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A Review to AC Modeling and Transfer Function of DC-DC Converters

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Abstract

In this paper, AC modeling and small signal transfer function for DC-DC converters are represented. The fundamentals governing the formulas are also reviewed. In DC-DC converters, the output voltage must be kept constant, regardless of changes in the input voltage or in the effective load resistance. Transfer function is the necessary knowledge to design a proper feedback control such as PID control to regulate the output voltage as linear PID and PI controllers are usually designed for DC-DC converters using standard frequency response techniques based on the small signal model of the converter.

Keywords: *DC-DC Converter, boost converter, buck converter, AC modeling, transfer function*

1. Introduction

DC-DC power converters are employed in a variety of applications, including power supplies for personal computers, office equipment, spacecraft power systems, laptop computers, and telecommunications equipment, as well as dc motor drives. In a DC-DC converter, the dc input voltage is converted to a dc output voltage having a magnitude differ from the input, possibly with opposite polarity or with isolation of the input and output ground references. Figure 1 shows a DC-DC converter as a black box. It converts a dc input voltage, v_g (t) , to a dc output voltage, v (t) , with a magnitude other than the input voltage. In a typical DC–DC converter application, the output voltage v(t) must be kept constant, regardless of changes in the input voltage v_g (t) or in the effective load resistance R. This is accomplished by building a circuit that varies the converter control input [i.e., the duty cycle d(t)] in such a way that the output voltage v(t) is regulated to be equal to a desired reference value.To design the control system of a converter, it is necessary to model the converter dynamic behavior. In particular, it is of interest to determine how variations in the power input voltage v_g(t), the load current, and the duty cycle d(t) affect the output voltage. Unfortunately, understanding of converter dynamic behavior is hampered by the nonlinear time-varying nature of the switching and pulse-width modulation process.

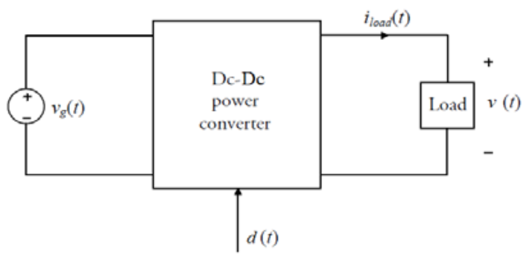

Figure 1. A DC-DC converter behavior

In particular, it is of interest to determine how variations in the power input voltage v_g(t), the load current, and the duty cycle d(t) affect the output voltage. Unfortunately, understanding of converter dynamic behavior is hampered by the nonlinear time-varying nature of the switching and pulse-width modulation process.

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These difficulties can be overcome through the use of waveform averaging and small signal modeling techniques.As illustrated in Figure 2, a controller block is an integral part of any power processing system. It is nearly always desired to produce a well-regulated output.

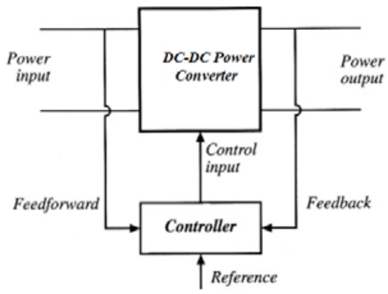

Figure 2. Required controller to DC-DC conveter

Since switching converters are nonlinear systems, it is desirable to construct small-signal linearized models. This is accomplished by perturbing and linearizing the averaged model about a quiescent operating point. Ac equivalent circuits can be constructed, in the same manner used in to construct dc equivalent circuits. The dc component of a converter waveform is given by its average value, or the integral over one switching period, divided by the switching period. Solution of a dc-dc converter to find its dc, or steady-state,voltages and currents therefore involves averaging the waveforms.

A typical DC–DC buck converter and feedback loop block diagram is illustrated in Figure 2 [11, 8, 2]. It is desired to design this feedback system in such a way that the output voltage is accurately regulated, and is insensitive to disturbances in $v_g(t)$ or in the load current. Additionally, the power stage and a feedback network feedback system should be stable, and characteristics such as transient overshoot, settling time, and steady-state regulation should meet specifications.

We are interested to design converters and their control systems such as Figure 3. To design the system of Figure 3, a dynamic model of the switching converter is required. What is the effect of variations in the power input voltage, the load current, or the duty cycle the output voltage? What are the small-signal transfer functions? To answer these questions, we will extend the steady-state models to include the dynamics introduced by the inductors and capacitors of the converter.

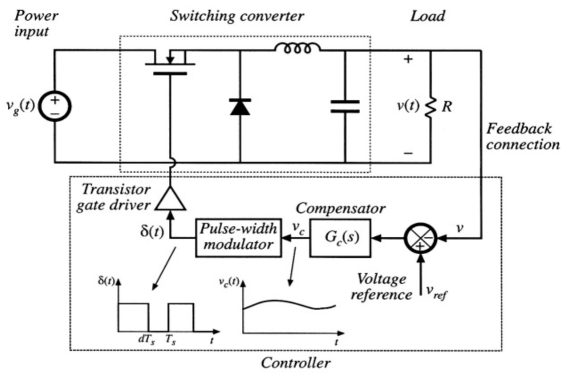

Figure 3. A simple dc-dc regulator system,including a buck converter

Modeling is the representation of physical phenomena by mathematical means. I n engineering, it is desired to model the important dominant behavior of a system, while neglecting other inconsequential phenomena. Simplified terminal equations of the component elements are used, and many aspects of the system response are neglected altogether, that is, they are "unmodeled."

The switching ripple is small in a well-designed converter operating in continuous conduction mode (CCM). Hence, we ignore the switching ripple, and model only the underlying ac variations in the converter waveforms. Suppose that some ac variation is introduced into the converter duty cycle d (t), such that:

$d(t)= D + D_m cos\ \omega_m t$

Where D and D_m are constants, | D_m << D | and the modulation frequency ω_m is much smaller than the converter switching frequency $\omega_x = 2\pi f_x$. The resulting transistor gate drive signal is shown in Figure 4(a), and a typical converter output voltage waveform v (t) is showen in Figure 4(b). The spectrum of v(t) is illustrated in Figure 4. This spectrum contains components at the switching frequency as well as its harmonics and sidebands; these components are small in magnitude if the switching ripple is small. In addition, the spectrum contains a low-frequency component at the modulation frequency ω_m.

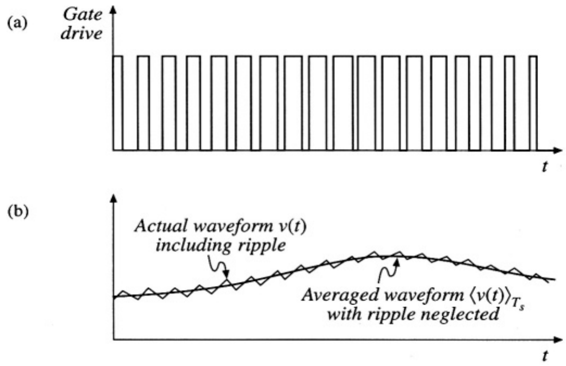

Figure 4. AC variation of the converter signals (a) transistor gate drive signal,and (b) the resulting converter output voltage waveform

The magnitude and phase of this component depend on the duty cycle variation, as well as the frequency response of the converter. If we neglect the switching ripple, then this low-frequency component remains [as illustrated in Figure 4(b)]. The objective of our ac modeling efforts is to predict this low-frequency component.

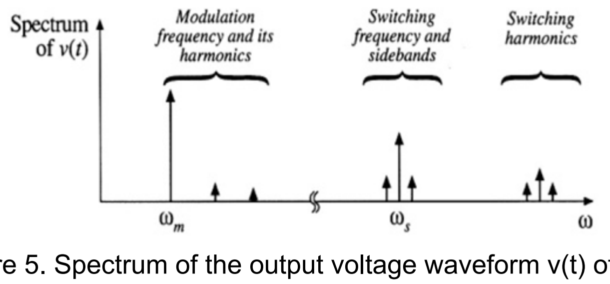

Figure 5. Spectrum of the output voltage waveform v(t) of Figure 4

The switching ripples in the inductor current and capacitor voltage waveforms are removed by averaging over one switching period. Hence, the low-frequency components of the inductor and capacitor waveforms are modeled by equations of the form:

$$L\frac{d\langle i_L(t)\rangle_{T_s}}{dt} = \langle v_L(t)\rangle_{T_s} \quad (1)$$

$$C\frac{d\langle v_C(t)\rangle_{T_s}}{dt} = \langle i_C(t)\rangle_{T_s}$$

Where $\langle x(t)\rangle_{T_s}$ denotes the average of x(t) over an interval of length T_s [11]:

$$\langle x(t)\rangle_T = \frac{1}{T_s}\int_t^{t+T_s} x(\tau)d\tau \quad (2)$$

So we will apply the basic approximation of removing the high-frequency switching ripple by averaging over one switching period.

In the next parts we will derive ac modeling of basic DC-DC converters and then by the approximation method mentioned above we will derive small signal transfer function of such converters.

2. AC Equivalent Circuit Modeling

2.1. The Basic AC Modeling Approach

Let us derive a small-signal ac model of the buck-boost converter of Figure 6 [9, 2]

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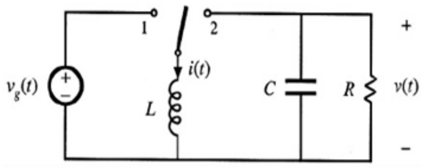


Figure 6. Buck-boost converter example

The analysis begins, by determining the voltage and current waveforms of the inductor and capacitor. When the switch is in position 1, the circuit of Figure 7(a) is obtained.

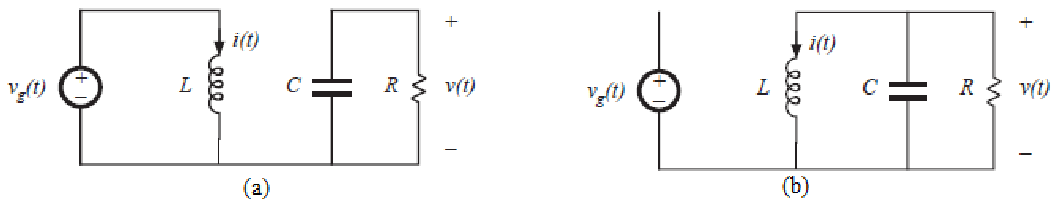


Figure 7. Buck-Boost Converter circuit: (a) switch is in position 1, (b) switch is in position 2

The inductor voltage and capacitor current are:

$$v_L(t) = L\frac{di(t)}{dt} = v_g(t)$$
$$i_C(t) = C\frac{dv(t)}{dt} = -\frac{v(t)}{R} \quad (3)$$

We now make the small-ripple approximation by replacing waveforms with their low-frequency averaged values

$$v_L(t) = L\frac{di(t)}{dt} \approx \langle v_g(t)\rangle_{T_s} \quad (4)$$
$$i_C(t) = C\frac{dv(t)}{dt} \approx -\frac{\langle v(t)\rangle_{T_s}}{R}$$

With the switch in position 2, the circuit of Figure 5(b) is obtained. Its inductor voltage and capacitor current are:

$$v_L(t) = L\frac{di(t)}{dt} = v(t)$$
$$i_C(t) = C\frac{dv(t)}{dt} = -i(t) - \frac{v(t)}{R} \quad (5)$$

Use of the small-ripple approximation, to replace $i(t)$ and $v(t)$ with their averaged values, yields:

$$v_L(t) = L\frac{di(t)}{dt} \approx \langle v(t)\rangle_{T_s}$$
$$i_C(t) = C\frac{dv(t)}{dt} \approx -\langle i(t)\rangle_{T_s} - \frac{\langle v(t)\rangle_{T_s}}{R} \quad (6)$$

Averaging the inductor voltage used Equation (2):

$$\langle v_L(t)\rangle_{T_s} = \frac{1}{T_s}\int_t^{t+T_s} v_L(\tau)d\tau \approx d(t)\langle v_g(t)\rangle_{T_s} + d'(t)\langle v(t)\rangle_{T_s} \quad (7)$$

By insertion this equation into Equation (1) leads to:

$$L\frac{d\langle i(t)\rangle_{T_s}}{dt} = d(t)\langle v_g(t)\rangle_{T_s} + d'(t)\langle v(t)\rangle_{T_s} \quad (8)$$

This equation describes how the low-frequency components of the inductor current vary with time. A similar procedure leads to the capacitor dynamic equation. Average capacitor current:

$$\langle i_C(t)\rangle_{T_s} = d(t)\left(-\frac{\langle v(t)\rangle_{T_s}}{R}\right) + d'(t)\left(-\langle i(t)\rangle_{T_s} - \frac{\langle v(t)\rangle_{T_s}}{R}\right) \quad (9)$$

Upon inserting this equation into Equation (1) and collecting terms, we will obtain:

$$C\frac{\langle v(t)\rangle_{T_s}}{dt} = -d'\langle i(t)\rangle_{T_s} - \frac{\langle v(t)\rangle_{T_s}}{R} \quad (10)$$

This is the basic averaged equation which describes dc and low-frequency ac variations in the capacitor voltage.

To derive a complete ac equivalent circuit model, it is necessary to write an equation for the average converter input current. Buck-boost input current waveform is:

$$i_g(t) = \begin{cases} \langle i(t)\rangle_{T_s} & during\ subinterval\ 1 \\ 0 & during\ subinterval\ 2 \end{cases}$$

Upon averaging over one switching period, we will obtain:

$$\langle i_g(t)\rangle_{T_s} = d(t)\langle i(t)\rangle_{T_s}$$

This is the basic averaged equation which describes dc and low-frequency ac variations in the converter input current. These equations are nonlinear because they involve the multiplication of time-varying quantities.

If the converter is driven with some steady-state, or quiescent, inputs:

$$d(t) = D$$
$$\langle v_g(t)\rangle_{T_s} = V_g$$

Then, after transients have subsided the inductor current, capacitor voltage, and input current:

$$\langle i(t)\rangle_{T_s}, \langle v(t)\rangle_{T_s}, \langle i_g(t)\rangle_{T_s}$$

Reach the quiescent values *I, V*, and *Ig*, given by the steady-state analysis as [8, 2]:

$$\boldsymbol{V} = -\frac{\boldsymbol{D}}{\boldsymbol{D}'}\boldsymbol{V_g}$$
$$\boldsymbol{I} = -\frac{\boldsymbol{V}}{\boldsymbol{D'R}}$$
$$\boldsymbol{I_g} = \boldsymbol{DI}$$

To construct a small-signal ac model at a quiescent operating point (*I*, *V*), one assumes that the input voltage $v_g(t)$ and the duty cycle *d(t)* are equal to some given quiescent values:

$$\langle \boldsymbol{v_g(t)} \rangle_{T_s} = \boldsymbol{V_g} + \boldsymbol{\hat{v}_g}$$
$$\boldsymbol{d(t)} = \boldsymbol{D} + \boldsymbol{\hat{d}(t)}$$

In response, and after any transients have subsided, the converter dependent voltages and currents will be equal to the corresponding quiescent values, plus small ac variations:

$$\langle \boldsymbol{i(t)} \rangle_{T_s} = \boldsymbol{I} + \boldsymbol{\hat{\imath}(t)}$$
$$\langle \boldsymbol{v(t)} \rangle_{T_s} = \boldsymbol{V} + \boldsymbol{\hat{v}(t)}$$
$$\langle \boldsymbol{i_g(t)} \rangle_{T_s} = \boldsymbol{I_g} + \boldsymbol{\hat{\imath}_g(t)}$$

If the ac variations are much smaller in magnitude than the respective quiescent values.

$$|\boldsymbol{\hat{v}_g(t)}| \ll |\boldsymbol{V_g}|$$
$$|\boldsymbol{\hat{d}(t)}| \ll |\boldsymbol{D}| \quad , \; |\boldsymbol{\hat{v}(t)}| \ll |\boldsymbol{V}|$$
$$|\boldsymbol{\hat{\imath}(t)}| \ll |\boldsymbol{I}| \qquad |\boldsymbol{\hat{\imath}_g(t)}| \ll |\boldsymbol{I_g}|$$

Then the nonlinear converter equations can be linearized. For the inductor equation, one obtains:

$$\boldsymbol{L}\frac{\boldsymbol{d(I+\hat{\imath}(t))}}{\boldsymbol{dt}} = \left(\boldsymbol{D} + \boldsymbol{\hat{d}(t)}\right)\left(\boldsymbol{V_g} + \boldsymbol{\hat{v}_g(t)}\right) + \left(\boldsymbol{D'} - \boldsymbol{\hat{d}(t)}\right)\left(\boldsymbol{V} + \boldsymbol{\hat{v}(t)}\right)$$

It should be noted that the complement of the duty cycle is given by:

$$d'(t) = \left(1 - d(t)\right) = 1 - \left(D + \hat{d}(t)\right) = D' - \hat{d}(t) \quad \text{with } D' = 1 - D$$

Multiply out and collect terms:

$$L\left(\frac{dI}{dt} + \frac{d\hat{\imath}(t)}{dt}\right) = \underbrace{\left(DV_g + D'V\right)}_{Dc\ terms}$$
$$+ \underbrace{\left(D\hat{v}_g(t) + D'\hat{v}(t) + \left(V_g - V\right)\hat{d}(t)\right)}_{\substack{1^{st} order\ ac\ terms \\ (linear)}} + \underbrace{\hat{d}(t)\left(\hat{v}_g(t) - \hat{v}(t)\right)}_{\substack{2^{nd}\ order\ ac\ terms \\ (nonlinear)}}$$

The derivative of *I* is zero, since *I* is by definition a dc (constant) term. For the purposes of deriving a small-signal ac model, the dc terms can be considered known constant quantities. On the right-hand side of equation three types of terms arise:

a) *Dc terms*: These terms contain dc quantities only.
b) *First-order ac terms*: Each of these terms contains a single ac quantity, usually multiplied by a constant coefficient such as a dc term. These terms are linear functions of the ac variations.
c) *Second-order ac terms*: These terms contain the products of ac quantities. Hence they are nonlinear, because they involve the multiplication of time-varying signals.

The second-order ac terms are much smaller than the first-order terms. So we will neglect second-order terms. Also, dc terms on each side of equation are equal. After discarding second-order terms, and removing dc terms (which add to zero), we will have:

$$L\frac{d\hat{i}(t)}{dt} = D\hat{v}_g(t) + D'\hat{v}(t) + \left(V_g - V\right)\hat{d}(t)$$

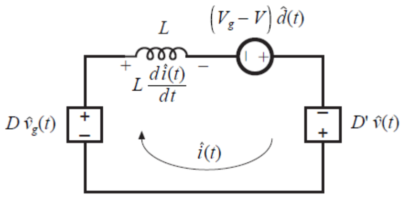


Figure 8. Circuit equivalent to the small-signal ac inductor loop equation

The capacitor equation can be linearized in a similar manner. So, we will have:

$$C\frac{d\hat{v}(t)}{dt} = -D'\hat{\imath}(t) - \frac{\hat{v}(t)}{R} + I\hat{d}(t)$$

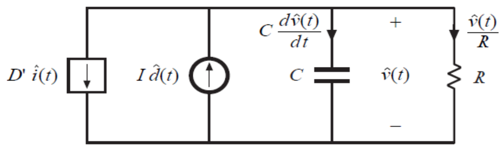


Figure 9. Circuit equivalent to the small-signal ac capacitor node equation

Finally, the equation of the average input current is also linearized.

$$I_g + \hat{\imath}_g(t) = \left(D + \hat{d}(t)\right)\left(I + \hat{\imath}(t)\right)$$

By collecting terms, we obtain:

$$\underbrace{I_g}_{Dc\ term} + \underbrace{\hat{\imath}_g(t)}_{1^{st}\ order\ ac\ term} = \underbrace{(DI)}_{Dc\ term} + \underbrace{\left(D\hat{\imath}(t) + I\hat{d}(t)\right)}_{1^{st}\ order\ ac\ terms(linear)} + \underbrace{\hat{d}(t)\hat{\imath}(t)}_{2^{nd} order\ ac\ term(nonlinear)}$$

We again neglect the second-order nonlinear terms. The dc terms on both sides of the equation are equal. The remaining first-order linear ac terms are:

$$\hat{\imath}_g(t) = D\hat{\imath}(t) + I\hat{d}(t)$$

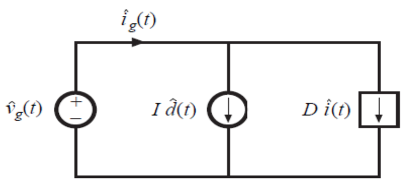


Figure 10. Circuit equivalent to the small-signal ac input source current equation

This is the linearized small-signal equation that describes the low-frequency ac components of the converter input current.

Collecting three circuit results:

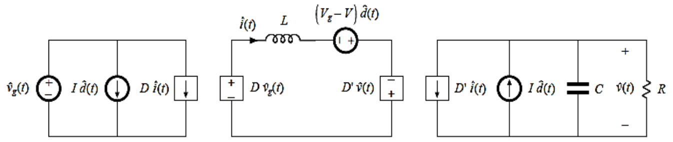


Figure 11. Buck- boost small-signal ac equivalent circuit

Combination of dependent sources into effective ideal transformer, leads to the final model.

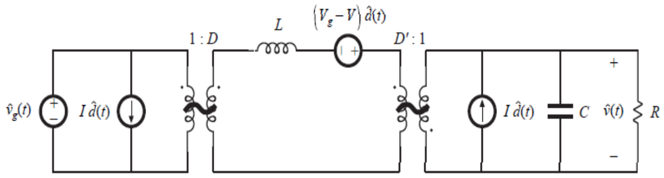
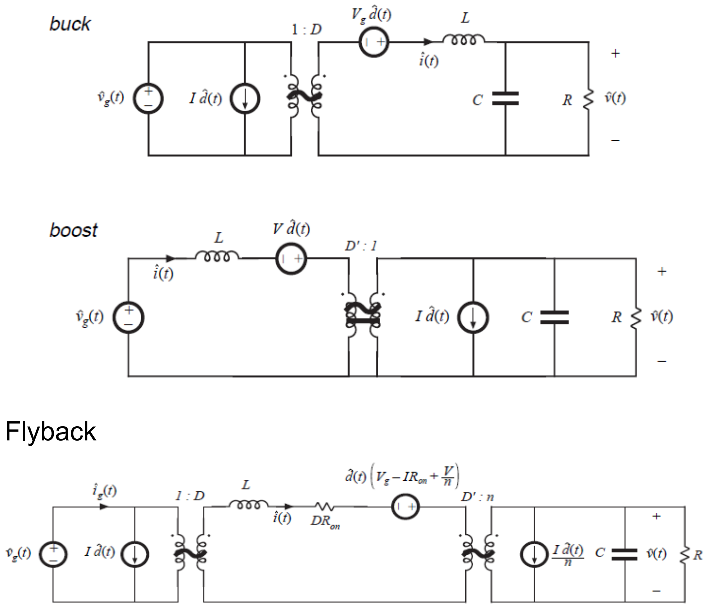


Figure 12. Final Small signal ac equivalent circuit model of the buck-boost converter

2.2. Results for Several Basic Converters



3. Converter Transfer Functions

3.1. Transfer Functions of the Buck-Boost Converter

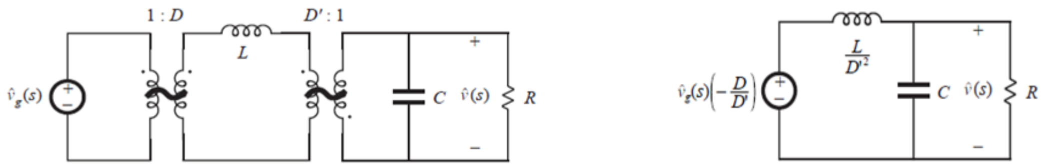
The converter contains two inputs $\hat{d}(s)$ and $\hat{v}_g(s)$ and one output, $\hat{v}(s)$.Hence, the ac output voltage variations can be expressed as the superposition of terms arising from the two inputs:

$$\hat{v}(t) = G_{vd}(s)\hat{d}(s) + G_{vg}(s)\hat{v}_g(s)$$

The control-to-output and line-to-output transfer functions can be defined as [5, 2], [13-14]:

$$G_{vd}(s) = \left.\frac{\hat{v}(s)}{\hat{d}(s)}\right|_{\hat{v}_s(s)=0} \quad and \quad G_{vg}(s) = \left.\frac{\hat{v}(s)}{\hat{v}_g(s)}\right|_{\hat{d}(s)=0}$$

To find the line-to-output transfer function we set the sources to zero as in Figure 13(a). We can then push the source and the inductor through the transformers, to obtain the circuit of Figure 13(b).



(a) Set $\hat{d}$ sources to zero (b) push inductor and $\hat{v}_g$ source through transformers

Figure 13. Manipulation of buck-boost equivalent circuit to find the line-to-output transfer function $G_{vg}(s)$

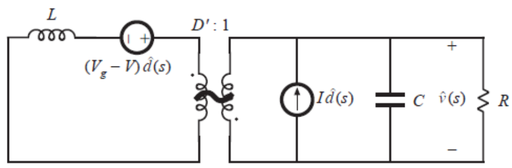
The transfer function is obtained using the voltage divider formula:

$$G_{vg}(s) = \frac{\hat{v}(s)}{\hat{v}_g(s)}\bigg|_{d(s)=0} = -\frac{D}{D'}\frac{\left(R\|\frac{1}{sC}\right)}{\frac{sL}{D'^2}+\left(R\|\frac{1}{sC}\right)} = \left(-\frac{D}{D'}\right)\frac{\left(\frac{R}{1+sRC}\right)}{\frac{sL}{D'^2}+\left(\frac{R}{1+sRC}\right)} = \left(-\frac{D}{D'}\right)\frac{R}{R+\frac{sL}{D'^2}+\frac{s^2RLC}{D'^2}} \quad (11)$$

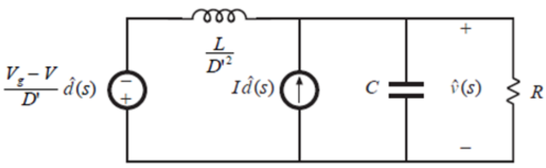
Which is the following standard form:

$$G_{vg}(s) = G_{s0}\frac{1}{1+\frac{s}{Q\omega_0}+\left(\frac{s}{\omega_0}\right)^2} \quad (12)$$

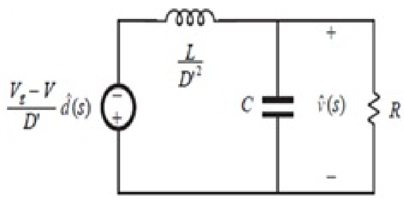
Derivation of the control-to-output transfer function $G_{vd}(s)$ is complicated. First, In small-signal model, set vg source to zero:



Then, push all elements to output side of transformer:

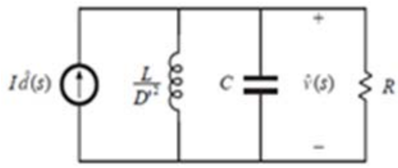


There are two d sources. One way to solve the model is to use superposition, expressing the output v as a sum of terms arising from the two sources. With voltage source only:



$$\frac{\hat{v}(s)}{\hat{d}(s)} = \left(-\frac{V_g - V}{D'}\right)\frac{\left(R\|\frac{1}{sC}\right)}{\frac{sL}{D'^2}+\left(R\|\frac{1}{sC}\right)}$$

With the current source alone:



$$\frac{\hat{v}(s)}{\hat{d}(s)} = \boldsymbol{I}\left(\frac{sL}{D'^2}\|\boldsymbol{R}\|\frac{1}{sC}\right)$$

The transfer function is the sum of above equations:

$$\boldsymbol{G_{vd}(s)} = \left(-\frac{V_{g-V}}{D'}\right)\frac{\left(R\|\frac{1}{sC}\right)}{\frac{sL}{D'^2}+\left(R\|\frac{1}{sC}\right)} + \boldsymbol{I}\left(\frac{sL}{D'^2}\middle\| \boldsymbol{R}\|\frac{1}{sC}\right)$$

By algebraic manipulation, we will have:

$$\boldsymbol{G_{vd}(s)} = \frac{\hat{v}(s)}{\hat{d}(s)}\bigg|_{\hat{v}_g(s)=0} = \left(-\frac{V_g - V}{D'^2}\right)\frac{\left(1 - s\frac{LI}{V_g - V}\right)}{\left(1 + s\frac{L}{D'^2 R} + s^2\frac{LC}{D'^2}\right)}$$

This equation is of the form:

$$G_{vd}(s) = G_{d0}\frac{\left(1-\frac{s}{\omega_0}\right)}{\left(1+\frac{s}{Q\omega_0}+\left(\frac{s}{\omega_0}\right)^2\right)} \tag{13}$$

The dc gain is:

$$\boldsymbol{G_{d0}} = -\frac{V_g - V}{D'} = -\frac{V_g}{D'^2} = \frac{V}{DD'}$$

The angular frequency of the zero is:

$$\boldsymbol{\omega_z} = \frac{V_g - V}{LI} = \frac{D'R}{DL} \qquad (\boldsymbol{RHD})$$

And:

$$\omega_0 = \frac{D'}{\sqrt{LC}}, Q = D'R\sqrt{\frac{C}{L}}$$

Simplified using the dc relationships:

$$V = -\frac{D}{D'}V_g, I = -\frac{V}{D'R}$$

3.2. Transfer Functions of some Basic CCM Converters

The prominent features of the line-to-output and control-to-output transfer functions of the basic buck, boost, and buck-boost converters are summarized in Table 1. In each case, the control-to-output transfer function is of the form of Equation (13) and the line-to-output transfer function is of the form of Equation (12).

Table 1. Salient features of the small-signal CCM transfer functions of some basic dc-dc converters

Converter	G_{g0}	G_{d0}	ω_0	Q	ω_z
buck	D	$\frac{V}{D}$	$\frac{1}{\sqrt{LC}}$	$R\sqrt{\frac{C}{L}}$	∞
boost	$\frac{1}{D'}$	$\frac{V}{D'}$	$\frac{D'}{\sqrt{LC}}$	$D'R\sqrt{\frac{C}{L}}$	$\frac{D'^2R}{L}$
Buck-boost	$-\frac{D}{D'}$	$\frac{V}{DD'^2}$	$\frac{D'}{\sqrt{LC}}$	$D'R\sqrt{\frac{C}{L}}$	$\frac{D'^2R}{DL}$

4. Conclusion

We represented the ac equivalent circuit modeling and small signal transfer function for DC-DC converters by illustrating fundamentals governing the formulas. The objective of our ac modeling efforts was to predict low frequency component. To achieve this purpose, we applied the basic approximation of removing the high-frequency switching ripple by averaging over one switching period and then we derived transfer functions of DC-DC converters.

References

[1] RP Severns, G Bloom. *Modern DC-To-DC Switchmode Power Converter Circuits.* Van Nostrand Reinhold. 1985.

[2] RW Erickson, D Maksimovic. Fundamentals of Power Electronics. Norwell, MA: Kluwer, 2001.

[3] L Guo, RM Nelms, JY Hung. Comparative evaluation of linear PID and fuzzy control for a boost converter. *Proc. 31st Annu. Conf. IEEE Ind. Electron. Soc.*, Raleigh, NC. 2005; 555–560.

[4] Liping Guo, John Y Hung, RM Nelms. Evaluation of DSP-Based PID and Fuzzy Controllers for DC–DC Converters. IEEE Trans. 2009; 2237.

[5] RD Middlebrook. Low Entropy Expressions: The Key to Design-Oriented Analysis. *IEEE Frontiers in Education Conference.* 1991; 399-403.

[6] F Barzegar, RD Middlebrook. Using Small Computers to Model and Measure Magnitude and Phase of Regulator Transfer Functions and Loop Gain. *Proceedings of Powercon.*1981

[7] A Kislovski, R Redl, N Sokal. *Dynamic Analysis of Switching-Mode DC/DC Converters.* New York: Van Nostrand Reinhold, 1994.

[8] R Tymerski, V Vorperian. Generation, Classification and Analysis of Switched-Mode DC-to-DC Converters by the Use of Converter Cells. *Proceedings of the 1986 International Telecommunications Energy Confere* (INTELEC'86). 1986; 181-195.

[9] V Vorperian, R Tymerski, FC Lee. Equivalent Circuit Models for Resonant and PWM Switches. *IEEE Transactions on Power Electronics*. 1989; 4(2): 205-214.

[10] V Vorperian. Simplified Analysis of PWM Converters Using the Model of the PWM Switch: Parts I and II. *IEEE Transactions on Aerospace and Electronic Systems*. AES-26. 1990; 490-505.

[11] PT Krein, J Bentsman, RM Bass, BC Lesieutre. On the Use of Averaging for the Analysis of Power Electronic Systems. *IEEE Transactions on Power Electronics*. 1990; 5(2):.182-190.

[12] Seth R Sanders, George C Vergese. Synthesis of Averaged Circuit Models for Switched Power Converters. *IEEE Transactions on Circuits and Systems*. 1991; 38(8): 905-915.

[13] RD Middlebrook. Low Entropy Expressions: The Key to Design-Oriented Analysis. *IEEE Frontiers in Education Conference.* 1991; 399-403.

[14] RD MIddlebrook. *Methods of Design-Oriented Analysis: The Quadratic Equation Revisited.* IEEE Frontiers in Education Conference. 1991; 95-102.