Boosting carbon removal efficiency in wastewater treatment systems using a fuzzy model predictive control stategy

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ABSTRACT

The efficient removal of carbon pollution has always presented a growing challenge facing wastewater treatment plants (WWTPs) operating with activated sludge process (ASP) technology. Enhancing pollution removal efficiency to meet standard wastewater quality limits remains a problematic in water pollution management. Recent progress in modeling and automatic control techniques can significantly improve the hydric pollution removal. In this paper, an effective carbon elimination strategy combining Takagi-Sugeno (TS) fuzzy modeling and model predictive control (MPC) is proposed to achieve high purification performance in terms of chemical oxygen demand (COD), biochemical oxygen demand (BOD5) and total suspended solids (TSS) indicators. A fuzzy TS model is established based on the concepts of quasi-linear parameter-varying (LPV) forms and convex polytopic transformations of the system nonlinearities. The concentrations of heterotrophic biomass, biodegradable substrate and dissolved oxygen as well as the effluent volume are controlled and maintained around their desired references with the aim of increasing pollution removal. Comparisons with the previously most used state-of-the-art parallel distributed compensation (PDC) are performed. High and competitive pollution removal percentages of 91% for COD and BOD5 indicators, and 92% for TSS metric, are achieved with the proposed MPC-based design, thus complying with the normative limits defined in WWTPs.

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1. INTRODUCTION

Hydric pollution is increasingly causing serious problems for human health, the ecosytem and the environment. Threats ralated to wastewaters containing harmful components, such as carbon and nitrogen among others, occur particularly in urban tributaries of industrial areas and rivers [1]–[4]. In wastewater treatment plants (WWTPs), the biological sanitation method acting with activated sludge processes (ASPs) remains the most commonly adopted solution to addess these environmental pollution problems [5]–[8]. Bacterial biomass suspensions are in charge of eliminating harmful and contaminating organisms. A typical architecture is used with anoxic and aerated bassins for chemical reactions and sludge growth, decanters for effluent purification and pipes for recycling microrganisms [9]–[11]. The main objectives in WWTPs aim to maintain an effluent quality complies with local regulations against constantly changes in influent composition and flow. Performance in terms of chemical oxygen demand (COD), biochemical oxygen demand (BOD5), total suspended solids (TSS), and total nitrogen (TN) indicators are commonly targeted in a typical wastewater pollution removal context [10], [11].

The nonlinearity of interconnected sub-processes, dependence on operating conditions and multiple coupling between pollution variables considerably increase the complexity of WWTPs. As an immediate consequence, the accurate modeling and robust control of pollution removal dynamics remains an ambitious task that requires advance and effective theories to comprehensively guarantee the desired sanitation performance. A review of the related literature shows the multitude of proposed control strategies and, more specifically, the strong dependence of pollution removal efficiencies on the used models describing WWTPs variables. The more descriptive the dynamic model, the more competitive the performance of pollution removal. On the other hand, the most of related studies consider reduced models with only the dynamics of dissolved oxygen while neglecting other influential variables, in this case the wastewater volume and the concentrations of biomass and substrate. Indeed, considering the changes in influent volume and the sensitivity of biochemical reactions among others, the use of a multivariable model of WWTPs becomes a necessity to effectively overcome the challenges of eliminating harmful susbstances. Up to now, there are no notable contributions in the related literature that consider a simultaneous manipulation of the entire variables of WWTPs to further boost the output performance indicators. Efforts in modeling and control must be carried out continuously to further improve the WWTPs efficiencies.

The relevant literature on the topic of polluants removal in WWTPs is constantly evolving. The related control techniques vary mainly from each other depending on the effluent treatment objectives and the type of toxic substances to be removed. In [12]–[15], a survey on various modeling and control strategies of WWTPs is addressed. A general framework for modeling methods and benchmarking of control techniques is proposed in terms of models selection, control parameters, control scheme, etc. In [16], parallel distributed compensation schemes (PDC and OPDC) are performed for an APS using the formalism of multimodeling and convex optimization under linear matrix inequalities constraints. In [17], a strategy for controlling the dissolved oxygen concentration in WWTPs is investigated. A scheme with an RLS identification and event-triggered sliding mode control is proposed to deal with the hydraulic retention delay problem that hinders the accurate control of such a concentration. In [18], a model-free deep reinforcement learning-based control strategy is performed to deal with the modeling complexity and the trade-off between operating costs an environmental conditions in WWTPs. In [19], a pre-compensation quantitative-based control approach is investigated to deal with the regulation of BOD5 and NH4+ concentrations. In [20], various classical and advanced control strategies addressing the dissolved oxygen dynamics, as one of the most important water quality factors, are reviewed and discusssed. In [21], the authors proposed a multiobjective technique to ensure performance in terms of energy consumption and effluent quality. In [22], [23], the authors studied the contribution of the main soft computing tools for the control and prediction of WWTPs. In [24], the variables of dissolved oxygen and substrate are regulated with fuzzy modeling and $H\infty$ observer based approach to meet performance in terms of tracking accuracy. In [25], a performance index measuring the ratio between the amount of removed nitrogenated compounds and energy consumption is retained to design an event-based cascaded PI controller for dissolved oxygen variables. In addition to the aforementioned related works, strategies using the model predictive control (MPC) framework are recently investigated and tried for different treatment architectures [26]–[28]. Findings are interesting but remain highly dependent on the differential equations used to describe the wastewater treatment process.

In this paper, a systematic procedure for modeling and effective control of all intervening dynamics in the carbon removal process of WWTPs, notably effluent volume (EV), heterotrophic biomass (HB), biodegradable substrate (BS) and dissolved oxygen (DO) concentrations, is proposed and successfuly applied in a numerical simulation framework. Such a wastewater pollution removal procedure combines the theories of Takagi-Sugeno fuzzy modeling and MPC design to meet standard regulatory performance in terms of the pollution indicators, i.e. COD, BOD5 and TSS. Techniques of LPV representation and convex polytopic transformation are incorporated and the variables of pollution removal process are controlled around corresponding set-point inputs. Comparisons with the widely used PDC design method, related to the TS fuzzy modeling formalism, are carried out and interpreted. Critical discussions are provided to highlight the superiority of the proposed approach and emphasize the implication of research findings.

The main contributions of this work are summerized as follows : (1) A comprehensive framework for modeling and effective control of WWTPs is proposed for carbon pollutants removal. (2) An equivalent TS fuzzy model, ensuring the transformation of nonlinear dynamics into a more easily handled linear-time-variant (LTI) form, is established and validated. Such a modeling procedure can be adopted as a systematic methodology while considering other types of pollution WWTPs variables. (3) An effective MPC strategy is designed based on such a developed TS fuzzy representation to deal with the complexity of the initial model governed by nonlinear and coupled differential equations. (4) A significant improvement in overall carbon removal efficiency is guaranteed in terms of COD, BOD5 and TSS indicators compared to previous works in the literature, especially with the PDC-based technique.

The rest of the work is arranged in the following manner. In Section 2, the proposed carbon removal method for WWTPs is presented. A detailed and systematic step-by-step description of the design procedure, including all information and materials required to reproduce the study in other applications, is highlighted. Starting from nonlinear differential equations of WWTPs, an equivalent TS fuzzy model is first established based on an LPV state-space representation and a convex polytopic transformation of premise variables. Then, a fuzzy MPC technique is elaborated on the basis of such an established model. In Section 3, numerical simulations are carried out and critical discussions are provided to highlight the effectiveness and superiority of the suggested TS fuzzy MPC-based carbon removal method in comparison with the competing PDC-based one. Section 4 ends the research paper by resuming the main findings of the work and highlights potential future directions and orientations.

2. METHOD

2.1. TS fuzzy modeling of the carbon removal process

Figure 1 depictes a typical layout of a WWTP equipped with aerobic and anaerobic bioreactor tanks, a clarifier, and a piping circuit for the recycling process [10]–[12]. After a pretreatment phase, the influent is mixed with the oxygen inside the bioreactor to favoriate the agglomeration and growth of microorganisms. The mixture is evacuated to the decanter where a separation of effluent and sludge is achieved by gravity. A portion of the settled activated sludge is returned back to the aeration tank to maintain a balanced population of microorganisms.

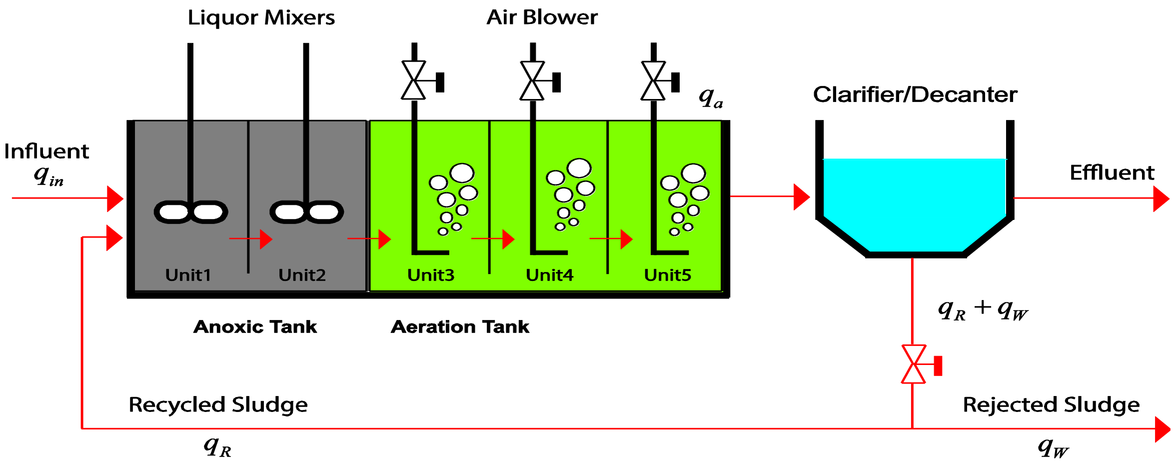

Figure 1. Layout of a WWTP with activated sludge treatment procedure

Focusing only on eliminating carbon pollution, the following nonlinear sub-model extracted from the well-known benchmark ASM1 (activated sludge model no. 1) is considered [12]:

$$\dot{V} = q_{in} + q_R - q_{out} = \kappa_V\left(V_{ref} - V\right) \tag{1}$$

$$\dot{X}_{BH} = \frac{q_{in}}{V} X_{BH,in} - \frac{q_{in}}{V}\frac{f_W(1+f_R)}{(f_R+f_W)} X_{BH} + \mu_H \frac{S_S}{\kappa_S+S_S}\frac{S_O}{\kappa_{OH}+S_O} X_{BH} - b_H X_{BH} \tag{2}$$

$$\dot{S}_S = \frac{q_{in}}{V} S_{S,in} - \frac{q_{in}}{V} S_S - \frac{\mu_H}{Y_H}\frac{S_S}{\kappa_S+S_S}\frac{S_O}{\kappa_{OH}+S_O} X_{BH} + (1-f) b_H X_{BH} \tag{3}$$

$$\dot{S}_O = -\frac{q_{in}}{V} S_O - \frac{1-Y_H}{Y_H}\mu_H \frac{S_S}{\kappa_S+S_S}\frac{S_O}{\kappa_{OH}+S_O} X_{BH} + \kappa_O q_a\left(S_{O,sat} - S_O\right) \tag{4}$$

where κ_V and V_{ref} are a control gain and a reference for the volume dynamics, respectively, f_R and f_W are the recycling and extraction coefficients, $q_R = f_R q_{in}$, $q_W = f_W q_{in}$, μ_H is the highest biomass growing percentage, κ_{OH} denotes the oxygen saturation rate, b_H is the biomass mortality rate, f denotes the biomass fraction, κ_O is the oxygen regulation gain, $S_{O,sat}$ is the oxygen saturation concentration.

Considering the nonlinear model in (1)-(4), a corresponding quasi-LPV state-space representation of the carbon removal process can be obtained as follows [29]:

$$\dot{X}(t) = A\big(z(X,u)\big)X(t) + B\big(z(X,u)\big)u(t); y(t) = C\big(z(X,u)\big)X(t) \tag{5}$$

where $z(X,u)$ is a parameters vector presenting the premise variables, $X = (V, X_{BH}, S_S, S_O) \in \mathbb{R}^4$ and $u = \big(X_{BH,in}, S_{S,in}, q_a, q_{in}\big) \in \mathbb{R}^4$ are the system state and input vectors, $A\big(z(X,u)\big) \in \mathbb{R}^{4\times4}$ and $B\big(z(X,u)\big) \in \mathbb{R}^{4\times4}$ are non-constant LPV state-space matrices expressed as follows:

$$A\big(z(X,u)\big) = \begin{bmatrix} -\kappa_V & 0 & 0 & 0 \\ 0 & \mu_H \frac{S_S}{\kappa_S+S_S}\frac{S_O}{\kappa_{OH}+S_O} - \frac{f_W(1+f_R)}{(f_R+f_W)}\frac{q_{in}}{V} - b_H & 0 & 0 \\ 0 & -\frac{\mu_H}{Y_H}\frac{S_S}{\kappa_S+S_S}\frac{S_O}{\kappa_{OH}+S_O} + (1-f)b_H & -\frac{q_{in}}{V} & 0 \\ 0 & \frac{Y_H-1}{Y_H}\mu_H \frac{S_S}{\kappa_S+S_S}\frac{S_O}{\kappa_{OH}+S_O} & 0 & -\kappa_O q_a - \frac{q_{in}}{V} \end{bmatrix} \tag{6}$$

$$B\big(z(X,u)\big) = \begin{bmatrix} 0 & 0 & 0 & \kappa_V \\ \frac{q_{in}}{V} & 0 & 0 & 0 \\ 0 & \frac{q_{in}}{V} & 0 & 0 \\ 0 & 0 & \kappa_O S_{O,sat} & 0 \end{bmatrix} \tag{7}$$

From the state-space matrices (6)-(7), a set of three non-constant terms, thus presenting the TS fuzzy premise variables of the nonlinear model (1)-(4), is summarized as:

$$\mathcal{S}_z = \left\{\frac{S_S(t)}{\kappa_S+S_S(t)}\frac{S_O(t)}{\kappa_{OH}+S_O(t)}, \frac{q_{in}(t)}{V(t)}, q_a(t)\right\} = \{z_1(t), z_2(t), z_3(t)\} \tag{8}$$

In this process modeling, TS fuzzy rules define local and linear input-output relations of the plant (1)-(4). A complete state-space representation of such a nonlinear plant is obtained as follows:

$$\dot{X}(t) = \sum_{i=1}^{r} h_i\big(z(t)\big)\{A_i X(t) + B_i u(t)\}\,; y(t) = \sum_{i=1}^{r} h_i\big(z(t)\big)C_i X(t) \tag{9}$$

where $z(t) = \big(z_1(t), z_2(t), z_3(t)\big)$ is the premise variables, $h_i(.)$ are the TS fuzzy activation functions, $A_i \in \mathbb{R}^{4\times4}$ and $B_i \in \mathbb{R}^{4\times4}$ are the state-space matrices with constant terms, $r = 2^m = 8$ is the number of local sub-models, with $m = 3$ the cardinality of the premise variables set, and $C_i = I_{4\times4} \in \mathbb{R}^{4\times4}$.

The convex transformation of the premise variables (8) leads to the following formula for the TS fuzzy activation functions used in state-space forme of (9):

$$h_i\big(z(t)\big) = \prod_{j=1}^{m} F_{j,\sigma_i^j}\left(z_j(X,u)\right) \tag{10}$$

where σ_i^j represents the index at the j[th] position in the m -tuple σ_i of sub-models indexing, $h_i(.) \geq 0$ and $\sum_{i=1}^{r} h_i(.) = 1$, and $F_{j,\sigma_i^j}(.)$ are the partition functions expressed with the upper and lower limits of premise variables $\bar{z}_j = \max\limits_{X,u}\{z_j(X,u)\}$ and $\underline{z}_j = \min\limits_{X,u}\{z_j(X,u)\}$, respectively, as follows [29]:

$$F_1^j\left(z_j(t)\right) = \frac{z_j(t)-\underline{z}_j}{\bar{z}_j-\underline{z}_j},\ F_2^j\left(z_j(t)\right) = \frac{\bar{z}_j-z_j(t)}{\bar{z}_j-\underline{z}_j}, \forall j = 1,2,3 \tag{11}$$

By making all possible combinations of upper and lower bounds on the premise variables (8), the constant state-space matrices of the TS fuzzy representation (9) can be obtained from (6) and (7) expressions as follows:

$$A_1 = A(\bar{z}_1, \bar{z}_2, \bar{z}_3); A_2 = A(\bar{z}_1, \bar{z}_2, \underline{z}_3); A_3 = A(\bar{z}_1, \underline{z}_2, \bar{z}_3); A_4 = A(\bar{z}_1, \underline{z}_2, \underline{z}_3)$$

$$A_5 = A(\underline{z}_1, \bar{z}_2, \bar{z}_3); A_6 = A(\underline{z}_1, \bar{z}_2, \underline{z}_3); A_7 = A(\underline{z}_1, \underline{z}_2, \bar{z}_3); A_8 = A(\underline{z}_1, \underline{z}_2, \underline{z}_3) \quad (12)$$

$$B_1 = B_2 = B_5 = B_6 = B(\bar{z}_2); B_3 = B_4 = B_7 = B_8 = B(\underline{z}_2) \quad (13)$$

2.2. MPC design for carbon removal

Using a discrete-time representation of the established TS fuzzy model (9), an MPC algorithm is designed. In this framework, a sequence of predictive control laws is computed and updated at the kth sampling times $t = kT_s$ to minimize the MPC quadratic criterion defined as follows [30]:

$$J(t) = \sum_{n=1}^{N_p} e^T(t+n|t)Qe(t+n|t) + \sum_{n=0}^{N_c-1}[\Delta u^T(t+n|t)R\Delta u(t+n|t)] \quad (14)$$

where $N_p \in \mathbb{N}$ and $N_c \in \mathbb{N}$ ($N_p \geq N_c$) are the MPC horizons for prediction and control, respectively, $Q = Q^T > 0$ and $R = R^T > 0$ are the weighting matrices, i.e. $Q = \lambda I_{4\times 4}$ with $\lambda \in \mathbb{R}_+$, and $e(t+n|t)$ is the error between the reference and predicted system outputs.

The designed local controllers are aggregated with the same modeling activation functions (10) and applied to the initial nonlinear system (1)-(4). A step-by-step procedure for the proposed carbon removal in WWTPs is finally summarized in Algorithm 1.

Algorithm 1: Proposed TS fuzzy MPC-based carbon removal in WWTPs.

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Step 1:   Modeling of nonlinear ASP dynamics
          Differential equations governing process behavior (1)-(4).
          Specification of standard quality limits for COD, BOD5 and TSS performance.
Step 2:   Equivalent TS fuzzy representation
          Rewriting the nonlinear model (1)-(4) into a quasi-LPV form (5)-(7).
          Characterization of all TS fuzzy premise variables (8) and their
          corresponding bounds.
          Convex partition of TS fuzzy premise variables according to (11).
          Calculation of activation functions (10) and constant state-space matrices in
          (12) and (13).
          Formation of the equivalent TS fuzzy representation (9).
Step 3:   Validation of TS fuzzy modeling stage
          Dynamics simulation and VAF (%) quantification of modeling deviations.
          Repeat Step 2 with new bounds of TS premise variables until VAF (%)
          performance is met.
Step 4:   MPC design for carbon removal enhancement
          Computation of local MPC laws for TS fuzzy model from criterion (14).
          Aggregation of all local MPC laws with the same activation functions (10).
          Application of the global MPC laws for nonlinear model (1)-(4).
          Repeat MPC design (Np,Nc, λ) until the desired COD, BOD5 and TSS performance
          are met.
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3. RESULTS AND DISCUSSION

3.1. Parameters setting and TS fuzzy modeling validation

Model parameters of the studied system are borrowed from the related literature [12], [16]. Standard regulatory limits for the carbon removal are considered as 90 [mg/l] for COD metric and 30 [mg/l] for both BOD5 and TSS indicators. Input profiles are considered as pseudo-random binary sequences (PRBS) over a simulation horizon of 300 hours. The simulation results for comparing the transient responses of nonlinear (1)-(4) and TS fuzzy (9) models are shown in Figures 2-3. The time-domain evolutions of modeled effluent volume (EV), heterotrophic biomass (HB), biodegradable substrate (BS) and dissolved oxygen (DO) dynamics are given in Figure 2(a), Figure 2(b), Figure 2(c) and Figure 2(d), respectively. The used activation functions are given in Figure 3(a) and the TS fuzzy modeling performance are evaluated based on the "Variance Accounted For" VAF (%) metrics as shown in Figure 3(b). Such an evaluation shows achieving high measures of VAF (%) indices, synonymous with competetive modeling accuracy. Indeed, a VAF value of 100% is guaranteed for EV, varying from 95% to 99% for BS, close to 99% for HB, and ranging from 87% to 97% for DO. Findings demonstrate the ability of the proposed TS fuzzy modeling tool (9) to accurately reproduce the nonlinear dynamics (1)-(4) of WWTPs. The results clearly show the correspondence and closness between the outputs of the two reported models. On the other hand, findings also contribute to understanding and examining the evolution policy of all carbon removal variables.

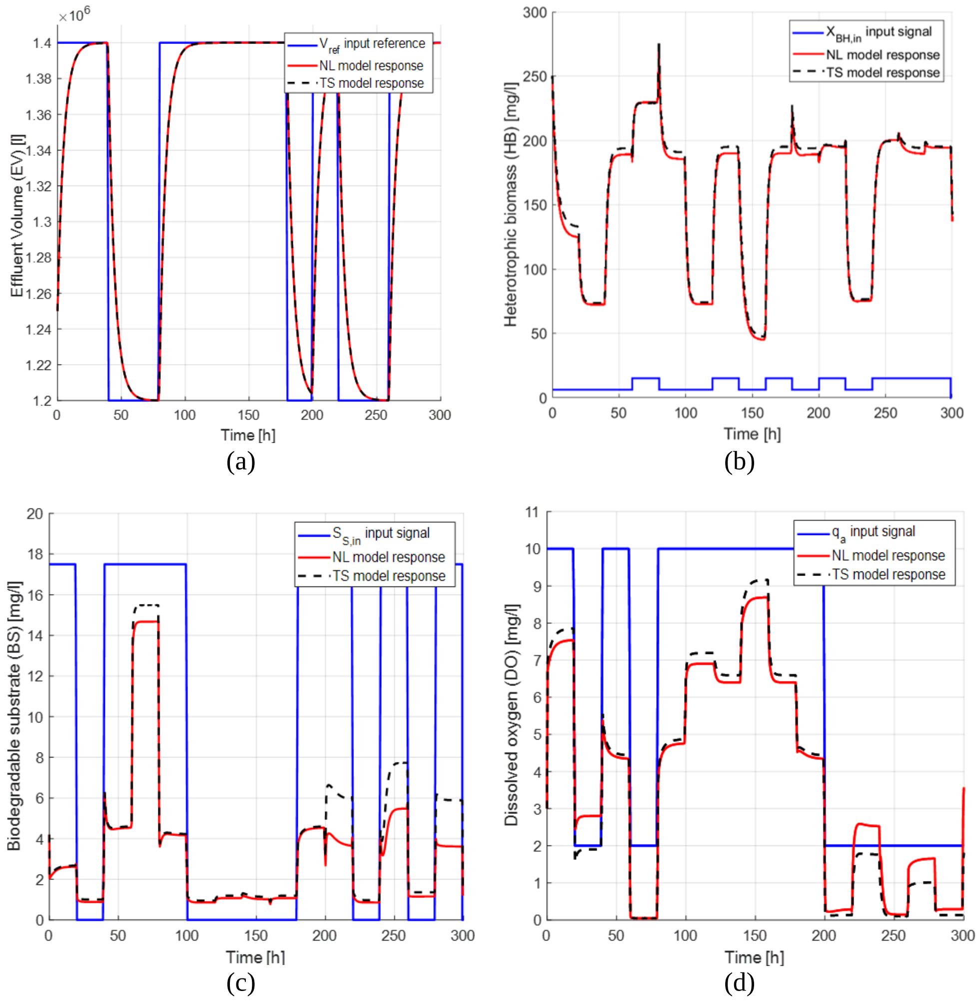


Figure 2. Comparisons of nonlinear and TS fuzzy modeling results: (a) EV dynamics, (b) HB concentration dynamics, (c) BS concentration dynamics, (d) DO concentration dynamics

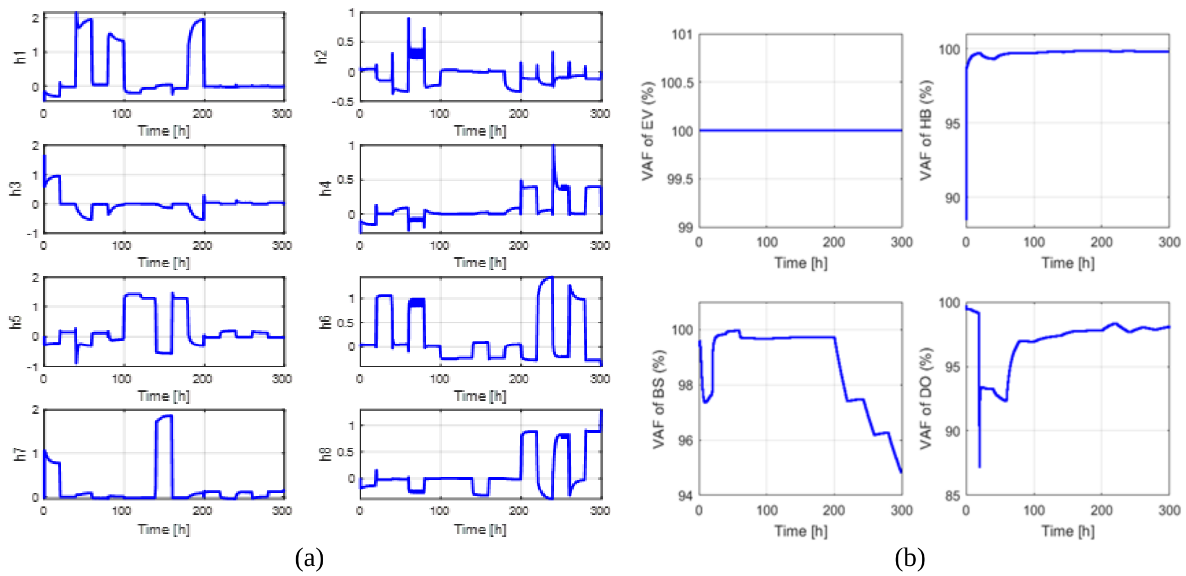


Figure 3. Performance evaluation of dynamic TS fuzzy modeling: (a) TS fuzzy activation functions distribution, (b) VAF (%) metrics quantification for WWTP's variables

3.2. TS fuzzy MPC design validation

The WWTP variables EV, HB, BS and DO are manipulated to boost the efficiency of the carbon pollutant removal process by controlling the COD, BOD5 and TSS indicators. In this work, the commonly used PDC form of static feedback TS fuzzy control is considered [29]. The corresponding PDC feedback gains are obtained by numerically solving a stabilization problem under LMIs convex constraints [16]. The MPC parameters as shown in (14) are selected through trial-and-error based procedures as follows: predictive horizon N_p equal to 10, control horizon N_c equal to 2, and weighting coefficients λ equal to 0.08. Those of PDC design, i.e. LMIs gain matrices, are obtained using the LMI Control Toolbox of MATLAB. Both MPC and PDC are designed based on the developed TS fuzzy model (9) and then tested and validated on the initial nonlinear model (1)-(4). For more scalability and reliability of the proposed MPC approach, other input profiles are considered as reference trajectories over a simulation horizon 300 hours. The simulation results of the controlled WWTP's dynamics which governing the carbon removal process are depicted and compared to those of PDC-based design in Figures 4-5.

Findings show high performance of the designed MPC in terms of responses fastness, steady-state accuracy and overshoots damping in comparison with the reported PDC-based results. Small steady-state errors and low rise/settling times are guaranteed at the MPC closed-loop responses compared to those of PDC design where the system responses exhibit jerky evolution and present low accuracy with significant tracking errors particularly for BS and DO dynamics, see Figure 4(a) and Figure 4(b). Undesirable jerky behaviors of the PDC design are also observed in EV and HB variables as shown in Figure 4(c) and Figure 4(d) in contrast to smoother responses in the case of MPC-based pollutants removal. Besides, MPC performances remain less sensitive to variations in system inputs and inlet volume transitions. A constructive comparison between transient performance of the reported MPC and PDC techniques operating on the initial nonlinear model (1)-(4) of WWTP is thus addreesed. Regarding the suitability of generated control outputs, Figure 5(a) and Figure 5(b) shown the signals amplitude variations for MPC and PDC, respectively. Based on these curves, one can observe the high amplitudes of PDC signals compared to those of MPC. These non moderated and overly energetic signals remain undesirable for practical implementations. Through all these demonstrative results, MPC improvements in terms of dynamics stabilization, desired references tracking and control signals moderation are significantly satisfied and outperform those of the PDC technique.

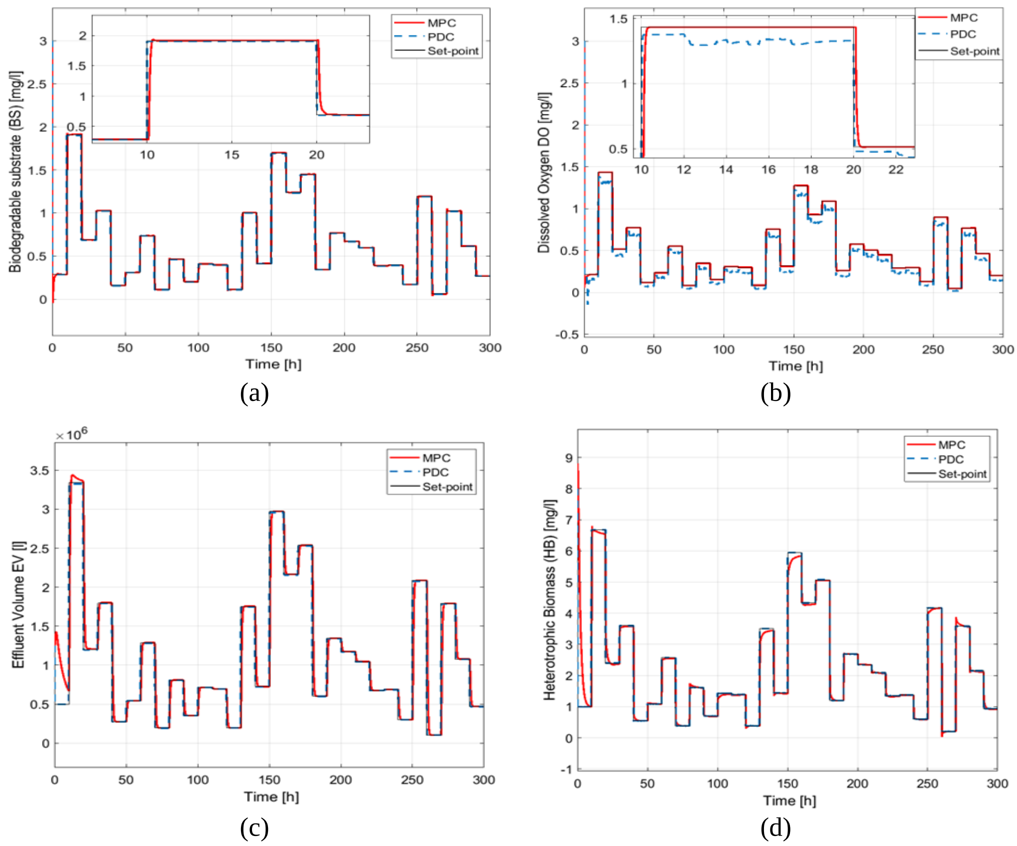
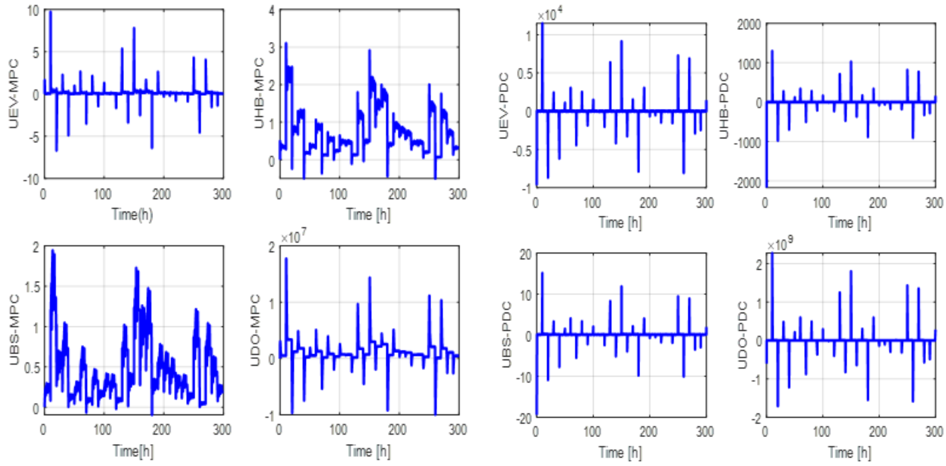


Figure 4. Comparisons of TS fuzzy MPC and PDC operating on nonlinear model (1)-(4): (a) EV dynamics, (b) HB concentration dynamics, (c) BS concentration dynamics, (d) DO concentration dynamics



(a) (b)

Figure 5. Comparions of control signal amplitudes: (a) TS fuzzy MPC-based design, (b) PDC-based design

3.3. Carbon removal efficiency evaluation

To assess the carbon removal efficiency based on the proposed MPC-based solution, the metrics of COD, BOD5 and TSS are considered for influent and effluent waters. Standard regulatory limits of 90 [mg/l] for COD, 30 [mg/l] for both BOD5 and TSS are defined for effluent qualities. Random profiles are considered to simulate the time variations of these input indicators as shown in Figure 6. The MPC-based temporal evolutions of the output indicators $COD_{effluent}$, $BOD5_{effluent}$ and $TSS_{effluent}$ are shown in Figure 6(a), Figure 6(b) and Figure 6(c), respectively. The entire of these simulation results demonstrates the high capabilities of the proposed TS fuzzy MPC-based carbon removal boosting in WWTPs. The MPC-based input-output evolutions depicted for the reported COD, BOD5 and TSS performances highlight significant improvements in effluent qualities. Indeed, decreasing removal rates form 300 [mg/l] to 27 [mg/l] for COD, from 72 [mg/l] to 6 [mg/l] for BOD5, and from 223 [mg/l] to 17 [mg/l] for TSS indicators are guaranteed. In Figure 6(d), bargraph plots show the average pollution removal efficiencies achieved with the proposed TS fuzzy MPC design. Improvements in carbon pollutant removal increase the COD, BOD5 and TSS reduction measures with the rates of 91%, 91%, and 92%, respectively. A remarkable superiority of the MPC-based pollutants removal is highlighted further justifying the potentials in solving problems of harmful substances elimination in WWTPs. The standard regulatory levels of effluent quality are widely met in this study, ensuring the effectiveness of the proposed method to improve wastewater treatment capacities.

In discussion of this research works, one can summarize that the proposed TS fuzzy modeling is valid in terms of reproducing the nonlinear dynamics of the carbon removal process. Transient responses of the WWTP variables are close and similar since using the initial nonlinear model (1)-(4) and TS fuzzy one (9). Indeed, using an LTI state-space form instead of nonlinear one further reduces the modeling complexity and contributes in the subsequent control design stage. The VAF (%) metrics provide a key piece of evidence supporting the dynamic modeling procedure thus proposed. The use of such a validated TS fuzzy model to design MPC-based wastewater treatment has clearly improved the carbon pollution removal efficiencies in terms of COD, BOD5 and TSS performance metrics. The MPC-controlled system exhibits steady-state accuracy, fastness and tracking capabilities for all intervening dynamics. The achieved performances support the evidence the carbon removal boosting in WWTPs by maintaining the controlled process variables around predefined set-point values. MPC-based efficiencies of 91% and 92% are achieved for COD, BOD5 and TSS reduction and regulatory limits of WWTPs are widely respected.

Comparisons and contrasts, carried out mainly with the previously published PDC technique [16], highlight the superior performance of the suggested MPC approach firstly in terms of achieved COD, BOD5 and TSS measures, and secondly in terms of complexity reducing in the design procedure. Indeed, only three design parameters have to be tuned in MPC algorithms instead of eight matrices gains in PDC case. The design and prototyping times will be significantly reduced. The main strengths of the study lie in the elaboration of a systematic and effective design procedure to deal with complexities and costs in the management of pollution removal problems. Such a study offers a comprehensive framework for harmful substances removal in WWTPs, compared to previous works of the related literature. However, minor limitations can be mentioned in the MPC design which still requires repetitive trial-and-error procedures for choosing its main parameters. Unexpected results regarding the high control signal amplitude for DO are observed. Future consideration of MPC input constraints in the formulation of removal carbon problem can correct such a shortcoming. The purposes of this study can be summarized around the proposal of an effective methodology for harmful substances elimination in WWTPs. Such an advanced pollution management policy promotes the carbon removal with increased purification efficiencies and reducing the complexity associated with tedious and time-consuming modeling and control procedures. The importance of the study lies in the design and prototyping of competive pollution removal technique in WWTPs promoting better performance in terms of most commonly used COD, BOD5 and TSS indicators. Potential future research could focus on considering other harmful substances to be removed in WWTPs such as nitrogen. The use of artificial intelligence tools for automatically tuning the MPC parameters will be investigated. Considering constraints on WWTPs' states and inputs presents another potential direction of this research.

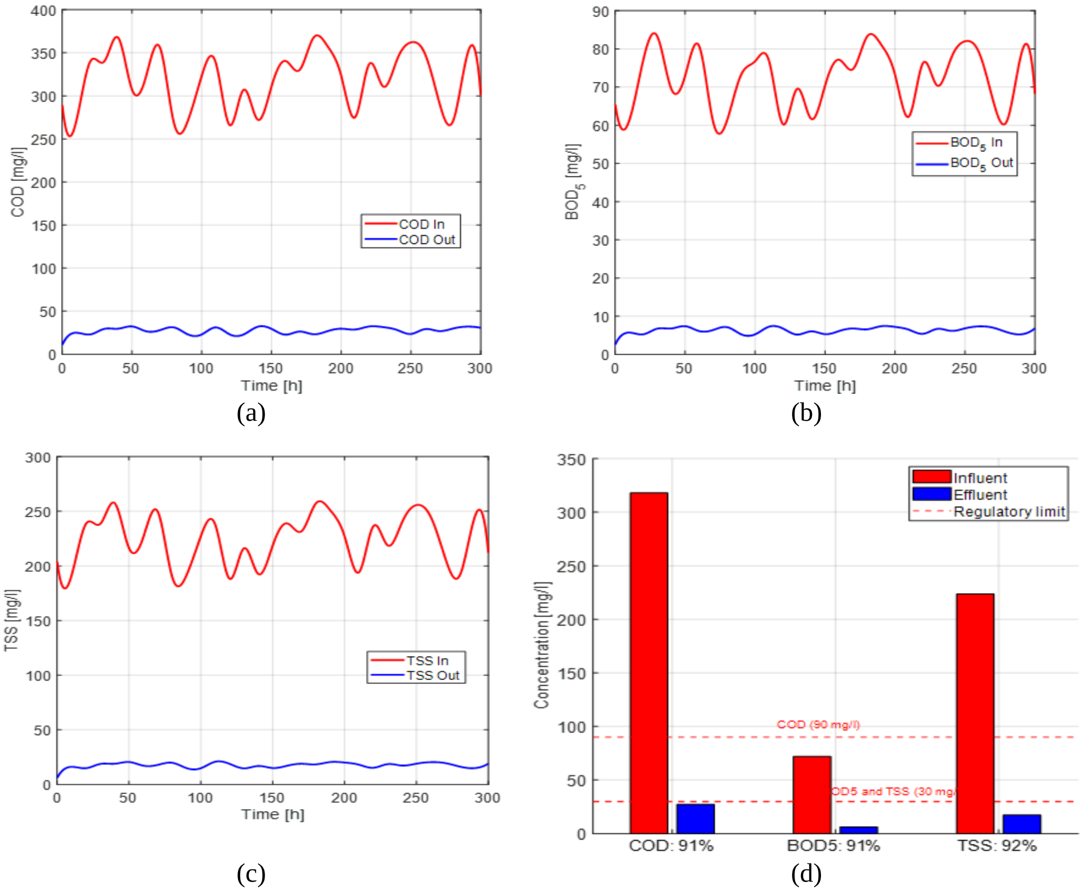


Figure 6. Carbon removal performance of the proposed TS fuzzy MPC-based approach: (a) Input-output variation of COD indicator, (b) Input-output variation of BOD5 indicator, (c) Input-output variation of TSS indicator, (d) Average pollution removal efficiencies.

4. CONCLUSION

In this work, an effective and competitive methodology for harmful carbon pollution removal in WWTPs is proposed and successfuly applied based on an advanced technique combining the advantages of TS fuzzy modeling and MPC design. Most commonly used pollution metrics namely COD, BOD5 and TSS are considered to assess the effectiveness of the proposed approach. The output wastewater quality is enhanced to reach purifying rates of 91% for the COD and BOD5 performance and 92% for the TSS indicator, thus widely respecting the normative limits defined in WWTPs. Main research findings in terms of accurately modeling of pollution variables and closed-loop performance of steady-state precision, responses fastness and control signals moderation are highlighted, compared and discussed to show the advantages in the proposed management policy of such harmful substnaces removal. A comprenhensive framework for water treatment against micopollutants is elaborated in which the overall results are satisfactory and very promising for the removal of other types of nocive components, especially those caused by nitrogen and phosphorous. The potential applications of such a proposed water pollution methodology are scalable and future extensions for other pollution management in wastewater plants are made possible by following the proposed step-by-step control procedure. The complexity of WWTPs, i.e. variables coupling and non-linearity, are managed by transforming dynamic models into an equivalent TS fuzzy form that remains beneficial for the efficient control of all pollution variables of EV, concentrations of HB, BS and DO. Advanced concepts of LPV representation, convex polytopic transformation of nonlinear variables and MPC design are combined to achieve high depollution efficiencies in terms of COD, BOD5 and TSS indicators. Future research orientations will focus on extending the proposed study, namely TS fuzzy modeling and MPC design, to consider in addition to carbon other harmful substances involved in WWTPs management. Additional constraints on inlet flow variations and weather conditions will be investigated. Anoter reaserch direction is the use of artificial intelligence techniques and advanded optimization theories to easly supervise the selection and tuning of all MPC parameters.

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CONFLICT OF INTEREST STATEMENT

The authors state no conflict of interest.

DATA AVAILABILITY

Data availability is not applicable to this paper as no new data were created or analyzed in this study.

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