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Solving Method of H-Infinity Model Matching Based on the Theory of the Model Reduction

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Abstract

People used to solve high-order H_∞ model matching based on H_∞ control theory, it is too difficult. In this paper, we use model reduction theory to solve high-order H_∞ model matching problem, A new method to solve H_∞ model matching problem based on the theory of the model reduction is proposed. The simulation results show that the method has better applicability and can get the expected performance.

Keywords: *high-order model, reduction theory, H_∞ model matching*

1. Introduction

H-infinity (H_∞) optimal control theory of linear systems is a new kind of design method developed in the end of 1980, and is the very active frontier subject in current control theory. In many control systems, in order to improve the steady and dynamic performance of system, the appropriate correction device needs to be added in the system, making the output characteristics of the system meet all of the demand for performance specifics. This is the model matching problem. In solving the model matching problem, it is mostly solved by converting to H_∞ standard control problem [1-2]. Chen Yongjin proposed a kind of upper bound method of searching for multi-blocks of model matching [3]. Zhuge Hai proposed an approximate method of imprecise model matching [4]. These methods are easy to be achieved for general systems, but these methods are more complicated for high order system model. Moore proposed the balance order reduction problem of system in 1981 [5], then the method is improved constantly [6], and some new reduction algorithms were put forward [7-9].

Due to the high order problem of system model in H_∞ model matching, combining with the model order reduction theory, H_∞ model matching resolving method is proposed based on model reduction theory. The analysis and simulation show that the method has good matching characteristics.

2. H_∞ Model Matching Problem

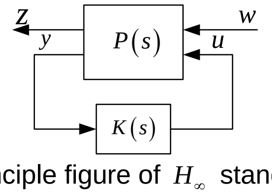

Figure 1. Principle figure of H_∞ standard problem

In control system, many H_∞ optimization problems of different requirements can be converted into H_∞ standard problem. As shown in Figure 1, w is the external input, z is control

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output, and u is the control input, y is the output of measurement. $P(s)$ is the generalized controlled object, $K(s)$ is designed controller.

State equation of the generalized object $P(s)$ is described as:

$$\dot{x} = Ax + B_1 w + B_2 u \tag{1}$$

$$z = C_1 x + D_{11} w + D_{12} u \tag{2}$$

$$y = C_2 x + D_{21} w + D_{22} u \tag{3}$$

Transfer function is:

$$P(s) = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \tag{4}$$

Using the linear fractional transformation (LFT), transfer function from w to z can be described as:

$$G = F_l(P, K) = P_{11} + P_{12} K (I - P_{22} K)^{-1} P_{21} \tag{5}$$

The H_∞ standard control problem is for a regular controller K, making the closed-loop of system stable, and $\|F_l(P,K)\|_\infty$ less than a given γ, $\gamma > 0$.

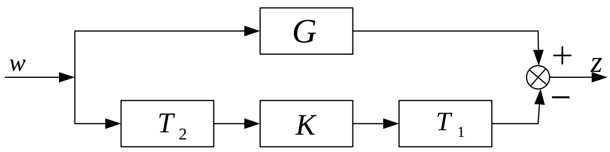


Figure 2. Matching principle figure of H_∞ standard control model

H_∞ standard control model matching is shown as fig.2. Using three transfer function matrix series T_1, K, T_2 to approach transfer function G, the approximation degree will be measured by $\|G - T_1 K T_2\|_\infty$. The generalized controlled object:

$$P(s) = \begin{bmatrix} G & T_1 \\ T_2 & 0 \end{bmatrix} \tag{6}$$

The controller is:

$$K = -K \tag{7}$$

A measure of model matching degree can be expressed as: $\|G - T_1 K T_2\|_\infty$. When T_1 and T_2 are reversible, then the expression of model matching measurement is: $\|T_1^{-1} G T_2^{-1} - K\|_\infty$. So $\hat{G} = T_1^{-1} G T_2^{-1}$, $G_r = K$, then, solving problem of H_∞ model matching can be transformed into solving the model reduction problems, making $\|\hat{G} - G_r\|_\infty$ within a required range.

To make $\hat{G}(s)=\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ a balance achievement.

Definition 1. Controllability and observability Gram matrix of system (A, B, C, D) are defined separately as follows:

$$P=\int_0^{\infty} e^{At}BB^T e^{A't}dt \tag{8}$$

$$Q=\int_0^{\infty} e^{A't}C^T Ce^{At}dt \tag{9}$$

A' denotes the transpose of matrix A. It can be seen that the two matrices are symmetric positive semi-definite matrixes, which satisfy the Lyapunov equation below:

$$AP+PA'+BB'=0 \tag{10}$$

$$QA+A'Q+C'C=0 \tag{11}$$

Diagonalization of the matrix P,Q, then:

$$TPT'=\left(T^{-1}\right)' QT^{-1}=\Sigma=diag(\sigma_1,\sigma_2,\cdots\sigma_k,\sigma_{k+1},\cdots\sigma_n) \tag{12}$$

Where $\sigma_1>\sigma_2>\cdots>\sigma_k>\sigma_{k+1}>\cdots>\sigma_n>0$.
The system (A, B, C, D) and Σ can be separated into blocks:

$$A=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, B=\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, C=\begin{bmatrix} C_1 & C_2 \end{bmatrix} \tag{13}$$

$$\Sigma=\begin{bmatrix} \Sigma_1 & \Sigma_2 \end{bmatrix} \tag{14}$$

Where $\Sigma_1\in R^{k\times k},\Sigma_2\in R^{(n-k)\times(n-k)}$.

Theorem 1 [6]. Given asymptotically stable minimum system $\hat{G}$ has Lyapunov equilibrium form as follows:

$$\hat{G}(s)=\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]=\left[\begin{array}{cc|c} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \\ \hline C_1 & C_2 & D \end{array}\right] \tag{15}$$

And there are:

$$P=Q=diag(\Sigma_1,\Sigma_2) \tag{16}$$

Where $\Sigma_1=diag(\sigma_1,\cdots\sigma_k)$, $\Sigma_2=diag(\sigma_{k+1},\cdots\sigma_n)$.

Reduced order model $G_r(s)=\left[\begin{array}{c|c} A_{11} & B_1 \\ \hline C_1 & D \end{array}\right]$ which is truncated is asymptotically stable and minimum system, and meet:

$$\left\|\hat{G}(s)-G_r(s)\right\|_{\infty}\le 2\left(\sigma_{k+1}+\cdots+\sigma_n\right) \tag{17}$$

The reduced order model $G_r(s)$ is the K in the matching model we are asking for.

3. Simulation Examples

The mathematical expressions for state equation model of DC motor drive system is [10]:

$$A=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.4 \\ 0 & -100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 130 & 0 & -100 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & -0.44 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 200 & -0.88 & 11.76 & -100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -100 & 0 & 0 & 1.4 \\ 0 & 0 & 0 & 0 & 100 & -10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 294.1 & -29.41 & 19.61 & -149.3 & 0 \\ -27.56 & 0 & 0 & 0 & 0 & 0 & 0 & 1.045\times10^{4} & -6.667 \end{bmatrix}$$

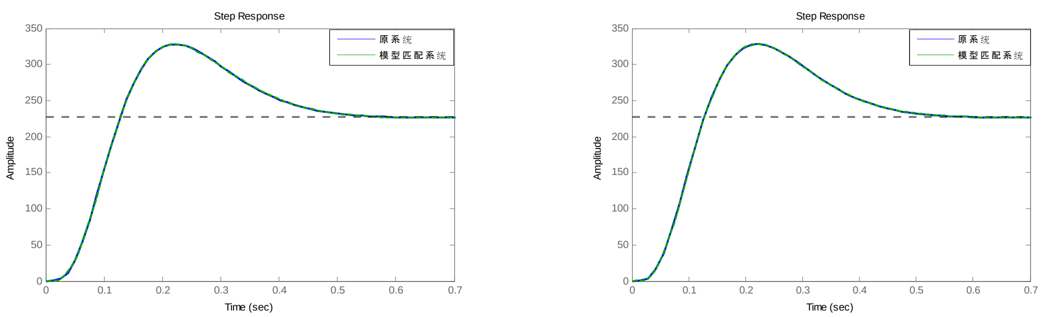
$$B^{T}=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C=\begin{bmatrix} 130 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D=0$$

As $T_1=I$ and $T_2=I$, output image for H_∞ model matching of system is shown as Figure 3(a), the model matching solution is:

$$K=\frac{152.9247(s+4.96)(s^2-255.7s+28050)}{(s^2+19.47s+141.7)(s^2+36.75s+659.7)}$$



(a) (b)

Figure 3. Output image of H_∞ model matching

As $T_1=\frac{1}{s+100}$ and $T_2=\frac{1}{s+5}$, step response for H_∞ model matching of the system is shown as fig.3 (b), the model matching solution is:

$$K=\frac{-12611.7073(s-3369)(s+2528)(s^2+7.705s+1639)}{(s+158.1)(s+41.78)(s+7.206)(s^2+27.46s+334.3)}$$

From the step response image of H_∞ model matching, it can be seen that the matching model got by order reduction method and the step response of the original system are completely consistent.

4. Conclusion

Using the principle of model order reduction to solve H_∞ model matching, from the step response curve, it can be seen that the system has good tracking ability. The controller got by this designing method has a certain practical application value, and model matching problem of high order system will be solved well.

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