Optimal Warranty Policy Considering Two-dimensional Imperfect Preventive Maintenance for Repairable Product

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Abstract

For some kinds of products, such as car, aircraft, government acquisition, the consumers have strict requirements to the reliability of these products. Then the manufacturer is inclined to provide the two-dimensional preventive maintenance policy to take the usage degree of the product into account. As a result, two-dimensional preventive maintenance policy in the warranty period has recently obtained increasing attention from manufacturers and consumers. In this paper, we focused on the optimization of based warranty cost and proposed a new expected cost model of the two-dimensional imperfect preventive maintenance policy from the perspective of the manufacture. The optimal two-dimensional preventive maintenance was obtained by minimizing based warranty cost. And asymmetric copula function was applied to modeling the failure function of the product. At last, numerical examples are given to illustrate the proposed models, of which the results prove the model effective and validate.

Keywords: two-dimensional warranty, two-dimensional imperfect preventive maintenance, based warranty, extended warranty, asymmetric copula function

1. Introduction

In general, warranty has two significant roles as "protector" and "promoter" to consumer and manufacturer when the products fail to perform their pre-specified functions during the warranty period [1-2]. According to Murthy and Blischke [3], there are only two kinds of warranty in the product life cycle including based warranty and extended warranty. Based warranty is sold with product. Its cost is usually included in the sale price. However, to have extended warranty, the customer should pay extra money to get the service after based warranty expires. Nowadays, either for based warranty or extended warranty, one key issue of interest that arises from warranty analysis is the modeling of warranty cost. By optimizing warranty cost, the manufacturer could price the warranty proper and make their product more attractive to consumers. For some kinds of products (such as aircraft, government acquisition), the consumers have strict requirements to the reliability of these products. So the manufacturer is inclined to provide the two-dimensional preventive maintenance policy to take the usage degree of the product into account, which has two limits of coverage in terms of age and usage. For example, a sold car may be covered a 3 year and 30,000 mile warranty, and the car should be check up in 4s(Sale, Spare part, Service and Survey) store at per year and 15,000 mile. From the perspective of the consumers, two-dimensional preventive maintenance policy might be better than one-dimensional warranty for its one more boundary that could make the products be maintained in time. However, it could also place heavier burden on the manufacturers. So the manufacturers should price the warranty properly to make their products more attractive and guarantee their profit. As a result, how to minimizing the warranty cost in the two-dimensional warranty becomes a practical requirement. To our knowledge, few analytical or numerical methods have been reported in the literature, which offers a decision framework to optimize the two-dimensional preventive maintenance period in the warranty period. In this perspective, we develop in this paper a new mathematical model to find the optimized preventive maintenance policy for based warranty.

The rest of this paper is organized as follows: Section 2 introduces the relative work existing in the literature with product warranty policies. Section 3 outlines the model assumptions and notation. Section 4 is dedicated to development of the mathematical model. A numerical example illustrates our approach to prove the model validate and effective in Section 5.

2. Literature Review

Different warranty policies has different warranty cost. The characteristics of warranty policies are essential factors in determining warranty cost including warranty service object, warranty renewal mechanism and warranty dimension etc. Blischke and Murthy [4] formulated a classic taxonomy for warranty policies. But when the products are becoming more complicate, the warranty policies have some changes. So based on the system of the warranty policies [5, 18, 24], we further develop the taxonomy for warranty policies, shown as Figure 1. At first, warranty policies could be divided into two types including single-component warranty [9, 19] and multi-component warranty. Multi-component warranty often considers a fleet of products. Then the warranty could be divided into two sub-groups as renewing warranty and non-renewing warranty. Renewing warranty policy means that the warranty period will restart at the failure time if the failure occurs in the warranty period [1, 21]. Non-renewing warranty policy will end at the warranty period W no matter how many failures happen in the warranty period [2, 27]. Simple warranty policy has only one warranty policy will be applied in the warranty period [22], and different warranty policies will be applied in the period of combined warranty policy [16]. According to the warranty boundaries, the warranty policies could be further divided into one-dimensional warranty, two-dimensional warranty and n-dimensional warranty ($n \geq 3$). One-dimensional warranty is limited by time [23], and two-dimensional warranty is usually limited by age and usage (such as miles and revolutions) [8, 10, 26]. For some products, which require high reliability, multi usage limits will be applied to restrain the warranty period. Then the warranty could be n-dimensional warranty. There are few literatures about n-dimensional warranty. However, along with the industrial product become more complex, the interesting of the n-dimensional warranty would increase. Obviously warranty policy is the result of the game between manufacturer and consumer. So the cost allocation mechanism will greatly influence the warranty decision and effect. And the warranty policy could be further divided into free-repair warranty, pro-rate warranty and rebate warranty etc by cost allocation mechanism. The manufacturer will take all expense in the free-repair warranty period. On the contrary, the warranty cost will be share by manufacturers and consumers in the period of pro-rata warranty or rebate warranty [7].
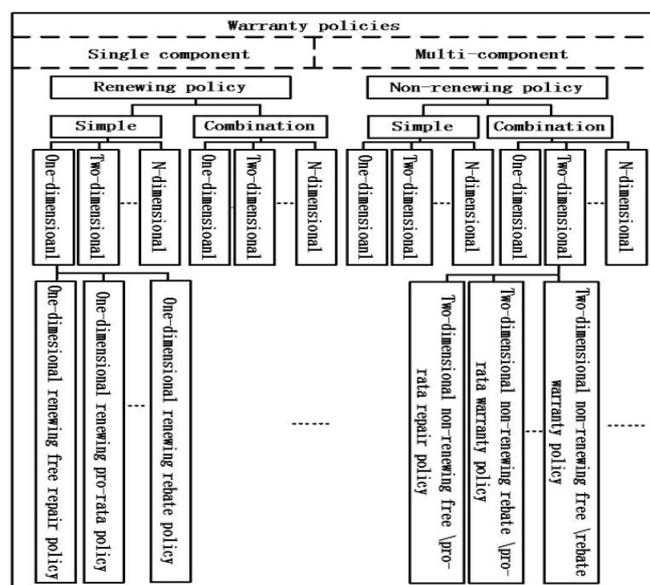

Figure 1. The classification of warranty policies

Obliviously, two-dimensional warranty is one of the main contents of the warranty policy For two-dimensional warranty, there are three methods that has been developed for analyzing two-dimensional warranty [29], ie univariate method [14, 19, 26], bivariate method [8] and time scale method [12]. Bivairate method is simpler and more straightforward. And one way to formulate the bivariate distribution is making full use of Copula function. Copulas are a tool for constructing multivariate distributions and describing the dependence between random variables Furthermore symmetric copula function were already applied to model the product reliability. In this paper, we propose the cost model of the two-dimensional preventive maintenance in the warranty period by utilizing asymmetric copula function. And the optimal two-dimensional preventive maintenance period is obtained by minimizing the warranty cost. The advantages of the model proposed are as follows.

      a) It is able to handle the tail dependence of age and usage to calculate two-dimensional warranty cost which could be more accurate. The asymmetric copula function is applied rather than assuming the age and usage have linear relationship.

      b) The two-dimensional preventive maintenance period could be obtained by minimizing based warranty cost. It is more realistic that the PM action is also two-dimensional to make the product more reliable.

3. Model Assumptions and Notation

This section provides model assumptions and notation.

(T_0, U_0)	The two-dimensional preventive maintenance (PM) period implemented
(W_B, U_B)	The based warranty time and usage limit $\frac{U_B}{W_B} = \frac{U_0}{T_0} = r_0$
$\Delta \ (0 \leq \Delta \leq 1)$	PM level. When $\Delta = 1$, PM is perfect maintenance. When $\Delta = 0$, PM is minimal repair.
C_p	The cost of PM
C_m	The cost of CM

Assuming the usage of the product is relatively steady for individual customer. It doesn't mean the usage for individual customer doesn't change but change little. The based warranty coverage area could be divided into two part: D_1 and D_2. For D_1, warranty is likely terminated at usage limit. On the contrary, warranty is likely terminated at time limit for D_2.
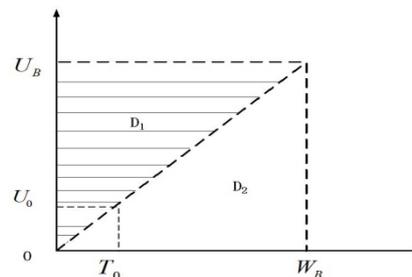

Figure 2. Two dimensional warranty coverage

Preventive maintenance is imperfect repair and the corrective maintenance (CM) is minimal repair. Compared to the mean time between failures, the maintenance time is negligible. A failure can be detected immediately which results in an immediate claim, then the manufacturer will respond all the claims. Assuming a failure could be detected immediately which results in an immediate claim and the manufacturer will respond all the claims. In this study, the warranty cost is composed of preventive maintenance cost and corrective maintenance cost. The cumulative failure distribution function, failure density function and failure rate of the product are [20]:

$$F(t,u) = \bar{C}(F_1(t), F_2(u), \Theta)$$

$$= p_0 C(F_1(t), F_2(u), \theta_1) + p_1 \left(F_2(u) - C(1 - F_1(t), F_2(u), \theta_2) \right) + p_2 \left(F_1(t) - C(F_1(t), 1 - F_2(u), \theta_3) \right) \tag{1}$$

$$f(t,u) = \frac{\partial F(t,u)}{\partial t \partial u} \, , \, h(t,u) = \left(\frac{\partial F(t,u)}{\partial t \partial u} \right) \Big/ \left(1 - F(t,u) \right) \tag{2}$$

$\bar{C}(F_1(t), F_2(u), \Theta)$ is the asymmetric copula function and $\sum_{i=0}^{2} p_i = 1$. $C(F_1(t), F_2(u), \theta_1)$ is symmetric copula function, which should be selected by historical warranty data. The common simple symmetric copula functions include Gaussian copula, t-copula, Gumbel copula, Clayton copula and Frank copula etc. Gumbel copula is:

$$C^{'}(F_1(t), F_2(u), \theta) = \exp\left\{ -\left[(-\ln(F_1(t))^\theta + (-\ln(F_2(u))^\theta \right]^{1/\theta} \right\} \tag{3}$$

For the two-dimensional imperfect preventive policy, the failure rate right after performing the jth PM action is given by Equation (4). Obviously, the PM action effect is the reduction of failure intensity [11]. $n_1 = \lfloor U_B/U_0 \rfloor, n_2 = \lfloor W_B/T_0 \rfloor (\lfloor * \rfloor$: Integer part of a real number).

$$\begin{cases} \text{PM at } jT_0 & h^+(jT_0, u) = (1-\Delta)h^-(jT_0, jT_0 r_0) \\ \text{PM at } jU_0 & h^+(t, jU_0) = (1-\Delta)h^-(jU_0/r_0, jU_0) \end{cases} \quad j=1,2,\dots \tag{4}$$

4. Analytical Warranty Cost Analysis

For two-dimensional warranty, there are two possible situations that basic warranty expires. One is the usage of the product exceeds the mileage limitation. And the other is the age of the product exceeds the time limitation. So the warranty cost includes two parts: D_1 and D_2. For the warranty coverage of D_1, let N_t be the number of PM observed up to time t. Then the failure rate between two PM action is:

$N_t = 0$, $h(t,u)$;

$N_t = 1$, $h(t,u) - \Delta h(U_0/r_0, U_0)$;

…

$N_t = j$, $h(t,u) - \Delta \sum_{i=0}^{j-1} (1-\Delta)^i h\left(\frac{(j-i)U_0}{r_0}, (j-i)U_0 \right)$

The cost of the warranty including two-dimensional imperfect preventive maintenance is:

$$C_{D_1} = C_p n_1 + \sum_{j=0}^{n_1-1} \int_{jU_0}^{(j+1)U_0} \int_0^{u/r_0} C_m \left[h(t,u) - \Delta \sum_{i=0}^{j-1} (1-\Delta)^i h\left(\frac{(j-i)U_0}{r_0}, (j-i)U_0 \right) \right] dt dt u$$

$$+ \int_{n_1 U_0}^{U_B} \int_0^{u/r_0} C_m \left[h(t,u) - \Delta \sum_{i=0}^{n_1-1} (1-\Delta)^i h\left(\frac{(n_1-i)U_0}{r_0}, (n_1-i)U_0 \right) \right] dt dt u \tag{5}$$

At the same principle, the cost of the warranty including two-dimensional imperfect preventive maintenance for the warranty coverage of D_2 is:

$$C_{D_2} = C_p n_2 + + \sum_{j=0}^{n_2-1} \int_{jT_0}^{(j+1)T_0} \int_0^{r_0 t} C_m \left[h(t,u) - \Delta \sum_{i=0}^{j-1} (1-\Delta)^i h\left((j-i)T_0, (j-i)T_0 r_0 \right) \right] du dt$$

$$+ \int_{n_2 T_0}^{W_B} \int_0^{r_0 t} C_m \left[h(t,u) - \Delta \sum_{i=0}^{n_2-1} (1-\Delta)^i h\left((n_2-i)T_0, (n_2-i)T_0 r_0 \right) \right] du dt \tag{6}$$

So the total expected based warranty cost is:

$$E(C_B) = C_{D_1} \frac{\iint\limits_{D_1} f(t,u)dtdu}{\iint\limits_{D_1} f(t,u)dtdu + \iint\limits_{D_2} f(t,u)dtdu} + C_{D_2} \frac{\iint\limits_{D_2} f(t,u)dtdu}{\iint\limits_{D_1} f(t,u)dtdu + \iint\limits_{D_2} f(t,u)dtdu} \tag{7}$$

5. Numerical Example

We use the asymmetric copula proposed by Wu(2014). The symmetric copula function which is applied to construction the asymmetric copula is:

$$C(F_1(t), F_2(u)) = F_1(t) + F_2(u) - 1 + C_0(1 - F_1(t), 1 - F_2(u))$$

$$C_0(x, y) = \exp\left\{-\left[\left(-\ln(x)\right)^\theta + \left(-\ln(y)\right)^\theta\right]^{1/\theta}\right\}$$ is the Gumbel copula. Suppose $C_p = 150$, $C_m = 820$ and $\Delta = 0.36$. The based warranty and extended warranty coverage are (5, 100000) and (10, 200000). The life of the product is(30, 600000). $p_0 = 0.9$, $p_1 = 0.1$, $\theta_1 = 3.76$ and $\theta_2 = 0.48$, $\alpha_1 = 0.78$, $\alpha_2 = 0.77$, $\beta_1 = 2.6$, $\beta_2 = 48400$. The marginal cumulative failure distributions are:

$$F_1(t) = 1 - \exp\left\{-\left(\frac{t}{\beta_1}\right)^{\alpha_1}\right\}, F_2(u) = 1 - \exp\left\{-\left(\frac{u}{\beta_2}\right)^{\alpha_2}\right\}$$

In the based warranty, the optimal preventive maintenance period could be obtained by minimize the based warranty shown as Figure 3. And the partial results showed as Table 1. The optimal warranty cost is 513.1398 when the preventive maintenance period is (1.2, 24000). From results below, we could find the warranty cost will decreases at first then increases with the increasing of T_0 or U_0.
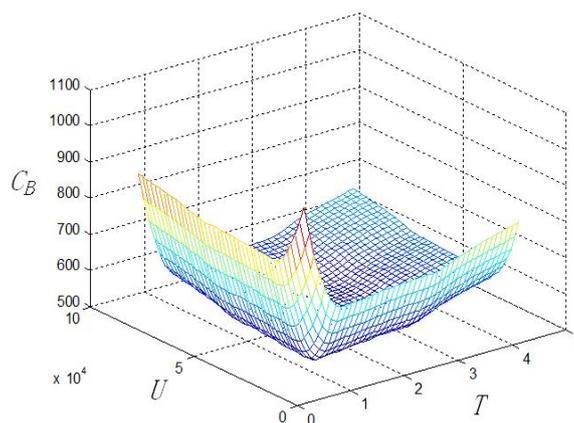

Figure 3. Warranty cost curve of different two-dimensional preventive maintenance period

Table 1. Warranty cost of different two-dimensional preventive maintenance period

Cost T_0									
U_0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
10000	1000.7	725.8	701.2	708.8	715.2	720.4	749.8	772.5	790.9
20000	835.2	560.3	535.6	543.3	549.6	554.8	584.3	607.0	625.4
30000	820.4	545.5	520.8	528.5	534.8	540.0	569.4	592.2	610.5
40000	825.0	550.1	525.5	533.2	539.5	544.7	574.1	596.9	615.2
50000	828.8	554.0	529.3	537.0	543.3	548.5	577.9	600.7	619.0
60000	832.0	557.1	532.4	540.1	546.4	551.6	581.1	603.8	622.2
70000	849.7	574.8	550.2	557.9	564.2	569.4	598.8	621.6	639.9
80000	863.4	588.6	563.9	571.6	577.9	583.1	612.5	635.3	653.6
90000	874.5	599.6	574.9	582.6	589.0	594.2	623.6	646.3	664.7

Then we analyze the sensitive of the based warranty cost model by changing α shown as Figure 4. We could notice some interesting phenomena. Warranty cost of different PM interval change more by increasing α_1 and α_2. And the optimal warranty cost is increasing when α_1 is increasing. On the contrary, it is decreasing when α_2 is increasing. We come to the conclusion that α_1 and α_2 have opposite influence on the based warranty cost.

$\alpha_1 = 0.58, \alpha_2 = 0.77$
$MinC_B = 505.0659 \, at \, (2.9, 58000)$

$\alpha_1 = 0.98, \alpha_2 = 0.77$
$MinC_B = 512.7420 \, at \, (1.3, 26000)$

$\alpha_1 = 0.78, \alpha_2 = 0.57$
$MinC_B = 553.5066 \, at \, (2.9, 58000)$

$\alpha_1 = 0.78, \alpha_2 = 0.97$
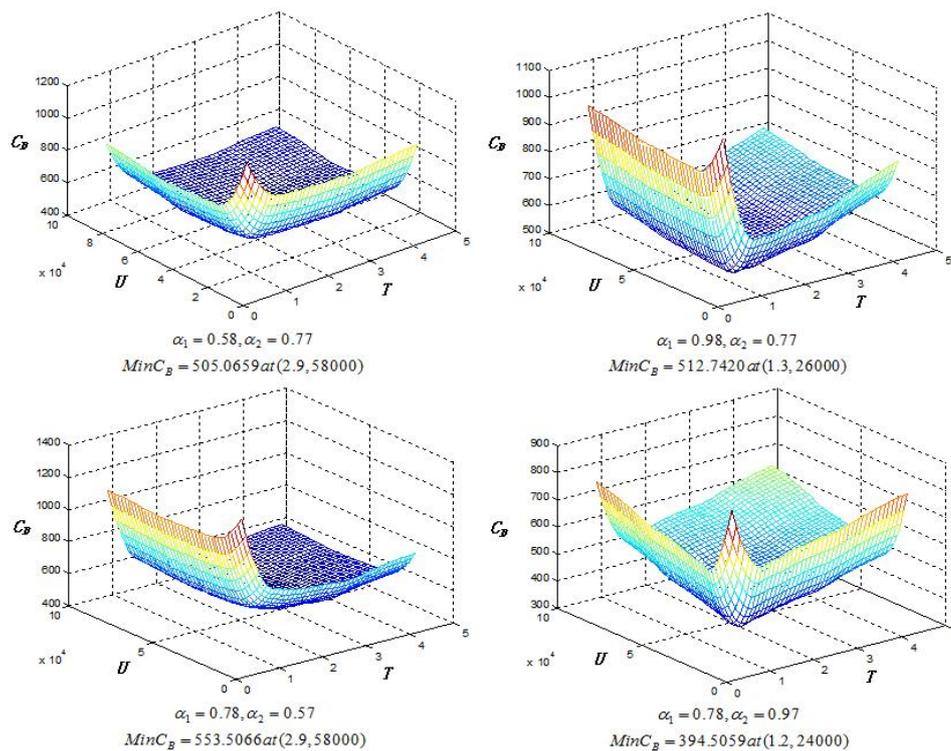$MinC_B = 394.5059 \, at \, (1.2, 24000)$

Figute 4. Sensitive analyze of based warranty cost model considering different α

Then we analyze the sensitive of the based warranty cost model by changing β shown as Figure 5. And the optimal warranty cost is decreasing when β_1 and β_2 is increasing. We come to the conclusion that β_1 and β_2 have the uniform influence on the based warranty cost.
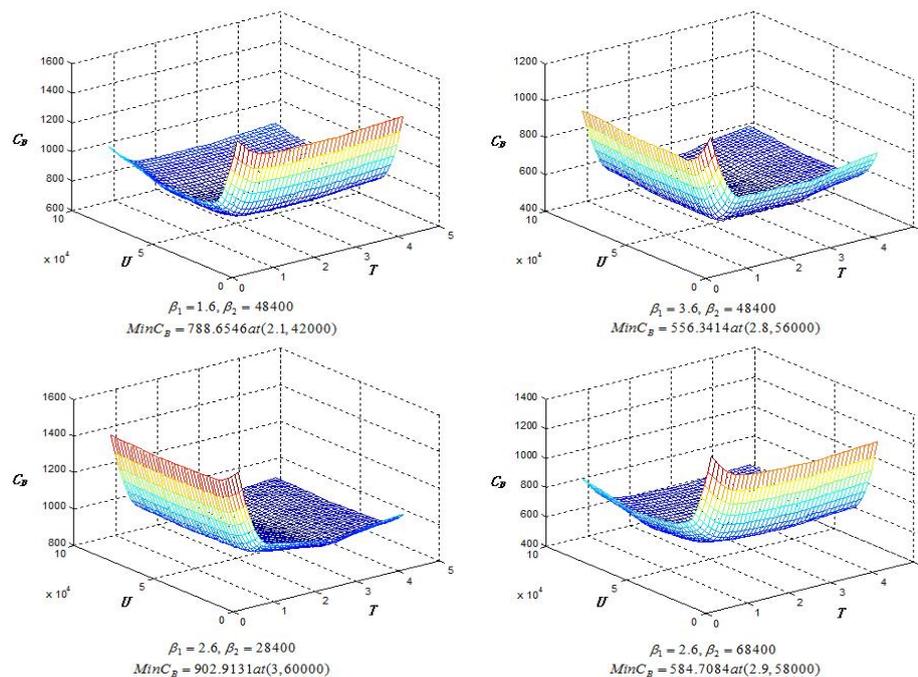
Figure 5. Sensitive analyze of based warranty cost model considering different β

6. Conclusion

This article proposes a method to find optimal two-dimensional preventive maintenance period by minimizing the warranty cost. The method proposed could help the manufacturer to price the based warranty properly. Many extensions to this work may be considered. The method of failure reduction models have to be further investigated. And the parameters of copula function have to be theoretically studied. In order to check models validity, some goodness-of-fit tests should be developed. At last, imperfect corrective maintenance should be included in these models.

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