

## Optimal Capacitors in Radial Distribution System for Loss Reduction and Voltage Enhancement

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### Abstract

The work presented in this paper is carried out with the objective of identifying the optimal location and size (Kvar ratings) of shunt capacitors to be placed in radial distribution system, to have overall economy considering the saving due to energy loss minimization. To achieve this objective, a two stage methodology is adopted in this paper. In the first stage, the base case load flow of uncompensated distribution system is carried out. On the basis of base case load flow solution, nominal voltage magnitudes and loss sensitivity factors are calculated and the weak buses are selected for capacitor placement. In the second stage, particle swarm optimization (PSO) algorithm is used to identify the size of the capacitors to be placed at the selected buses for minimizing the power loss. The developed algorithm is tested for 10-bus, 34-bus and 85-bus radial distribution systems. The results show that there has been an enhancement in voltage profile and reduction in power loss thus resulting in much annual saving.

**Keywords:** Capacitor placement, Loss sensitivity factors, Particle Swarm Optimization (PSO), Radial Distribution System (RDS)

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### 1. Introduction

Electric power industry is advancing rapidly and the environment is moving to a competitive power supply market. One of the most important concerns in today's life is to minimize the power loss and increase the overall efficiency of the system. Furthermore, the voltage profile of the system is to be kept within a prescribed limit. The electrical energy produced at the generating station is delivered to the consumer through a network of transmission and distribution system. Distribution system is one of the main three parts of a power system, responsible for transferring electrical energy to the end users. The analysis of a distribution system is an important area of activity, as distribution systems provide the vital link between the bulk power system and the consumers. A distribution circuit normally uses primary or main feeders and lateral distributors. Many distribution systems used in practice have a single circuit main feeder and are defined as radial distribution systems (RDS). Radial Distribution Systems are popular because of their simple design and generally low cost.

The distribution networks have a typical feature that the voltages at buses (nodes) reduce if moved away from substation. This decrease in voltage is mainly due to insufficient amount of reactive power. It is also well known that losses in a distribution system are significantly higher compared to that in a transmission system. Most of the loads are inductive in nature and require reactive power, if reactive power is fed to them locally than the line current can be reduced. Reduced current frees up capacity; the same circuit can serve more loads and also significantly lowers the  $I^2R$  line losses. Hence, in order to improve the voltage profile and to minimize the losses proper reactive power compensation is required. One such example of reactive power compensation in distribution system is shunt capacitors.

The shunt capacitors supply part of the reactive power demand, thereby reducing the current and power flow in lines. Installation of shunt capacitors on distribution network will help in reducing energy losses, peak demand losses and also helps in improving the system voltage profile, system stability and power factor. However, to achieve these objectives, keeping in mind the overall economy, an optimal size and location of capacitors need to be decided. The system benefits attained due to the application of shunt capacitors include:

- Reactive power support.
- Voltage profile improvements.
- Line loss reductions.
- Release of power system capacity.
- Savings due to decreased energy loss.

Due to the need of improving the overall efficiency of the system, the loss minimization in distribution systems is considered highly significant since about 13% of total power generated is wasted in the form of losses at the distribution level [1]. Since, the optimal capacitor placement is a complicated combinatorial optimization problem, many different optimization techniques and algorithms have been proposed in the past. Neagle and Samson [4] developed a capacitor placement approach for uniformly distributed lines and showed that the optimal capacitor location is the point on the circuit where the reactive power flow equals half of the capacitor var rating. From this, they developed the 2/3 rule for selecting and placing capacitors. For a uniformly distributed load, the optimal size capacitor is 2/3 of the var requirements of the circuit. The optimal placement of this capacitor is 2/3 of the distance from the substation to the end of the line. For this optimal placement for a uniformly distributed load, the substation source provides vars for the first 1/3 of the circuit, and the capacitor provides vars for the last 2/3 of the circuit. Grainger and Lee [5] provided another simple and optimal method for capacitor placement. This method is useful for circuits with any load profile, not just for uniformly distributed load profile. Here also the main principle is to place the capacitor at the point of circuit where the reactive power equals one half of capacitor rating. With this 1/2-kVAR rule, the capacitor supplies half of its VARs downstream and half are sent upstream.

Baran et al [6] proposed a mixed integer programming technique for capacitor placement problem, in which the problem is decomposed into two levels. The problem at the top level is called the master problem which is an integer programming problem and is used to place the capacitor (i.e. to determine the number and the location of the capacitors). The problem at the bottom level is called the slave problem and is used by the master problem to determine the types and the settings of capacitors placed. Here the cost of the capacitor is taken as a differentiable function of its size. Baran et al [7] also developed a solution algorithm for the capacitor based on a feasible solution approach. Also a new power flow equations and a solution method called 'Disc flow' is proposed.

Sundharajan and Pahwa [8] formulated a design methodology for determining the size, location, type and number of capacitors to be placed on radial distribution system. Sensitivity analysis is used to select the candidate locations for placing the capacitor in the distribution system. Ying-Tung Hsiao et al [10] considered three objective functions and a non-differential optimization problem for minimizing the total cost for energy loss. A combination of fuzzy and genetic algorithm was used to resolve the capacitor placement problem. The objective function was formulated in fuzzy sets to assess their imprecise nature. Das [11] presented the problem by using Fuzzy-GA method, in that sensitivity analysis has been used to identify the candidate buses for shunt capacitor placement. Only three load levels were considered and system voltage improvement analysis was not carried out.

Injeti et al [12] implemented two bio-inspired algorithms (Bat Algorithm and Cuckoo Search Algorithm) to solve optimal capacitor placement problem in two ways that is, Variable Locations Fixed Capacitor banks (VLFC) and Variable Locations Variable Sizing of Capacitors (VLVQ) for real power loss minimization and network savings maximization. Wu et al [13] proposed the dispatch of capacitors in distribution systems for daily operation, based on loop-analyzing methods. Here switching of capacitors for varying load is optimized.

Rao et al [14] developed a two stage methodology for capacitor placement in radial distribution systems. In part one, they calculated loss sensitivity factors to select the candidate locations for the capacitor placement and in part two they employed Plant growth Simulation Algorithm (PGSA) to estimate the optimal size of capacitors at the optimal buses determined in part one. Elsheikh et al [26] presented the problem as discrete optimization problem of fixed shunt capacitor placement and sizing and implemented Clustering Based Optimization (CBO) for minimizing the power loss and capacitor costs, considering over-compensation and voltage constraints.

In this paper an attempt is made to reduce the losses and to improve the system voltage profile by placement of capacitors at the candidate buses selected by Loss Sensitivity Factor and the sizing of optimal capacitor is done by Particle Swarm Optimization. The loss

sensitivity factor is a very important tool for prediction of buses where placing capacitor will produce best results i.e. maximum loss reduction. Therefore, these sensitive buses can serve as candidate locations for the capacitor placement. To estimate the required level of shunt capacitive compensation to minimize the losses and to improve the voltage profile of the system PSO is used. The capacitor placement problem in distribution system needs repeated load flow solutions. MATPOWER [17] version 3.2 package is used for Newton-Raphson (NR) load flow analysis in this paper. The proposed method is tested on 10-bus, 34-bus and 85-bus Radial Distribution Systems and results show the effectiveness of the proposed method.

The PSO method is becoming very popular because of its simplicity of implementation as well as ability to swiftly converge to a good solution. As compared with other optimization methods, it is faster, cheaper and more efficient. In addition, there are few parameters to adjust in PSO.

That's why PSO is an ideal optimization problem solver in optimization problems.

The remaining part of the paper is structured as follows: Section 2 gives the problem formulation; Section 3 describes sensitivity analysis and loss sensitivity factors to determine the optimal location of shunt capacitors and Sections 4 gives brief description of the particle swarm optimization and also the algorithm for capacitor placement using PSO. In section 5 results on the 10-bus, 34-bus and 85-bus Radial Distribution Systems radial distribution system are presented and finally the conclusion is given in Section 6.

## 2. Problem Formulation

Shunt capacitors placed in distribution system can provide reactive power and also reduces the voltage drop in the radial distribution system. Optimal capacitor placement is a complex optimization problem in which we try to "optimally" set the values of control variables i.e. reactive power output of shunt compensators (capacitors) to minimize the total active power losses while satisfying a given set of constraints.

### 2.1. Assumptions

There are many variables which are to be considered for capacitor placement problem including size of the capacitors, locations where capacitors are to be placed, cost of the capacitor. For simplicity only the fixed type capacitors are taken into consideration while following assumptions are made:

- The system is balanced
- All loads are time invariant

### 2.2. Objective Function

The objective function of the optimal capacitor placement is to minimize the total active power losses and thus minimizing the total annual cost due to energy loss while considering the cost of capacitor placement.

### 2.3. Constraints

While doing a capacitor placement problem there are a number of constraints which are to be taken into account

- Voltage at the buses must remain within the permissible limits before and after the capacitor placement.
- Reactive power compensation by capacitor placement at a bus is limited and is available in discrete sizes.

For practical use there exist a finite number of standard discrete size capacitors and there cost is not linearly proportional to size of the capacitor bank.

### 2.4. Mathematical Representation

Mathematically, the objective function of the problem is described as:

$$\min \sum_{k=1}^{N_B} P_{kloss} = \sum_{k=1}^{N_B} G_{ij} (V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij}) \quad (1)$$

Where

- $k$  The branch between bus  $i$  and  $j$   
 $N_B$  Total no. of branches  
 $\sum_{k=1}^{N_B} P_{kloss}$  Total power loss in radial distribution system  
 $G_{ij}$  Conductance of branch  $k$  (p.u.)  
 $V_i, V_j$  Magnitude (p.u.) of bus  $i$  and  $j$  respectively  
 $\theta_{ij}$  Load angle difference between bus  $i$  and  $j$  (rad)

With constraints:

$$V_i^{\min} \leq V_i \leq V_i^{\max} \quad (2)$$

$$Q_i^C \leq Q_{\max}^C \quad (3)$$

Where,

- $V_i$  The voltage magnitude of bus  $i$   
 $V_i^{\min}$  Minimum voltage limit  
 $V_i^{\max}$  Maximum voltage limit  
 $Q_i^C$  The reactive power compensation at bus  $i$   
 $Q_{\max}^C$  The maximum amount of reactive power compensation at any bus

### 3. Sensitivity Analysis and Loss Sensitivity Factors

Consider a distribution line with an impedance  $R + jX$  and a load of  $P_{eff} + jQ_{eff}$  connected between 'p' and 'q' buses as given below:

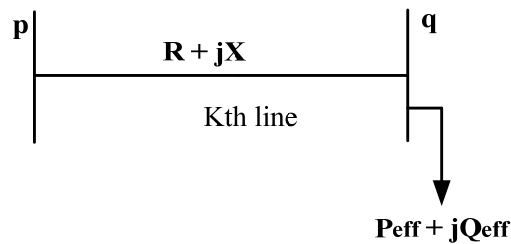


Figure 1. Electrical equivalent of one branch of RDS

Active power loss in the  $k^{th}$  line is given by,  $[I_k^2] * R[k]$  which can be expressed as,

$$P_{line\ loss}[q] = \frac{(P_{eff}^2[q] + Q_{eff}^2[q])R[k]}{(V[q])^2} \quad (4)$$

Similarly the reactive power loss in  $k^{th}$  line is given by

$$Q_{line\ loss}[q] = \frac{(P_{eff}^2[q] + Q_{eff}^2[q])X[k]}{(V[q])^2} \quad (5)$$

Where,

$P_{eff}[q]$  Total effective active power supplied beyond the node 'q'

$Q_{eff}[q]$  Total effective reactive power supplied beyond the node 'q'.

Now, both the Loss Sensitivity Factors can be obtained as shown below:

$$\frac{\partial P_{line\ loss}}{\partial Q_{eff}} = \frac{2 * Q_{eff}[q] * R[k]}{(V[q])^2} \quad (6)$$

$$\frac{\partial Q_{line\ loss}}{\partial Q_{eff}} = \frac{2 * Q_{eff}[q] * X[k]}{(V[q])^2} \quad (7)$$

For calculating the Loss Sensitivity Factors ( $\partial P_{line\ loss} / \partial Q_{eff}$ ) the base case load flows are considered and is calculated for all the lines of the given system, these values of ( $\partial P_{line\ loss} / \partial Q_{eff}$ ) are arranged in decreasing order.

The sequence in which buses are arranged decides the sequence in which the buses are to be considered for capacitor placement. The Loss Sensitivity Factors is solely responsible for the sequence in which buses are to be considered therefore it is very powerful and useful in capacitor placement. It is important to find the candidate buses where capacitors can be placed in order to reduce the power losses in radial distribution system. Since capacitors cannot be placed at a bus with healthy voltage, some method is required to find the weak buses in the radial distribution system. Normalized voltage magnitude is one such method to find the weak buses in the radial distribution network.

Normalized voltage magnitude,  $norm[i]$  for bus  $i$  can be calculated by considering the base case voltage magnitudes and is given by:

$$norm[i] = \frac{|V_i|}{0.95} \quad (8)$$

The buses with  $norm[i]$  value greater than 1.01 are healthy buses and are not considered for Capacitor Placement. The weak buses in the sequence ( $norm[i] < 1.01$ ) needs compensation and are considered for the capacitor placement.

#### 4. Particle Swarm Optimization

In this paper, Particle Swarm Optimization is used to identify the sizes of the capacitor for minimizing the cost of energy loss. The PSO algorithm motivated by social activities of individuals such as schooling of fishes and bird flocking was proposed by Eberhart and Kennedy in 1995 and since then, it has been successfully utilized in different practical optimization. Suppose a group of birds is randomly searching for food in an area. There is only one piece of food in the area being searched. All the birds do not know where the food is, but they know how far the food is in each iteration. So the best strategy to find the food is to follow the bird which is nearest to the food. The flocks simultaneously achieve their best condition through communication among members who already have a better condition. This happens repeatedly until the piece of food is discovered.

##### 4.1. Mathematical Model of PSO

In PSO, each single solution is a "bird" in the search space and it is called a 'particle'. Each particle has its fitness value and the velocity. The velocity directs the flying of the particle and influences its position. Initialization of PSO is done by group of random particles and then by means of updating their generation they find the optimal solution. In every iteration, each

particle update their fitness - *local best* (*pbest*) i.e. the best solution it has achieved so far and the best fitness - *global best* (*gbest*) achieved so far by any particle in the population.

The particle updates its velocity  $v_i(t)$  as:

$$v_i(t) = wv_i(t-1) + c_1r_1(pbest(t) - x_i(t-1)) + c_2r_2(gbest(t) - x_i(t-1)) \quad (9)$$

Where,  $c_1$  and  $c_2$  are acceleration coefficients and  $r_1$  and  $r_2$  are random vectors and  $w$  is adaptive inertia weight and is given by:

$$w = w_{\max} - \frac{(w_{\max} - w_{\min})}{itr_{\max}} iter \quad (10)$$

Where,  $w$  varies from  $w_{\max}$  to  $w_{\min}$ ,  $itr_{\max}$  is total number of iterations and  $itr$  is the number of current iteration.

Let  $x_i(t)$  denote the position of particle  $i$  in the search space at time step. The change in the particle's position is done by adding a velocity,  $v_i(t)$  to the current position:

$$x_i(t+1) = x_i(t) + \chi v_i(t+1) \quad (11)$$

Where,  $\chi$  is the constriction factor.

#### 4.2. Parameters of PSO

There are some parameters in PSO algorithm that may affect its performance. For any given optimization problem, some of these parameter's values and choices have large impact on the efficiency of the PSO method, and other parameters have small or no effect [19].

- Population Size: Population size or swarm size is the number of particles  $n$  in the swarm. A big swarm generates larger parts of the search space to be covered per iteration. A large number of particles may reduce the number of iterations need to obtain a good optimization result. In contrast, huge amounts of particles increase the computational complexity per iteration, and more time consuming. From a number of empirical studies, it has been shown that most of the PSO implementations use an interval of  $n \in [20,60]$  for the swarm size.
- Iteration Number: The number of iterations to obtain a good result is also problem-dependent. A too low number of iterations may stop the search process prematurely, while too large iterations has the consequence of unnecessary added computational complexity and more time needed [20].
- Acceleration Coefficients: The acceleration coefficients  $c_1$  and  $c_2$ , together with the random values  $r_1$  and  $r_2$ , maintain the stochastic influence of the cognitive and social components of the particle's velocity respectively. The constant expresses how much confidence a particle has in itself, while expresses how much confidence a particle has in its neighbours [20]. Normally,  $c_1$  and  $c_2$  are static, with their optimized values being found empirically. Wrong initialization of  $c_1$  and  $c_2$  may result in divergent or cyclic behaviour [20]. From the different empirical researches, it has been proposed that the two acceleration constants should be  $c_1 = c_2 = 2$ .
- Inertia weight: The inertia weight plays a very important role in the convergence behaviour of the PSO algorithm. The inertia weight is employed to control the impact of the previous history of velocities on the current one. Usually the best choice of the inertia weight is around 1.2, and as the algorithm progresses this value is gradually decreased to 0.
- Constriction factor: The constriction coefficient was developed by Clerc. This coefficient is extremely important to control the exploration and exploitation trade-off, to ensure convergence behaviour. The constriction coefficient guarantees convergence of the particles over time and also prevents collapse [21].

- **Maximum Velocity:** Maximum velocity  $v_{\max}$  controls the granularity of the search space by clamping velocities and creates a better balance between global exploration and local exploitation. If a particle's velocity goes beyond its specified maximum velocity, this velocity is set to the value  $v_{\max}$ . If the maximum velocity  $v_{\max}$  is too large, then the particles may move erratically and jump over the optimal solution. On the other hand, if  $v_{\max}$  is too small, the particle's movement is limited and the swarm may not explore sufficiently or the swarm may become trapped in a local optimum.

The flowchart for the basic PSO algorithm is shown in figure 2.

#### 4.3. Algorithm of Capacitor Placement Using PSO

The Computational steps involved in finding the optimal locations and sizes of the capacitors to minimize the losses in a radial distribution system are summarized in following:

1. Input load and line data of the test case and run the NR load flow program. Evaluate real and reactive power flows in lines and as well as losses.
2. Calculate Loss Sensitivity Factors and Normalized Voltage Magnitude. And find the candidate buses for capacitor placement in the radial distribution system.
3. Define variables (capacitors to be placed at the candidate buses) within their permissible range, define population size, no. of iteration and assume suitable values of PSO parameters.
4. Take iter=0
5. Randomly generate the population of particles and their velocities
6. For each particle run NR load flow to find out losses.
7. Calculate the fitness function of each particle using equ. (1)
8. Find out "personal best (pbest)" of all particles and "global best (gbest)" particle from their fitness.
9. Increment iteration count
10. Calculate the velocity of each particle using equation (9) and adjust it if its limit gets violated
11. Calculate the new position of each particle using equation (11)
12. For each particle run NR load flow to find out losses.
13. Calculate the fitness function of each particle using equation (1)
14. For each particle if current fitness(P) is better than pbest then pbest = p
15. Set best of pbest as gbest
16. Go to step no. 9, until maximum number of iterations is completed.
17. Coordinate of gbest particle gives optimized values of control variables and its fitness gives minimized value of losses.

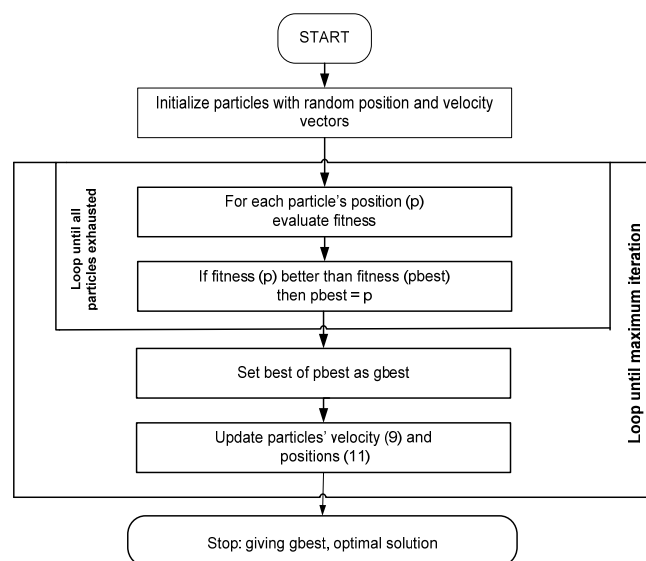


Figure 2. Flow chart depicting the PSO Algorithm

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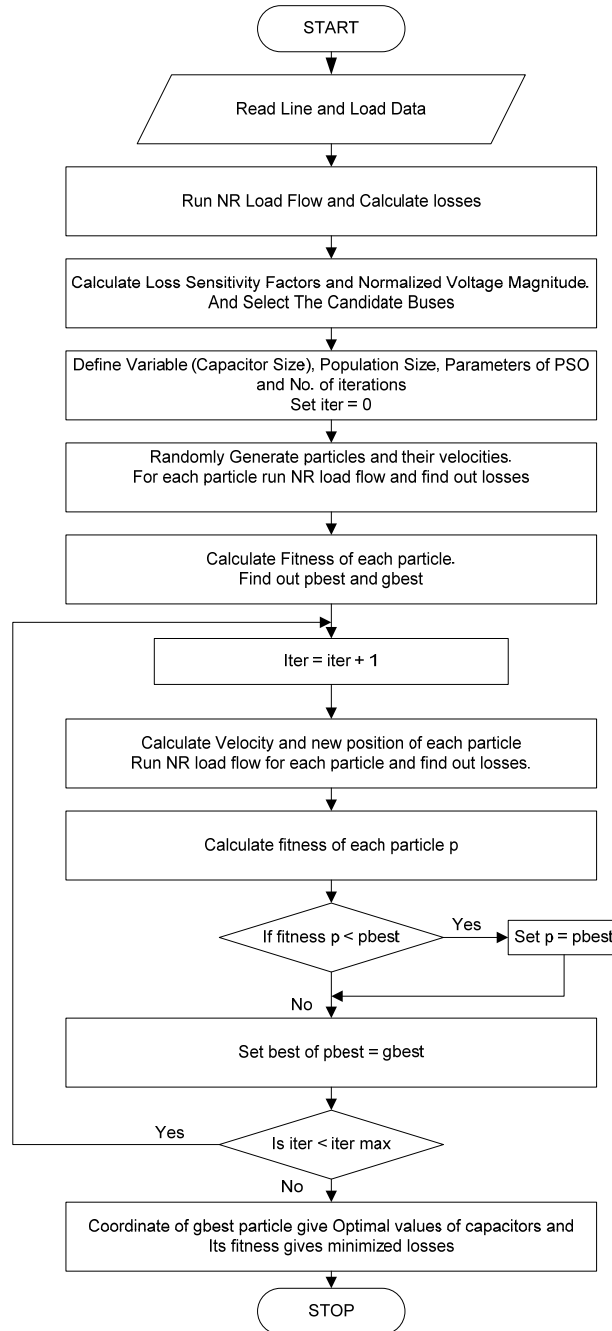


Figure 3. Flow Chart of Optimal Capacitor Placement through PSO

## 5. Test Results

The Algorithm given in Section IV has been programmed using MATLAB and run on an Intel Core i3, 2.20-GHz personal computer with 2.00 GB RAM and is tested on 10-bus, 34-bus and 85-bus radial distribution systems and the obtained results are explained in this section to demonstrate the effectiveness of this method. For calculation of the cost, the \$ rate has been considered in order to meet the international standards. The cost value can be converted into any currency values with the use of respective multiplication factor. In this dissertation, the equivalent ₹ for \$, multiplication factor has been assumed as ₹62 /\$.

Commercially available capacitors sizes with real costs/KVar are used in the analysis. Table I shows the example of such data. It was decided that the maximum permissible capacitor

size  $Q_c^{\max}$  placed at any bus should not exceed 1200 KVar. The equivalent annual cost per unit of power loss in \$/ (kW-year) is selected to be 168 \$/ (kW-year) [23]. The fixed cost of the capacitors is taken as \$1000 [9] and the data given in Table II is used to calculate the annual installation cost of the capacitor.

Table 1. Available Three Phase Capacitor Sizes and Costs

Size (KVAR)	150	300	450	600	900	1200
Cost (\$)	750	975	1140	1320	1650	2040

Table 2. Possible Sizes of Capacitors and Sizes In \$/KVAR

j	1	2	3	4	5	6	7
$Q_j^c$	150	300	450	600	750	900	1050
\$/KVAR	0.50	0.35	0.25	0.22	0.27	0.18	0.28
	0	0	3	0	6	3	8
j	8	9	10	11	12	13	14
$Q_j^c$	1200	1350	1500	1650	1800	1950	2100
\$/KVAR	0.17	0.20	0.20	0.19	0.18	0.21	0.17
	0	7	1	3	7	1	6
j	15	16	17	18	19	20	21
$Q_j^c$	2250	2400	2550	2700	2850	3000	3150
\$/KVAR	0.19	0.17	0.18	0.18	0.18	0.18	0.19
	7	0	9	7	3	0	5
j	22	23	24	25	26	27	--
$Q_j^c$	3300	3450	3600	3750	3900	4050	--
\$/KVAR	0.17	0.18	0.17	0.18	0.18	0.17	--
	4	8	0	3	2	9	--

The selected parameters of PSO are shown in Table 3.

Table 3. Selected Parameters of PSO

Population Size	50
Acceleration Constants ( $c_1, c_2$ )	2.1 and 2.0
Inertia weights ( $w_{max}, w_{min}$ )	1 and 0.2
Max. and min. velocity of particles	0.003 and -0.003
Constriction Factor	0.729

**A. 10-Bus RDS**

The test case I is a 10-bus, 9-line, single feeder radial distribution system [23] shown in figure 4 with the rated line voltage of 23 kV, active and reactive power load on the system are 12368KW and 4186 KVAR respectively. The line and load data of the 10-bus radial distribution system is given in Table IV.

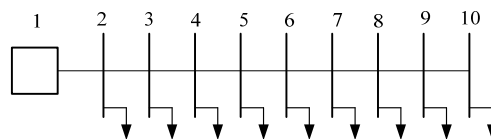


Figure 4. 10-Bus Radial Distribution System

Table 4. Load and Line Data for 10-Bus Radial Distribution System

Line No.	From Bus,i	To Bus,i+1	$R_{i,i+1}$ ( $\Omega$ )	$X_{i,i+1}$ ( $\Omega$ )	$P_L$ (KW)	$Q_L$ (KVAR)
1	1	2	0.1233	0.4127	1840	460
2	2	3	0.0140	0.6057	980	340
3	3	4	0.7463	1.2050	1790	446
4	4	5	0.6984	0.6084	1598	1840
5	5	6	1.9831	1.7276	1610	600
6	6	7	0.9053	0.7886	780	110
7	7	8	2.0552	1.1640	1150	60
8	8	9	4.7953	2.7160	980	130
9	9	10	5.3434	3.0264	1640	200

For the Test Case I (10-Bus RDS) NR load flow is applied using MATPOWER 3.2 and the voltages at each bus and the active power losses are calculated. The overall active power losses in the system are 783.643 KW and the lowest and highest voltage magnitudes calculated are 0.838 p.u at bus 10 and 0.993 p.u at bus 2 respectively.

The optimal capacitor locations on the radial distribution system are identified by Loss sensitivity factors and normalized voltage magnitudes. The initial configuration loss sensitivity factor and normalized voltage magnitude are calculated using base case load flow and the values are given in Table 5.

From Table 5 it is clear that normalized voltage at buses 5-10 is less than 1.01 therefore these buses are considered as weak buses and they are arranged in decreasing order of their loss sensitivity factor values. And the sequence is given by:

$$\text{Bus } 6 > 5 > 9 > 10 > 8 > 7$$

The first four buses of the sequence are considered for capacitor placement i.e. buses 6, 5, 9 and 10. The optimal size of the capacitors to be placed at these buses is calculated using PSO algorithm.

After capacitor placement, the optimal size of capacitors placed at selected buses i.e. 6, 5, 9 and 10 is calculated using PSO and are found to be 1175, 1190, 582 and 294 KVar respectively. The active power losses in the system are reduced to 696.38 KW and the lowest and highest voltage magnitudes calculated are 0.872 p.u at bus 10 and 0.995 p.u. at bus 2 respectively.

Table 4 summarizes the percentage of loss reduction from the initial configuration to the final configuration of the Test Case I. The optimized result achieved through this method has been compared with the exiting Fuzzy reasoning method proposed by Su and Tsai [24] (1996) and Discrete PSO method proposed by Elsheikh and Helmy [26] (2014).

Table 5. Loss Sensitivity factors and Normalized Voltage of 10-Bus RDS

Branch No.	Start Bus	End Bus	Loss Sensitivity Factor ( $\times 10^{-3}$ )	Normalized Voltage Magnitude ( $ V_i /0.95$ )
1	1	2	1.98	1.0451
2	2	3	0.20	1.0393
3	3	4	10.29	1.0141
4	4	5	8.64	0.9979
5	5	6	9.80	0.9654
6	6	7	2.08	0.9549
7	7	8	3.83	0.9357
8	8	9	8.11	0.9039
9	9	10	5.76	0.8816

From the initial and final configuration load flow results it is also clear that there is an enhancement in voltage profile of the system. The minimum voltage recorded at bus 10 is

enhanced from 0.838 p.u. to 0.872 p.u. The comparison of the voltage profile before and after capacitor placement is shown in figure 5.

The convergence characteristic of power loss by this method on 10-Bus RDS is shown in figure 6.

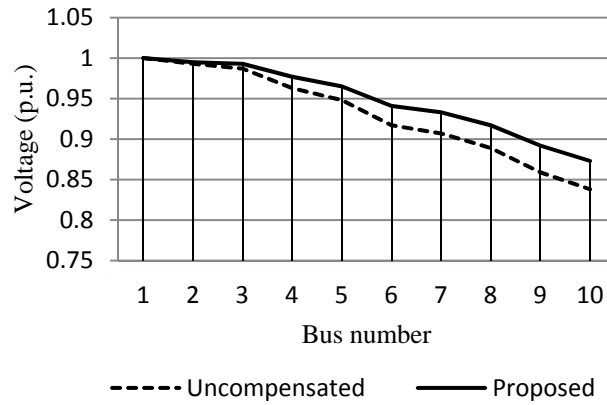


Figure 5. Voltage Profile of 10-Bus RDS before and after Compensation

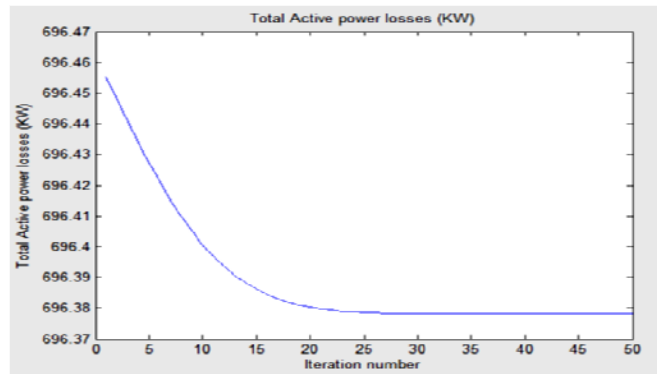


Figure 6. Convergence Curve

Table 6. Results of 10-Bus Radial Distribution System

Items	Un-compensated	Fuzzy Reasoning [24]	Discrete PSO [26]	PSO (Proposed)	Method
Total losses (KW)	783.64	704.883	701.2	696.38	
Loss reduction (%)	---	10.05	10.52	11.13	
Optimal Locations and size of capacitors in KVar	---	4 1050 5 1050 6 1950 10 900	4 1050 5 1500 6 150 7 900 8 600 9 300	6 1175 5 1190 9 582 10 294	
Total KVar		4950	6000	3241	
Annual Cost (\$/year)	131,652	119,420	119,352	118,556	
Annual Cost (₹/year)	81,62,424	74,04,040	73,99,824	73,50,472	
Net Savings (\$/year)	-	12,232	12,325	13,096	
Net Savings (₹/year)	-	7,58,384	7,64,150	8,11,952	
% Saving	-	9.29	9.34	9.95	

### B. 34 – Bus Radial Distribution System

The Test Case II is a 34-bus, 33-line radial distribution system. The single line diagram is shown in fig. 7 [25] with the rated line voltage of 11 kV, active and reactive power load on the system are 4636.50 KW and 2873.50 KVar respectively.

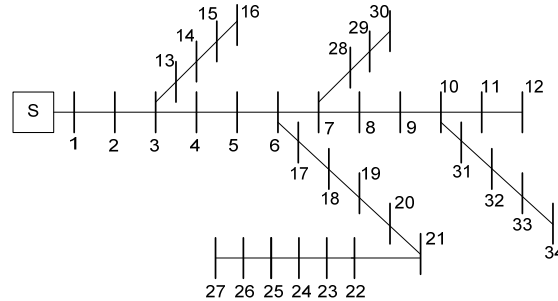


Figure 7. 34-Bus Radial Distribution System

For the Test Case II (34-Bus RDS) NR load flow is applied using MATPOWER 3.2 and the voltages at each bus and the active power losses are calculated. The overall active power losses in the system are 4636.50 KW and the lowest and highest voltage magnitudes calculated are 0.942 p.u at bus 27 and 0.994 p.u. at bus 2 respectively.

The optimal capacitor locations on the radial distribution system are identified by Loss sensitivity factors and normalized voltage magnitudes. The initial configuration loss sensitivity factor and normalized voltage magnitude are calculated using base case load flow.

For this test system buses 19, 22 and 20 are selected as optimal locations for the capacitor placement.

The optimal size of capacitors placed at selected buses i.e. 9, 22 and 20 is calculated using PSO and are found to be 1022, 845 and 304 KVar respectively.

After capacitor placement, the active power losses in the system are reduced to 168.867 KW and the lowest and highest voltage magnitudes recorded are 0.949 p.u at bus 27 and 0.995 p.u. at bus 2 respectively.

Table 7 summarizes the percentage of loss reduction from the initial configuration to the final configuration of the Test Case II. The optimized result achieved through this method has been compared with the exiting Fuzzy reasoning method proposed by Su and Tsai [24] (1996).

From the initial and final configuration load flow results it is also clear that there is an enhancement in voltage profile of the system. The minimum voltage recorded at bus 27 is enhanced from 0.942 p.u. to 0.949 p.u.

Table 7. Results of 34-Bus Radial Distribution System

Items	Un-compensated	Fuzzy Reasoning [24]	PSO Method (Proposed)
Total losses (KW)	221.747	168.47	168.867
Loss reduction (%)	---	24.02	23.85
Optimal Locations and size of capacitors in KVar	---	26 1400 11 750 17 300 4 250	19 1022 22 845 20 304
Total KVar		2700	2171
Annual (\$/year)	37,254	33,128	29,797
Cost (₹/year)	23,09,748	20,53,936	18,47,414
Net (\$/year)	-	4,126	7,457
Savings (₹/year)	-	2,55,812	4,62,334
% Saving	-	11.07	20.01

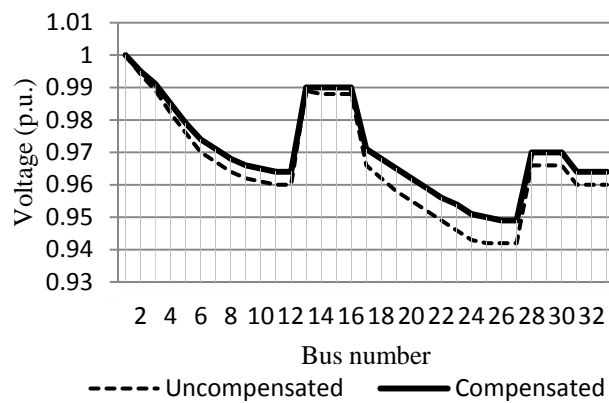


Figure 8. Voltage Profile of 34-Bus RDS Before and After Compensation

The comparison of the voltage profile before and after capacitor placement is shown in figure 8.

**C. 85 – Bus Radial Distribution System**

The Test Case III is an 85-bus, 84-line radial distribution system. The single line diagram is shown in figure 9 [16] with the rated line voltage of 11 kV, active and reactive power load on the system are 2514.28 KW and 2565.04 KVar respectively.

For the Test Case III (85-Bus RDS) NR load flow is applied using MATPOWER 3.2 and the voltages at each bus and the active power losses are calculated. The overall active power losses in the system are 300.872 KW and the lowest and highest voltage magnitudes calculated are 0.874 p.u at bus 54 and 0.996 p.u. at bus 2 respectively.

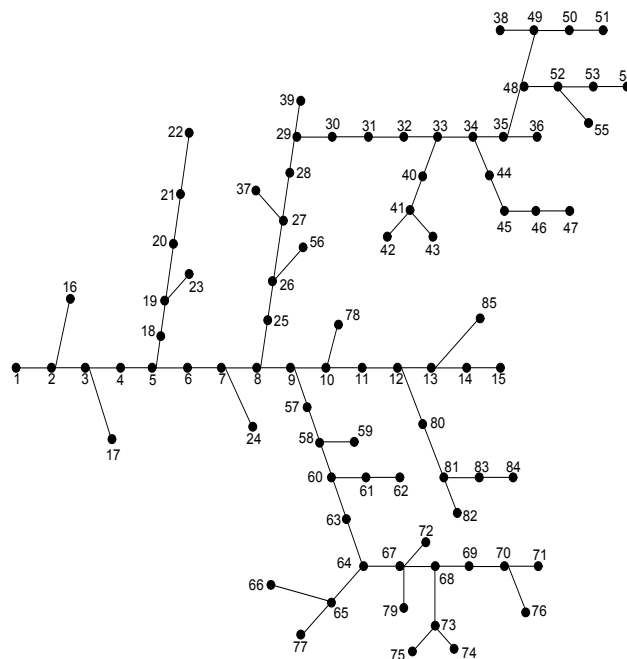


Figure 9. 85-Bus Radial Distribution System

For this test system buses 8, 58, 7 and 27 are selected as optimal locations for the capacitor placement.

Table 8. Results of 85-Bus Radial Distribution System

Items	Un-compensated	PGSA [14]	PSO Method (Proposed)
Total losses (KW)	300.872	161.40	161.37
Loss reduction (%)	---	46.35	46.37
Optimal Locations and size of capacitors in KVar	---	8 1200	8 124
		58 908	58 895
		7 200	7 113
Total KVar	-	2308	2103
Annual (\$/year)	50,547	28,508	28,480
Cost (₹/year)	31,33,914	17,67,496	17,65,760
Net (\$/year)	-	22,039	22,067
Savings (₹/year)	-	13,66,418	13,68,154
% Saving	-	43.60	43.66

The optimal size of capacitors placed at selected buses i.e. 8, 58, 7 and 27 is calculated using PSO and are found to be 124, 895, 113 and 971 KVar respectively. The active power losses in the system are reduced to 161.37 KW and the lowest and highest voltage magnitudes calculated are 0.91 p.u at bus 54 and 0.997 p.u. at bus 2 respectively.

Table 8 summarizes the percentage of loss reduction from the initial configuration to the final configuration of the Test Case I. The optimized result achieved through this method has been compared with the exiting PGSA method proposed by Rao et al [14] (2011).

From the initial and final configuration load flow results it is also clear that there is an enhancement in voltage profile of the system.

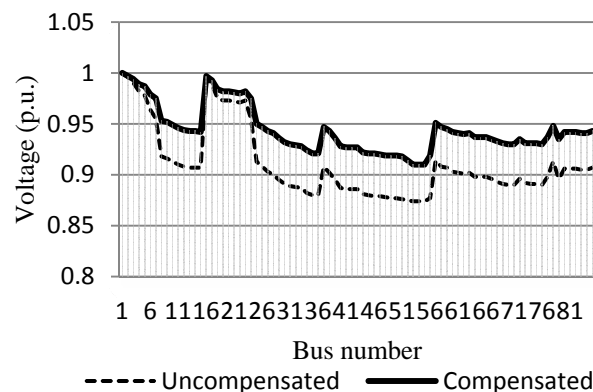


Figure 10. Voltage Profile of 34-Bus RDS Before and After Compensation

The minimum voltage recorded at bus 54 is enhanced from 0.874 p.u. to 0.91 p.u. The comparison of the voltage profile before and after capacitor placement is shown in figure 10.

## 6. Conclusion

The work has been carried out to find the optimal locations and sizes (KVar) of capacitors to be placed in radial distribution system to maximize the saving after considering the energy loss cost, capacitor installation and capacitor cost. The above problem has been solved by two step methodologies, the candidate locations for compensation are found using Loss Sensitivity Factors, calculated from the base case load flow. The sizing has been attempted using PSO algorithm. The coding scheme is developed identifying the optimal size of capacitor.

The study has been carried out on 10-bus, 34-bus and 85-bus Radial Distribution System. The results obtained from this method on various radial distribution systems have been compared with existing methods. From the results, the active power loss of the radial

distribution system has been reduced without compromising the constraints and also the voltage profile of the radial distribution system has been enhanced.

The main advantages of this method are that the selection of optimal locations by loss sensitivity factor reduce the search space of the optimization problem and thus evades heavy numerical computing and saves time. It also promises global optimum and recommends a solution for the control parameters.

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