum (SNR=25 dB) difference between these two curves. In no ordering IKSD algorithm searches about (61 and 181) nodes, and in combine ordering and MM needs (30 and 42) nodes visited respectively. So the proposed IKSD with combine ordering and MM needs 51% to 77% fewer complexities than IKSD with no ordering. Also, can compare the computations between no ordering and ordering of IKSD algorithm, at a minimum (SNR=0 dB) and the maximum (SNR=25 dB) difference between these two curves, the no ordering IKSD algorithm searches about (61 and 181) nodes, and in ordering needs (44 and 48) nodes visited respectively. So the IKSD algorithm with ordering needs 28% to 73% fewer complexities than IKSD with no ordering. From figure (4) we can do the same calculation as in figure (3), for compare between two curves of no ordering and combine ordering with MM at SNR=0dB and SNR=25, it's need (1179 and 9323) nodes and (87and 87) respectively. So the combine ordering with MM needs 92% to 99% fewer complexities than no ordering, and the comparison between no ordering and ordering needs (1179 and 9323) and (599and 1123) nodes respectively. So the ordering needs 49% to 88% fewer complexities than no ordering.

In figure (3) and figure (4) can see that the complexity of traditional KSD algorithm in the cases of no ordering and ordering is the same and it's different compared with IKSD algorithm, this is due to the way of account the visited nodes in each algorithm, to clarify that, the visited nodes in the traditional KSD algorithm are calculated from all child nodes that extend in every layer and also the value of K. But in IKSD algorithm the visited nodes calculated from all child nodes that extend in every layer and restricted with the value of K and fixed threshold [27]. As depicted in figures (1)and(2), we can note that as it has happened in other works [28], [29], when using MM the performance suffer from a slight degradation, but in our proposed work, with using of channel ordering approach the SER performance does not suffer any degradation until with using MM.

In figure (5) note that the visited nodes of IKSD with 64-QAM is much larger than the visited nodes in 16-QAM, this is due to the different in constellation size between 16-QAM and 64-QAM, and also the visited nodes is directly proportional to increasing the size of the constellation.The 16-QAM and 64-QAM modulation schemes achieve different performing in the presence of noise. In particular, 64-QAM (higher order) modulation scheme is able to achieve higher data rates but it's not robust in the presence of noise. 16-QAM (Lower order) modulation scheme give fewer data rates but it is more robust in the presence of noise. So, figure (6) show us the variation in the performance of the IKSD algorithm between two types of modulation 16-QAM and 64-QAM, we can see that the performance of 16-QAM better than the performance of 64-QAM.

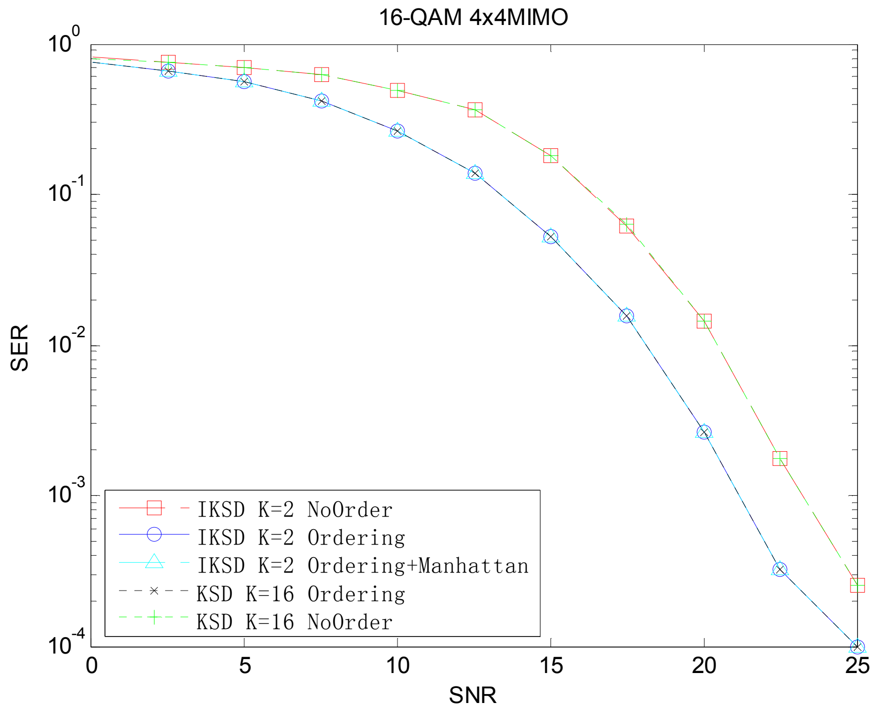

Figure 1. Performance of KSD and IKSD with No ordering, ordering, and combine ordering with MM for uncoded 4x4 MIMO 16-QAM system

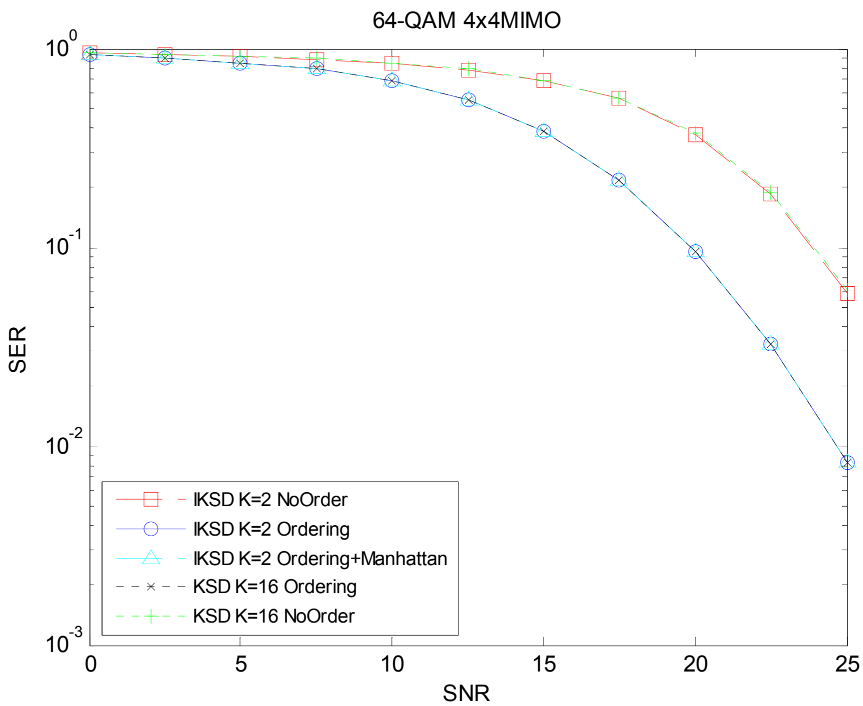

Figure 2. Performance of KSD and IKSD with No ordering, ordering, and combine ordering with MM for uncoded 4x4 MIMO 64-QAM system

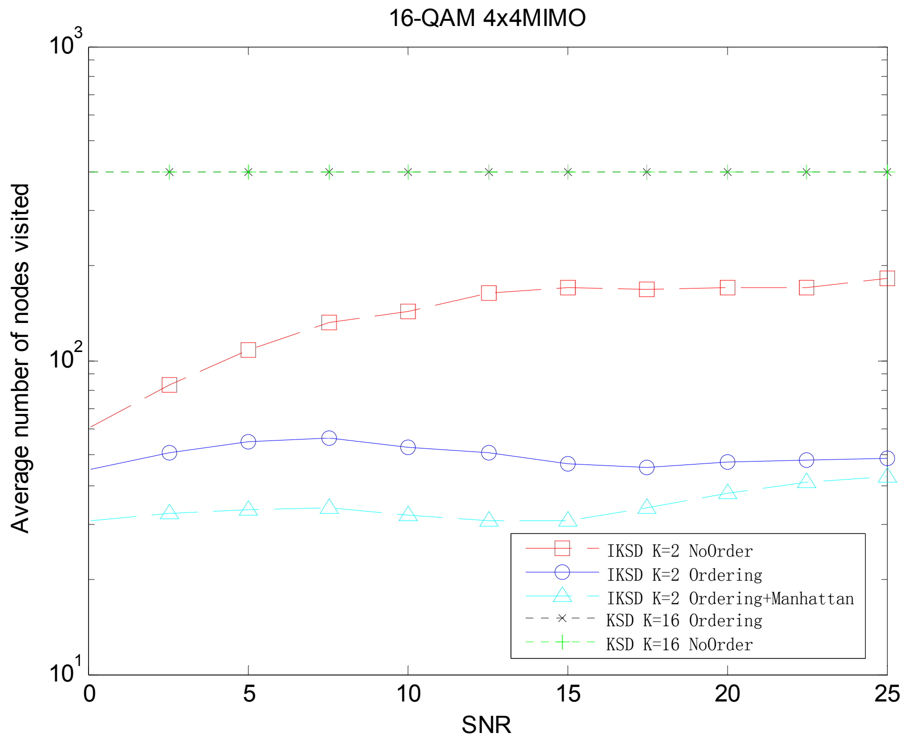

Figure 3. Complexity of KSD and IKSD with No ordering, ordering, and combine ordering with MM for uncoded 4x4 MIMO 16-QAM system

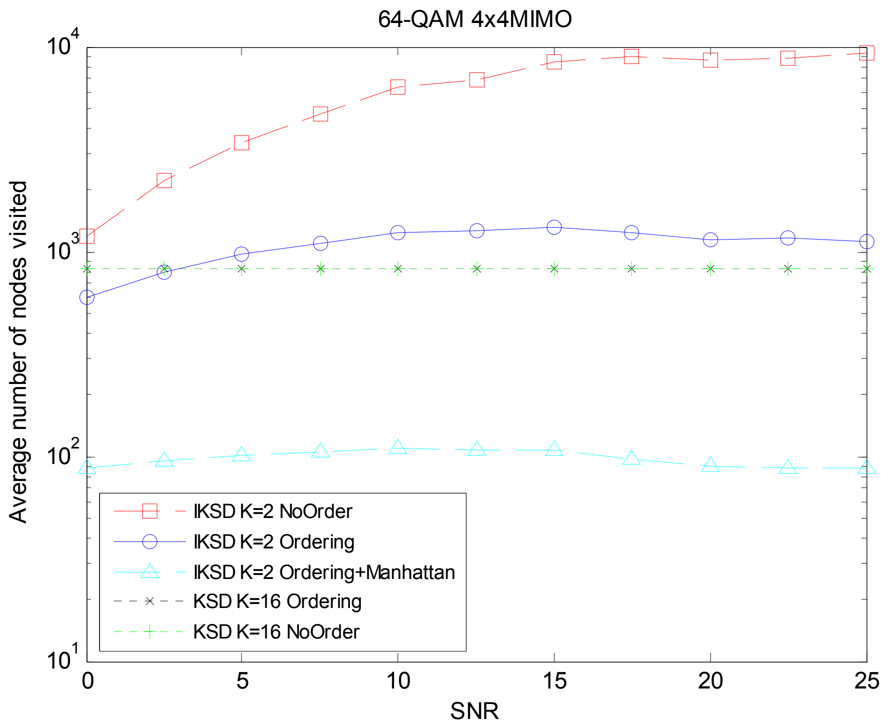

Figure 4. Complexity of KSD and IKSD with No ordering, ordering, and combine ordering with MM for uncoded 4x4 MIMO 64-QAM system

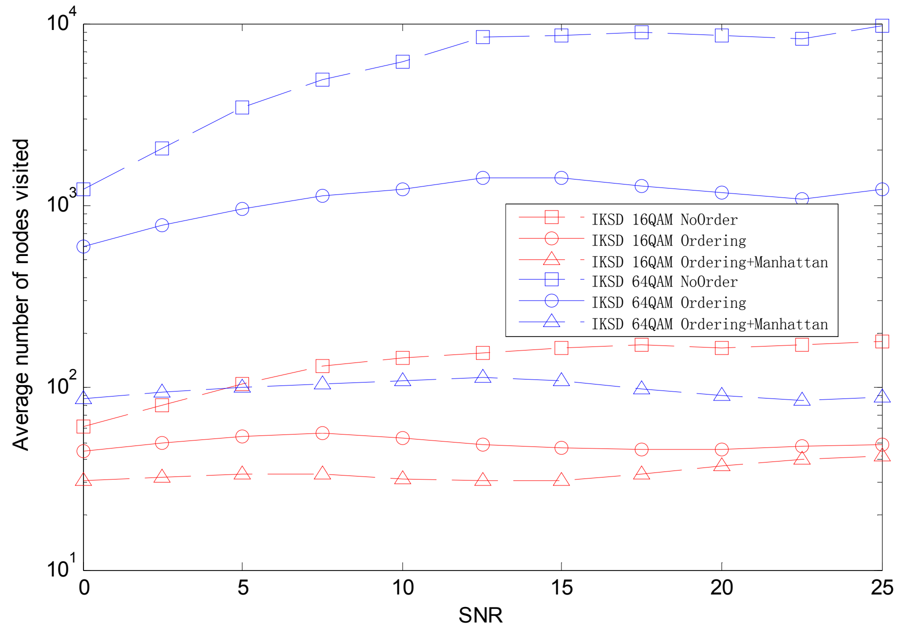

Figure 5. Compare the complexity of IKSD with No ordering, ordering, and combine ordering with MM for uncoded 4x4 MIMO 16-QAM and 64-QAM system

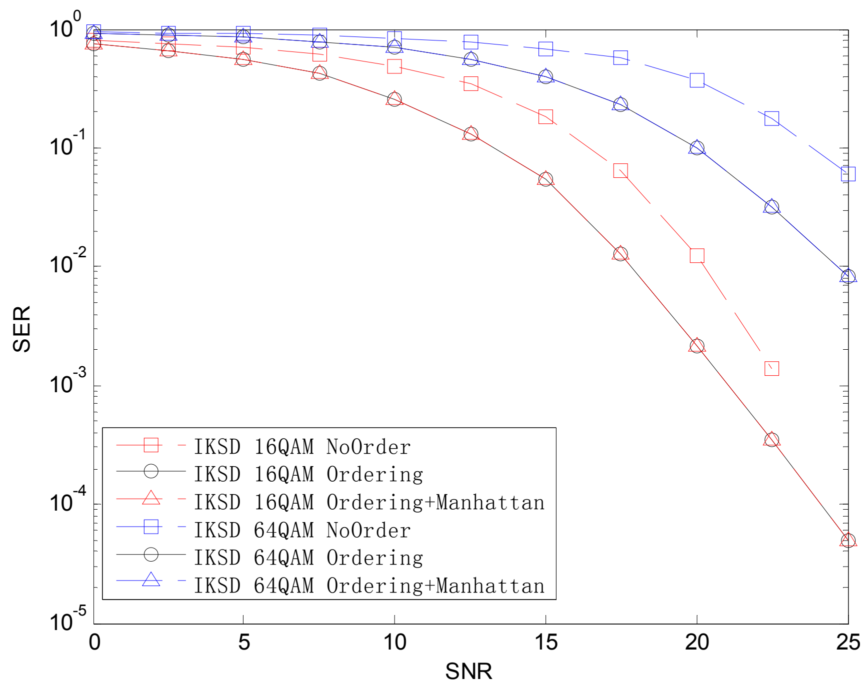

Figure 6. Compare the SER performance of IKSD with No ordering, ordering, and combine ordering with MM for uncoded 4x4 MIMO 16-QAM and 64-QAM system

5. Conclusions

In this paper, we propose an enhancing IKSD by adding the combining of column norm ordering and Manhattan metric to enhance the performance and reduce the computational complexity, in order to make this kind of algorithms more suitable in term of FPGA implementation.From the simulation results that appear in this work, we can see that the column norm ordering method is simple to implement but very effective in term of enhancing the performance and reduce the complexity. And we can also see the effect of combining the column norm ordering and Manhattan metric in term of enhancing the performance and more reducing of complexity than the column norm ordering method alone.

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