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Two Wheeled Robot Self Balancing Control Research

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Abstract

According to movement balancing and position control problem of Self Balancing Two Wheeled Robot, a method based on H∞ Robust Control was proposed. We apply it onto the MIMO nonlinear model of robot, and simulated it in the MATLAB environment The simulation results shows that the robot can be balanced in fixed position well by this method, and also it have the ability to anti interference.

Keywords: *self balancing, two wheeled robot, balancing contron, robust control, MIMO nonlinear system*

1. Introduction

Self Balancing Two Wheeled Robot is a kind of absolutely unstable under-actuated system with high-rank unstable multi-variable strongly coupling complicated dynamic nonlinear property. The two wheeled robots are common in the form of two coaxial driving wheels and their associated robot bodies. All of such robots are based on the design of an inverted pendulum [1], which are essentially different from the traditional multi wheel robots bringing such robots obvious advantages. Because of the characteristics of this type of robots, they have attracted much attention of researchers on designing, controlling and application of the two wheeled inverted pendulum robots. In 2002, the Swiss Federal Institute of Technology have developed a mobile robot called JOE [2]. It has proved that such robots have considerable performances with multi wheel robot under certain control systems. JOE can run smoothly on a flat surface and even on an inclined surface, it showed satisfactory operating characteristics. A suppression system has been designed and used in a low cost, two wheeled, self balancing robot for detecting and reporting fire and intruders in a small home/office setting [3]. SASAKI et al [4, 5] have proposed a personal riding type wheeled mobile platform, of which the two wheels are driven independently, and the robot is steered to move forward and backward by changing the relative position of the operator on the base. Recently, a robotic mobility platform (RMP) has been developed by Segway Company after Human Transporter, and this platform can be used in various applications [6]. Based on Segway's RMP, several robots have been built by researchers all over the world. SAWATZKY et al [7] have introduced a method to meet the mobility requirements of disable persons by using the Segway platform. BROWNING et al [8] have adopted the Segway RMP to build soccer robot, which is capable of playing soccer autonomously. Furthermore, they have presented a game called Segway Soccer in which a human plays soccer riding on Segway platform cooperating with Segway RMP robots [9]. The capability of this type robot for cooperation with human in complex tasks has been demonstrated [9].

Many similar systems, such as JOE, nBot, etc. [10, 11], have been studied by researchers in the literature at home and abroad. In an earlier work [12], wheeled inverse pendulum type mobile robot (WIPMR) was studied and a trajectory-tracking algorithm was found using a linear state-space model. F. Grasser applied Newton approach to derive the system's dynamic model [13], and the equations were linearized around an operating point to design a controller. These six state-space variables fully describe the dynamics of the 3-DOF system. We just use a six state-space model to control our robot. And we will use a kind of H_∞ robust control method to implement the movement balancing control and fixed position contro [14].

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2. Research Method

2.1. H∞ Robust Control

We consider the problem of an n-order generalized MIMO system, which is represented by following equation.

$$\begin{cases} \dot{\mathrm{x}} = \mathrm{Ax} + \mathrm{B}_1\omega + B_2 u \\ z = C_1 x + D_{11}\omega + D_{12} u \\ y = x \end{cases} \tag{1}$$

Where $\mathrm{x} \in R^{n\times 1}, \mathrm{z} \in R^{m\times 1}, \omega \in R^{r\times 1}, \mathrm{x} \in R^{p\times 1}, A, B_1, B_2, C_1, D_{11}, D_{12}$ are respectively constant matrix with proper dimension, is the modelling error and disturbance in addition.

To above equation, we propose γ>0, there exist matrix P_1 and P_2, in which, P_1 is symmetrical positive defined matrix. If the following inequation is satisfied:

$$\begin{bmatrix} \mathrm{AP}_1 + \mathrm{P}_1 A^T + B_2 P_2 + (B_2 P_2)^T + \gamma^{-2} B_1 B_1^T & (C_1 P_1 + D_{12} P_2)^T \\ C_1 P_1 + D_{12} P_2 & -I \end{bmatrix} < 0 \tag{2}$$

we can conclude that the states feedback controller is

$$\mathrm{u} = \mathrm{Kx} \tag{3}$$

Where $\mathrm{K} = \mathrm{P}_2 P_1^{-1}$

We have two control objective. The first is that x=0 is the locally asymptotically stable point of closed no disturbance system, i.e. to the initial state x(0), we have x(t)→0. The second objective is that $\forall \omega \in L_2[0,+\infty]$, the initial state x(0)=0, closed-loop system has the ability to restrain the disturbance, and the robust performance can be denoted as follow.

$$\int_0^\infty \mathrm{x}^{\mathrm{T}} Q x dt < \gamma^2 \int_0^\infty \omega^2(t) dt \tag{4}$$

Where $\mathrm{Q} = \begin{bmatrix} q_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & q_n \end{bmatrix}$ is a diagonal matrix, $\mathrm{x} = [x_1, x_2, \cdots, x_n]^T, \gamma > 0$.

2.2. Self Balancing Two Wheeled Robot

B-TWR System Description: Here we study the B-TMR (BALANCING TWO WHEELED ROBOT) with two control inputs, which are the torques of the two wheels. And the system degree of the freedom is three, which is more than the control inputs. So the system belongs to under-actuated system.

B-2WMR is a nonholonomic system, and here the rigid model is discussed, which has two coaxial driving wheels. And there is an internal body which can install some subsystems, such as controllers, sensors, etc. The internal body is said to be Intermediate Body (i.e. IB). The holistic physical framework is shown in Figure 1 (left). The states of the robot can be detected by gyroscope, inclinometer and coder shown in Figure 2.

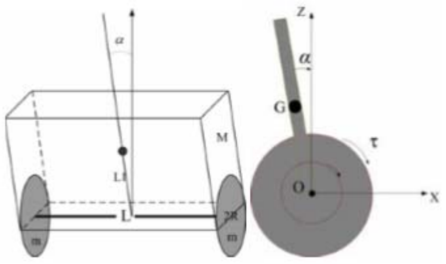

Figure 1. Left: system whole structure;
Right: side elevation of robot after simplification

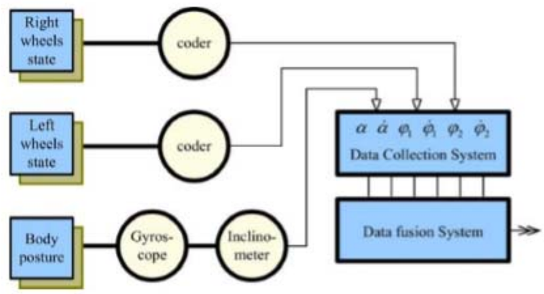

Figure 2. Robot states detecting

B-TWR Mathematical Model : System parameters are all shown in Table I.

Table 1. The Parameters of the Robot System

Parameter	Unit	Value
ml, mr: the mass of the left and the right wheel	kg	1
M: the mass of the Intermediate Body	kg	10
R: radius of wheel	m	0.15
L: the distance between the center point of two wheels	m	0.44
L1: the distance between the barycentre of IB and O	m	0.4
$J_l = J_r = J_w$: the moment of inertia of (left and right wheel) about its axis	$kg \cdot m^2$	1.125e-2
J_y: the moment of inertia of the IB about the y –axis	$kg \cdot m^2$	1.5417
J_z: the moment of inertia of the IB about the z –axis	$kg \cdot m^2$	0.5893
g: acceleration of gravity	m/s^2	9.8
µl，µr frictional coefficient of two wheel with ground	$N.\ s^2/rad^2$	10,10

The math model of two-wheeled self-balancing robot has been built, and the correctness has been validated. The dynamic model can be described as follow.

$$\mathrm{M}(q)\ddot{q} + D(q,\dot{q}) = E\tau + u_d \tag{5}$$

where $\mathrm{M}(q) \in \mathrm{R}^{n\times n}$ is the symmetrical matrix, $\tau \in \mathrm{R}^{r}$ is the input vector, $\mathrm{E} \in \mathrm{R}^{n\times r}$ is the transition matrix of the input vector, $\mathrm{F}^{T}(q) \in \mathrm{R}^{n\times m}$ is the matrix about constraints. $\omega = u_d \in \mathrm{R}^{r\times 1}$ is the disturbance put on the body of robot.

$$\mathrm{M}(q)=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \mathrm{D}(q,\dot{q})=\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix},$$

$$\mathrm{D}(q,\dot{q})=\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}, \quad E=\begin{bmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad u_d=\begin{bmatrix} \tau_d \\ 0 \\ 0 \end{bmatrix}, \quad \tau=\begin{bmatrix} \tau_l \\ \tau_r \end{bmatrix}$$

$$a_{11}=(ML_1^2+J_y)$$

$$a_{12}=a_{21}=a_{13}=a_{31}=\frac{1}{2}ML_1R\cos\alpha$$

$$a_{22}=a_{33}=(m_lR^2+J_l)+\frac{1}{4}MR^2+\frac{R^2}{L^2}(ML_1^2\sin^2\alpha+J_z)$$

$$a_{23}=a_{32}=\frac{1}{4}MR^2-\frac{R^2}{L^2}(ML_1^2\sin^2\alpha+J_z)$$

$$d_1=-MgL_1\sin\alpha-M\dot{\theta}^2L_1^2\sin\alpha\cos\alpha$$

$$d_2=2ML_1^2\frac{R^2}{L^2}(\dot{\phi}_l-\dot{\phi}_r)\sin\alpha\cos\alpha-\frac{1}{2}ML_1R\dot{\alpha}^2\sin\alpha+u_lRsign(\dot{\phi}_l)(\dot{\phi}_l)^2$$

$$d_3=-2ML_1^2\frac{R^2}{L^2}(\dot{\phi}_l-\dot{\phi}_r)\sin\alpha\cos\alpha-\frac{1}{2}ML_1R\dot{\alpha}^2\sin\alpha+u_rRsign(\dot{\phi}_r)(\dot{\phi}_r)^2$$

The model is different from others. The friction of two wheels is considered, and the frictional coefficient could be different.

It's assumed that the two wheels are restricted by the restriction of the pure rolling. Referring to Figure 1(a), L is the distance OO_w, where O is the point midway between the two wheel centres. R is the radius of the wheels. The coordinate of point O is (x,y) , α is the inclination angle of the Intermediate Body. Φ_r, ϕ_l are respectively the rotary angle of the right and left wheel.

q is the generalized coordinate of system, and can be defined as $q=(\alpha,\phi_l,\phi_r)^T$, τ_d is the disturbance put on the body of robot. τ_l and τ_r are respectively the torque provided by left and right motor. We choose the Maxon DC motor, and the torque is limited in the bound of ±5Nm. The kinematics model of robot can be described as follow.

$$\begin{cases} \dot{x}=\frac{1}{2}(\dot{\phi}_l+\dot{\phi}_r)R\cos\alpha \\ \dot{y}=\frac{1}{2}(\dot{\phi}_l+\dot{\phi}_r)R\sin\alpha \\ \dot{\theta}=\frac{R}{L}(\dot{\phi}_r-\dot{\phi}_l) \end{cases} \tag{6}$$

Where (x,y) denotes the position of robot in the Cartesian coordinate, and $\dot{\theta}=\frac{R}{L}(\dot{\phi}_r-\dot{\phi}_l)$ is the yaw angle velocity of robot.

Linearization of B-TWR mathematical model : The MIMO system model is linearized in the balancing position, that is to say, $\alpha\approx 0$;$\sin\alpha\approx\alpha$;$\cos\alpha\approx 1$; $\alpha=\dot{\alpha}=0$; $\dot{\theta}=0$, and we can get the linear model of robot as the form of equation (1).

$$\begin{cases} \dot{x}=Ax+B_1\omega+B_2u \\ y=x \end{cases} \tag{7}$$

Where $x=[\alpha,\dot{\alpha},\dot{\phi}_l,\dot{\phi}_r,\phi_l,\phi_r]^T, u=[\tau_l,\tau_r]^T, \omega=\tau_d$.

$$\mathbf{A}=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 20.5139 & 0 & 0 & 0 & 0 & 0 \\ -42.0798 & 0 & 0 & 0 & 0 & 0 \\ -42.0798 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{aligned} \mathbf{B}_1 &= \begin{bmatrix} 0 & 0.5233 & -1.0735 & -1.0735 & 0 & 0 \end{bmatrix}^T \\ \mathbf{B}_2 &= \begin{bmatrix} 0 & -1.5968 & 9.6228 & 3.7657 & 0 & 0 \\ 0 & -1.5968 & 3.7657 & 9.6228 & 0 & 0 \end{bmatrix}^T \end{aligned}$$

Considering equation (7), we design the control output as follow equation (8).

$$z = C_1 x + D_{11}\omega + D_{12}u \tag{8}$$

Where $\mathrm{C}_1 = \begin{bmatrix} \mathrm{Q}_{6\times 6} \\ 0_{2\times 6} \end{bmatrix}, \mathrm{D}_{11} = 0_{5\div 1}, D_{12} = \begin{bmatrix} 0_{6\times 2} & \begin{matrix} \rho_1 & 0 \\ 0 & \rho_2 \end{matrix} \end{bmatrix}^T, Q = diag(q_1, q_2, \cdots, q_6),$ $q_j > 0 (j = 1,2,\cdots,6)$ are the weighted coefficient of every state. We can get that $\mathrm{z} = [q_1\alpha, q_2\phi_l, q_3\phi_r, q_4\dot{\alpha}, q_5\dot{\phi}_l, q_6\dot{\phi}_r, \rho_1\tau_l, \rho_2\tau_r]^T$ and then

$$\| z \|_2^2 = \int_0^\infty z^T z dt \tag{9}$$

In order to guarantee the balancing of the robot, the equation (4) must be satisfied. We get two LMIs, the first is the inequation described in (2), and the other inequation is as follows.

$$\text{-P}_1 < 0 \tag{10}$$

3. Results and Analysis

We choose $\rho_1 = 1;, \rho_2, q_1 = 2, q_2 = 1_l, q_3 = 10, q_4 = 10, q_5 = 10, q_6 = 10, \gamma = 120$ and utilizing the LMI toolbox in MATLAB, we can get the feedback gain K.

$$\mathrm{P1} = \begin{matrix} 0.1276 & -0.4188 & 0.0243 & 0.0243 & 0.0000 & 0.0000 \\ -0.4188 & 1.3949 & -0.1693 & -0.1693 & 0.0004 & 0.0004 \\ 0.0243 & -0.1693 & 0.5667 & 0.2236 & -0.0098 & -0.0001 \\ 0.0243 & -0.1693 & 0.2236 & 0.5667 & -0.0001 & -0.0098 \\ 0.0000 & 0.0004 & -0.0098 & -0.0001 & 0.0059 & 0.0001 \\ 0.0000 & 0.0004 & -0.0001 & -0.0098 & 0.0001 & 0.0059 \end{matrix}$$

$$\mathrm{P2} = \begin{matrix} 0.0531 & 1.2147 & -8.6719 & -2.9541 & -0.0000 & -0.0000 \\ 0.0531 & 1.2147 & -2.9541 & -8.6719 & -0.0000 & -0.0000 \end{matrix}$$

$$\mathrm{K} = \begin{matrix} 781.2958 & 240.3425 & 11.5193 & 29.0085 & -1.2760 & 27.7715 \\ 781.2958 & 240.3425 & 29.0085 & 11.5193 & 27.7715 & -1.2760 \end{matrix}$$

x_r=[0;0;0;0;0;0;0], We simulate it in MATLAB7.1 shown in Figure 3.

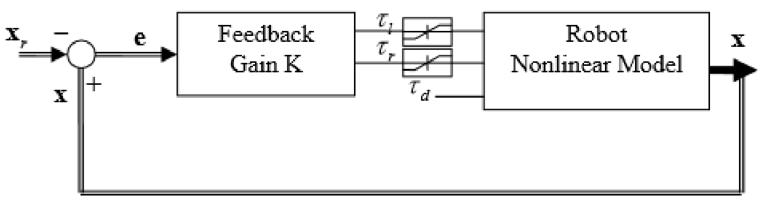

Figure 3. SIMULATION in MATLAB

In the initial state of x_0=[pi/20;0;0;0;0;0], the balancing results are shown in Figure 4 and Figure 5.

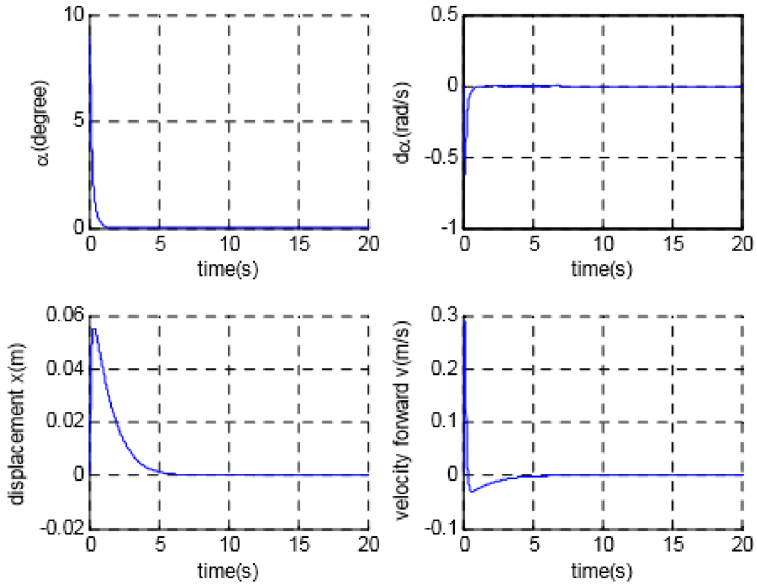

Figure 4. Dynamic response under H_∞ method

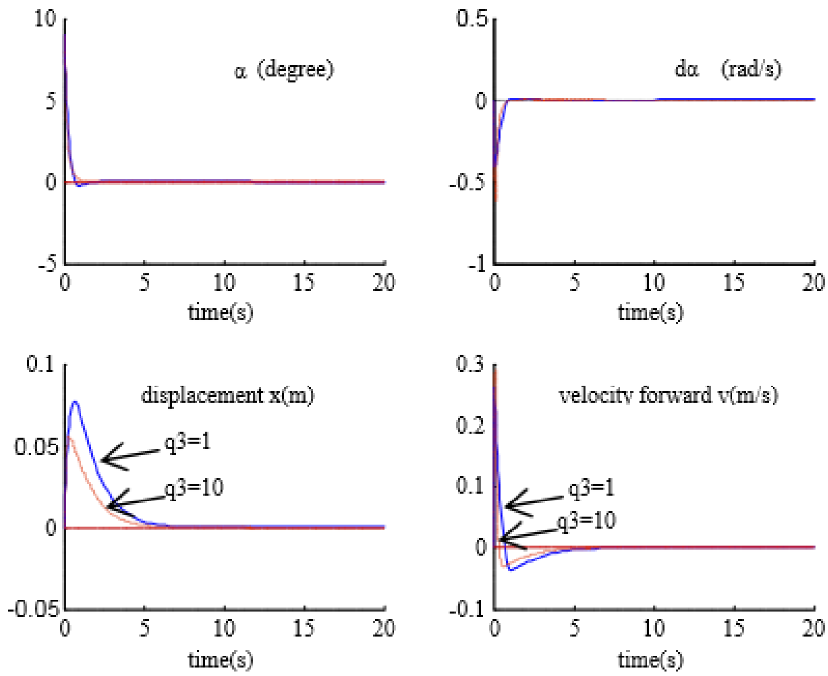

Figure 5. Dynamic response with different q value

We also can achieve the position control. The expected state is x_r=[0;0;0;0;1;1], and the initial state x_0=[pi/20;0;0;0;0;0]. Through the feedback control, the robot can stop in the position we expected. The result is shown in Figure 6.

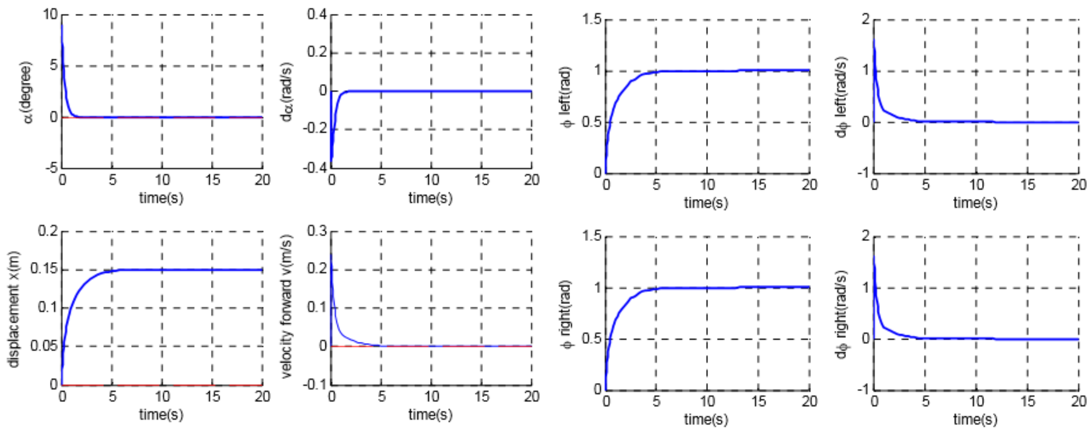

Figure 6. Dynamic response of position control

Let's look at the anti-jamming effectiveness of this control scheme. We give a force 5Nm lasting 0.05s onto the body of robot. The result is shown in Figure 7.

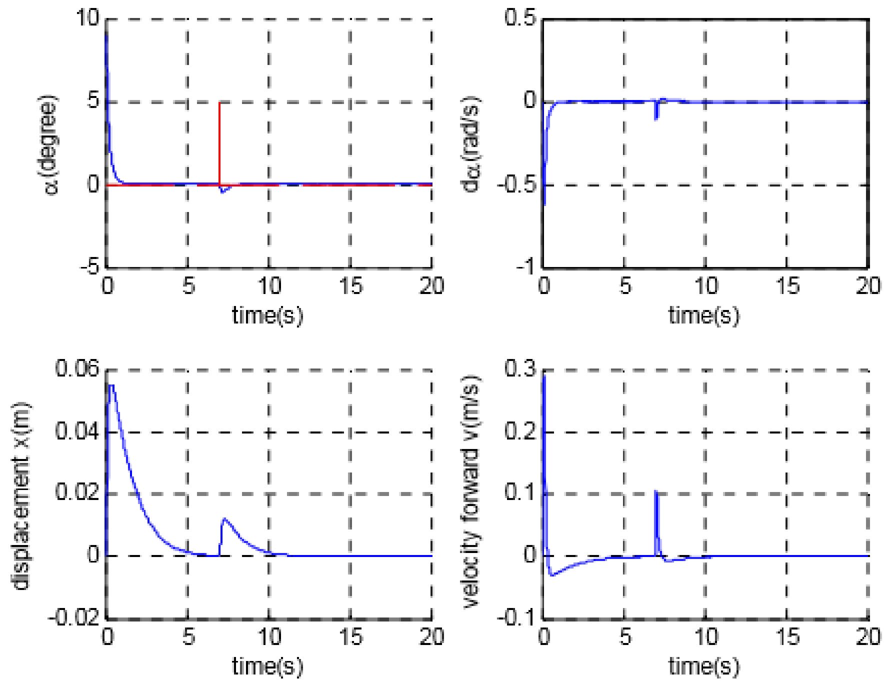

Figure 7. Dynamic response with disturbance

From the figure, we can see that the robot can walk back to zero point quickly and keep balancing.

4. Conclusion

A method based on H_∞ Robust Control was proposed to control a self-balancing two wheeled robot. From the simulation, we can see that the robot can be balanced in fixed position well by this method, and also it have the ability to anti interference. The next step we should do about robust control is the utilization of nonlinear robust control in the twowheeled robot. And we would find some adaptive or learning method which don't depend much on the parameter of robot and can balance the robot well.

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