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A Nonlinear System of Generalized Predictive Control

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Abstract

Generalized predictive control (GPC) algorithm has been applied to all kinds of industry control systems. But systemic and effective method for nonlinear system has not been found. To this problem, this paper integrates the characteristics of PID technology and GPC, present a PID generalized predictive control algorithm for a class of nonlinear system, and improves the control quality of the system.

Keywords*: generalized predicitive control, PID, nonlinear system, simulation*

1. Introduction

Since the 1990s, the modern industrial become quickly to complex, large-scale and automation development, therefore many industrial control systems with a high degree of non-linearity, coupling, when the variability and large time-delay characteristics, and the existence of demandingthe constraint conditions. These require the automatic control technology to provide important technical guarantee for the realization of efficient, safe, high-quality mass production. Only rely on traditional control techniques such as PID control algorithm can not solve all these problems. Therefore, we must seek more advanced control methods to meet the high requirements of modern industrial automatic control technology. The continuous development of computer technology brought great changes to the control system hardware, and on this basis, the new control algorithm - predictive control slowly developed.

Generalized predictive control algorithm with a feedback correction, multi-step prediction, rolling optimization control method, and thus control the effect of good, strong robustness, this control algorithm can be used to control complex industrial processes [1], or difficult to establish precise the mathematical model, and has been used in the control system of the chemical, petroleum, metallurgy, machinery and other industrial sectors, and reflects the characteristics of the a superior traditional control system, is a promising new class of computer control algorithms.

From the above, we can see that, although predictive control has many advantages, however, the traditional PID control algorithm because of its structure is simple, clear and concise algorithm robustness, low control algorithm model accuracy requirements and operation easily accepted in actual industrial process control or occupy a dominant position, constrained generalized predictive control to the application of the actual industrial field.

Relatively According to the above, we can see that the PID control technology with predictive control combination is a new research direction; the integration of the respective characteristics of this new control algorithm has certain significance.

2. Generalized Predictive Control

2.1. Basic Principle of Generalized Predictive Control

Generalized Predictive Control (GPC) [2] was proposed in 1984 by Clarke et al. GPC based on generalized minimum variance control, the introduction of a multi-step prediction idea to make it a random noise, significantly improved the ability of anti-disturbance and delay variation. GPC's basic structure as shown by the prediction model, GPC is rolling optimization and feedback correction consists of three parts.

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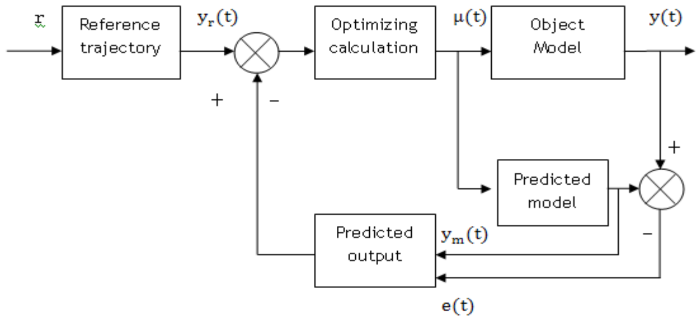

Figure 1. GPC structure schematic

2.2. Generalized Predictive Control Model

In 1987, Clarke et al generalized parametric model based control. Its essence is based on generalized minimum variance introduced the idea of the multi-step prediction, random noise, anti-load disturbance and delay variation capability has been significantly improved robustness suitable for open-loop unstable, non-minimum phase delay system. The basic form of the algorithm, the following discrete differential equations to describe the mathematical model of the controlled object:

$$A(z^{-1})y(t) = B(z^{-1})\mu(t-1) + C(z^{-1})\omega(t)/\Delta \quad (1)$$

Where $A(z^{-1})$、$B(z^{-1})$、$C(z^{-1})$ are the backward shift operator z^{-1} polynomial. $\mu(t)$ and $y(t)$ is respectively represent the input and output of the controlled object,Δ=z^{-1}, is difference operator.$\omega(t)$ is not related to the sequence of random variables.Constant n_a and n_b is not related to the sequence of random variables.Constant $C(z^{-1})$=1.

3. A Class of Nonlinear Generalized Predictive Control

3.1. Nonlinear System Model

General nonlinear system can be used to describe the following I / O model with first-order delay:

$$y(t) = f(y(t-1), \ \ldots, \ y(t-n), \ u(t-1), \ \ldots, \ u(t-m)) \quad (2)$$

Wherein m, n, respectively, is known as a system input output gradation times or the ceiling in order: f () is the unknown, and is $y(t-1), \ \ldots, \ y(t-n), \ u(t-1), \ \ldots, \ u(t-m)$'s nonlinear function.The following condition is satisfied:

a) f(0,0, …0)=0,
b) f() is about $\mathbf{y(t-1)}, \ \ldots, \ \mathbf{y(t-n)}, \ \mathbf{u(t-1)}, \ \ldots, \ \mathbf{u(t-m)}$ continuously differentiable, and various partial derivatives bounded.

3.2. Equivalent Time-varying Linear Systems

In order to facilitate the analysis, we consider only the input-output system, the system eq. (2), we obtain the following theorem by analyzing:

Theorem 3.1 [3]: satisfy the condition (1), (2) non-linear system eq. (2) can be equivalently expressed as follows when denatured system:

$$\begin{aligned} y(t) &= f(y(t-1), \ \ldots, \ y(t-n), \ u(t-1), \ \ldots, \ u(t-m)) \\ &= a_1(t)y(t-1) + \cdots + a_n(t)y(t-n) + b_1(t)u(t-1) + \cdots + b_m(t)u(t-m) \end{aligned} \quad (3)$$

Formula's a_i,b_j is bounded time-varying coefficients.

3.3. Time-varying Parameter Identification

Theorem 3.1 only given the existence of the equivalent linear system of the original nonlinear system, did not give the specific parameters of the corresponding linear system in every moment. Since the nonlinear function f () of the partial derivative is unknown, i.e. the linear representation of the respective coefficients in the Equation (3) is unknown. And can be seen from the above derivation, the coefficient $a_i(i = 1, \ldots n)$, $b_j(j = 1, \ldots m)$ is not only with the nonlinear function f (), also associated with the input and output of the system, that is constantly changing with time. Need online identification of these time-varying coefficients, on this basis, re-design of the controller.

Seen by Theorem 3.1, the system Equation (2) can be expressed as:

$$y(t) = \phi^T(t)\theta(t) + \xi(t) \tag{4}$$

Where

$$\phi(t) = [y(t-1), \ldots, y(t-n), u(t-1), \ldots, u(t-m)]^T \tag{5}$$
$$\theta(t) = [a_1(t), a_2(t), \ldots a_n(t), b_1(t), b_2(t), \ldots, b_m(t)]^T$$

3.4. Controller Design

By the formula (3) shows, the w. Charged object type eq.(2) is equivalent linear time-varying systems:

$$y(t) = a_1(t)y(t-1) + \cdots + a_n(t)y(t-n) + b_1(t)u(t-1) + \cdots + b_m(t)u(t-m) + \xi(t) \tag{6}$$

Theorem 3.1 shows that $|a_i|$、$|b_j| \leq r$.

Assume 1: $a_i(t) = \sum_{j=0}^{\infty} a_{i、j}F_j(t) \qquad i = 0, \ldots n$

$b_i(t) = \sum_{j=0}^{\infty} b_{i、j}F_j(t) \qquad i = 0, \ldots m$

Here a basis function approximation with variable coefficients. Actual application, the interception of a finite number of terms, that:

$$a_i(t) = \sum_{j=0}^{q} a_{i、j}F_j(t) \qquad i = 1, \ldots n$$
$$b_i(t) = \sum_{j=0}^{q} b_{i、j}F_j(t) \qquad i = 1, \ldots m$$
$$Y(t) = a_{1,0}F_0(t)y(t-1) + \cdots a_{1,q}F_q(t)y(t-1) + \cdots a_{n,0}F_0(t)y(t-n) + \ldots + a_{n,q}F_q(t)y(t-n) + b_{1,0}F_0(t)u(t-1) + \cdots + b_{1,q}F_q(t)u(t-1) + \ldots + b_{m,0}F_0(t)u(t-m) + \cdots + b_{m,q}F_q(t)u(t-m) + \varepsilon(t) \tag{7}$$

$$y(t) = \phi^T(t)\theta + \varepsilon(t) \tag{8}$$

Where,

$$\theta = \begin{matrix}[a_{1,0}, & \ldots, a_{1,q}, & \ldots, & a_{n,0}, & \ldots, & a_{n,q}, \\ b_{1,0}, & \ldots, b_{1,q}, & \ldots, & b_{m,0}, & \ldots, & a_{m,q}]\end{matrix}^T$$
$$\phi(t) = [y(t-1)F_0(t), \ldots, y(t-1)F_q(t), \ldots y(t-n)F_0(t) \ldots y(t-n)F_q(t) \; u(t-1)F_0(t), \ldots, u(t-1)F_q(t), \ldots u(t-m)F_0(t) \ldots u(t-m)F_q(t)]$$

Here is including noise and unmodeled dynamics.

Assume 2: $\varepsilon(t)$ is bounded, $|\varepsilon(t)| \leq k_1$, k_1 is the normal number. For solving control law convenience, (3-6) can be expressed as:

$$A(t, z^{-1})y(t) = B(t, z^{-1})u(t-1) + \varepsilon(t) \tag{9}$$

$$A(t, z^{-1}) = [a_{1,0}F_0(t) + \cdots + a_{1,0}F_0(t)]z^{-1} + \cdots + [a_{n,0}F_0(t) + \cdots + a_{n,q}F_q(t)]z^{-n}$$

$$B(t, z^{-1}) = [b_{1,0}F_0(t) + \cdots + b_{1,q}F_q(t)]z^{-1} + \cdots + [b_{m,0}F_0(t) + \cdots + b_{m,q}F_q(t)]z^{-m+1}$$

So the time-varying parameter estimation in the original system Equation (3) into Equation (4) in the fixed-length parameters estimated. Parameter identification using adaptive algorithm as follows:

$$\hat{\theta}(t) = P_r\left\{\theta(t-1) + \frac{\phi(t-1)e(t)}{1+\phi^T(t-1)\phi(t-1)}\right\}$$
$$e(t) = y(t) - \phi^T(t-1)\hat{\theta}(t-1)$$

The above formula, P_r is projection operand,and is used to locate $\hat{\theta}(t)$ on a compact set c.

$$\hat{A}(t,\ z^{-1})y(t) = \hat{B}(t,\ z^{-1})u(t-1) + \varepsilon(t) \tag{10}$$

Estimation model Equation (10) can be used as the GPC plant model, the design of generalized predictive controller, as the controller of the original nonlinear system.

4. The Generalized Predictive of a Class of Nonlinear Systems PID Control Algorithm

4.1. Plant model

By theorem 3.1, spline basis functions [4] better than other polynomial basis functions smooth and simple calculation, using this time-varying coefficients of the cubic spline basis function approximation. Finally, we can calculate the estimated model:

$$\hat{A}(t,\ z^{-1})y(t) = \hat{B}(t,\ z^{-1})u(t-1) + \varepsilon(t) \tag{11}$$

Where eq.(11) can be used as the GPC charged like a model, the design of generalized predictive controller and controller as an element nonlinear systems.

4.2. Controller Design

Formula (11) was charged with the object model, more than one of a class of nonlinear systems GPC principle design PIDGPC controller. The performance index function we get:

$$J(t) = E\left\{\sum_{j=1}^{N}[k_p\left(\Delta e(t+j)\right)^2 + k_i e(t+j)^2 + k_d\left(\Delta^2 e(t+j)\right)^2] + \sum_{j=1}^{N_u}\lambda(j)[\Delta u(t+j+1)]2et{=}\Delta et{=}0\right.$$

$$e(t+j) = \omega(t+j) - \hat{y}(t+j) \tag{12}$$

Where E is the mathematical expectation, N is the prediction horizon, N_u is controlling a time domain, λ(j) is a weighting coefficient, we set the constant λ、k_p、k_i、k_d, respectively, for the coefficient of the proportional term, the integral term coefficient and derivative coefficient, $\hat{y}(t+j)$ is $y(t)$ forward j–th step prediction, the set value of the $\omega(t+j)$ for a given softening sequence.

$$\omega(t) = y(t)$$
$$\omega(t+j) = \eta\omega(t+j) + (1-\eta)y_r(t+j)\quad j = 1,2,\ \ldots,\ N$$

Wherein $y(t)$ is the set value of the time point t, $\eta(0 < \eta < 1)$ is soften factor.

Introduction of Diophantine equations.

$$\begin{cases} 1 = E_j A\Delta + Z^{-1}F_j \\ E_j B = G_j + Z^{-1}H_j \end{cases} \tag{13}$$

Wherein, E_j, F_j, G_j, H_j is a polynomial for Z^{-1}. Through a series of calculations and derivations eventually came to the following formula:

$$\Delta u(t) = P^T y_r(t) - P^T(F_j - \eta^j)y(t) - P^T H_j \Delta u(t-1) \tag{14}$$

$$u(t) = u(t-1) + \Delta u(t) \tag{15}$$

Be seen from the above derivation of the plant parameters are known, the PID indirect generalized predictive controller is designed as follows:

To a specified forecast domain N, control of time-domain N_u, the weighted constant λ, as well as the number of PID control parametersk_p、k_i、k_d.

(1) The linearization of nonlinear systems, have to the GPC model parameters matrix A, B;

(2) The polynomial E_j, F_j, G_j, H_j is solved by the the Diophantine equation;

(3) The control amount is solved by the formula (14), (15);

(4) $t = t + 1$, and returns to (1).

5. Simulation and Conclusions

Generalized Predictive Control using multi-step prediction, domain N, as well as the control domain N_u forecast increase of these two parameters with a single step prediction, PID-GPC added parameter k_p、k_i、k_d, these parameters and control weighting constants λ,soften the selection factor η control performance will have an important impact.

Controlled object:

$$y(t)\frac{5y(t-1)y(t-2)}{1+y(t-1)^2+y(t-2)^2+y(t-3)^2} + u(t-1) + 1.1u(t-2)$$
$$y(1) = y(2) = 0\ ;\ y(3) = 1,\ \ u(1) = -1,\ \ u(2) = 1$$

The GPC parameters taken prediction field N = 20, control the time domain $N_u = 1$, $\eta = 0.95$, $\lambda = 0.3$, $\rho = 0.6$, PID parameters is $k_p = 0.5$, $k_i = 0.8$, $k_d = 5.6$.

The following plans were the GPC and PID-GPC tracking $y_r(t) = 1$ step response simulation results.

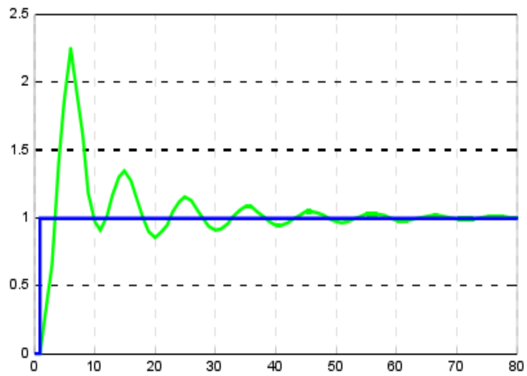

Figure 2. When λ = 0.3, The simulation results of the conventional GPC

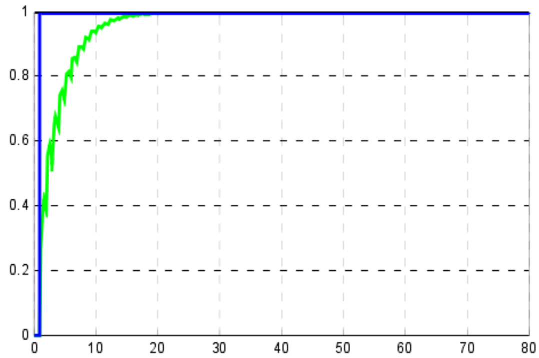

Figure 3. When $\lambda = 0.3$, $k_p = 0.5$, PID-GPC simulation results

The simulation results show that the improved controller PID-GPC the control effect than traditional GPC control effect, can effectively shorten the track time, and enhance the robustness of the control, suppress overshoot smoother control action, to achieve the control effect.

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