Indonesian Journal of Electrical Engineering and Computer Science
Vol. 2, No. 3, June 2016, pp. 486 $\sim$ 500
DOI: 10.11591/ijeecs.v2.i3.pp486-500

Routh Approximation: An Approach of Model Order Reduction in SISO and MIMO Systems

D. K. Sambariya* and Omveer Sharma
Department of Electrical Engg., Rajasthan Technical University
Rawatbhata Road, Kota, 324010, India
*Corresponding author, e-mail: dsambariya_2003@yahoo.com

Abstract

In this paper the Routh Approximation method is explored for getting the reduced order model of a higher order model. The reduced order modeling of a large system is necessary to ease the analysis of the system. The approach is examined and compared to single-input single-output (SISO) and multi-input multi-output (MIMO) systems. The response comparison is considered in terms of step response parameters and graphical comparisons. It is reported that the reduced order model using proposed Routh Approximation (RA) method is almost similar in behavior to that of with original systems.

Keywords: *Model order reduction (MOR), Single-input single-output (SISO), Multi-input multi-output (MIMO), Routh Approximation method.*

1. Introduction

The analysis of high order systems (HOS) is generally very much complicated and costly. On other hand it became easy to analysis of lower order system [1, 2]. The reduced models for the original high order system is achieved by using mathematical optimization procedures or simplification procedures based on physical considerations [3]. Thus analysis, synthesis and simulation of reduced low order systems is easier and practicable as compared to it's high order systems [4]. An approach to get reduced order model of a higher order system using time-moments method is presented by [5]. The reduced model have a problem of stability because of mathematical approximation in model reduction technique. The reduced model may be unstable even though the high order system is stable [6].

The instability problem of reduced models was studied by Hutton [7], Shamash [8], Gutman et. al. [9] and Wan [10]. Some method based on stability criterion and other not based on stability criterion but the reduced model for a stable high order system (HOS) is always stable [11, 12]. Different methods give different approach some gives batter result in rise time, some gives batter results in settling time [13]. The combination of these methods gives batter results. The reduced model using combination of methods is nearly with its higher order system. The combination of Routh approximation and particle swarm optimization (PSO) is presented in [14]. The concept of preservation of stability is presented in [15]. The differentiation method for reduction of systems is presented in [16]. The differentiation method is used to derive reduced order model of single machine infinite bus power system in [17]. The application of Routh stability algorithm is presented in [18, 19].

The application of soft computing techniques have been presented in literature in the field of model order reduction [20]. The concept used is minimization of integral squared error using bat algorithm [20]. The application of fire fly algorithm in model order reduction is presented in [21]. The application of particle swarm optimization (PSO) is presented in [22]. The application of Routh approximation with Cuckoo search algorithm for model order reduction is presented in [23]. The hybrid application of stability equation method with self-adaptive bat algorithm to reduce power system to a reduced model is presented in [24].

Received January 1, 2016; Revised April 24, 2016; Accepted May 10, 2016

In this paper the application of Routh approximation method is presented for deriving reduced order model of the higher order LTI systems which includes bechmark problems. The statement of problem is presented in section 2.. The detailed procedural steps on Routh approximation method are included in section 3.. The systems under consideration and their reduced order models are presented in section 4.. The results of original system and reduced models are subjected to step input and compared in this section. Finally the manuscript is concluded in section 5. and followed by references.

2. Problem Formulation

Consider a high order transfer function of a system represented as in Eq. 1.

$$G(s) = \frac{\sum\limits_{i=0}^{n-1} b_i s^i}{\sum\limits_{i=0}^{n} a_i s^i} \tag{1}$$

where, the $G(s)$ represents a high order system with the order of n. The purpose of manuscript is to reduce the order of such high order system to r. The reduced order model may be represented as in Eq. 2.

$$R(s) = \frac{\sum\limits_{j=0}^{r-1} d_j s^j}{\sum\limits_{j=0}^{r} c_j s^j} \tag{2}$$

where, a_i, b_i, c_j and d_j are the scalar constants of original high order system and the reduced order system. The objective is to find a reduced r^{th} order system model $R(s)$ such that it retains the important properties of $G(s)$ for the same types of inputs.

3. Review on Routh approximation

This method number of useful properties like if original system is stable then reduce model will be stable,converge monotonically of original system in terms of step and impulse response. By increase order of approximation poles and zeros of the approximants move towards the poles and zeros of the original. In this method Routh Table for original system is use to construct the approximate in a manner that it will stable for stable original system [22].

3.1. Description of Method

$$G(s) = \frac{b_n s^{(n-1)} + b_n s^{(n-2)} + \ldots + b_1}{a_n s^n + a_{(n-1)} s^{(n-1)} + \ldots + a_0} \tag{3}$$

By taking reciprocal of Eq. 3 and shown in Eq. 4

$$\hat{G}(s) = \frac{1}{s} G\left(\frac{1}{s}\right) = \frac{b_1 s^{(n-1)} + \ldots + b_n}{a_0 s^n + a_1 s^{(n-1)} + \ldots + a_n} \tag{4}$$

If s_i, represents the i^{th} pole/zeros of the original system then $1/s_i$, the i^{th} poles/zeros of the reciprocal system.

3.2. Alpha-Beta expansion

The transfer function of Eq. 4 can be expanded in the canonical form as presented in Eq. 5.

$$\begin{aligned}\hat{\mathrm{G}}(\mathrm{s}) &= \begin{cases} \beta_1\mathrm{F}_1(\mathrm{s})+\beta_2\mathrm{F}_1(\mathrm{s})\mathrm{F}_2(\mathrm{s})+\beta_3\mathrm{F}_1(\mathrm{s})\mathrm{F}_2(\mathrm{s})\mathrm{F}_3(\mathrm{s}) + \dots \\ +\beta_\mathrm{n}\mathrm{F}_1(\mathrm{s})\mathrm{F}_2(\mathrm{s})\mathrm{F}_3(\mathrm{s})\dots\dots\mathrm{F}_\mathrm{n}(\mathrm{s}) \end{cases} \\ &= \sum_{\mathrm{i}=1}^{\mathrm{n}} \beta_\mathrm{i} \prod_{i=1}^{i} \mathrm{F}_\mathrm{j}(s)\end{aligned} \tag{5}$$

The $F_i(s)$ can be defined by the continued fraction expansions as shown in Eq. 6.

$$F_i(s) = \cfrac{1}{\alpha_i s + \cfrac{1}{\alpha_{i+1}s + \cfrac{1}{\alpha_{i+2}s+\cdots} \\ \vdots \\ \alpha_{n-1}s + \frac{1}{\alpha_n s}}} \tag{6}$$

In Routh Table 1, the first two rows of table are formed by coefficients of the denominator of function $\hat{G}(s)$ and taking assumption that the entries of $a_J^0 = a_{(J-1)}^I = 0$ for $j > n$.

$$\begin{aligned} a_0^{i+1} &= a_2^{i-1} - \alpha_i a_2^i \\ a_2^{i+1} &= a_4^{i-1} - \alpha_i a_4^i \\ &\vdots \\ a_{n-i-1}^{i+1} &= a_{n-i}^{i-1} - \alpha_i a_{n-i}^i \end{aligned} \tag{7}$$

where, Eq. 7 stands for $i = 1, 2, 3, \dots, n-1$. If the value of $n - i$ as odd, the last term in Eq. 7 is replaced by as shown in Eq. 8.

$$a_{n-i-1}^{i+1} = a_{n-i-1}^{i-1} \tag{8}$$

For $i = 1, 2, 3, \dots, n$, the marginal entries for α_i are calculated as in Eq. 9.

$$\alpha_i = \frac{a_0^{i-1}}{a_0^i} \tag{9}$$

The β_i coefficients of the canonical form Routh table are determined using coefficients of the numerator of $\hat{G}(s)$ and is shown in Eq. 10.

$$\beta_i = \frac{b_0^i}{a_0^i} \tag{10}$$

$$b_{j-2}^{i+2} = b_j^i - \beta_i a_j^i \tag{11}$$

The Routh Table 1 is equivalent to construction of following finite continued fraction expansion as shown in Eq. 12.

$$\hat{D}(s) = \frac{\alpha_1}{s} + \cfrac{1}{\frac{\alpha_2}{s} + \cfrac{1}{\frac{\alpha_3}{s} + \cdots} \\ \vdots \\ \frac{\alpha_{n-1}}{s} + \frac{1}{\frac{\alpha_n}{s}}} \tag{12}$$

It could be easy to say that the system with all α parameters being positive refers to an asymptotically stable system [1].

Table 1. Alpha table

I^{st} row	$a_0^0 = a_0$	$a_2^0 = a_2$	$a_4^0 = a_4$
2^{nd} row	$a_0^1 = a_0$	$a_2^1 = a_3$	$a_4^1 = a_5$
$\alpha_1 = \frac{a_0^0}{a_0^1}$	$\mathrm{a}_0^2 = \mathrm{a}_2^0 - \alpha_1 \mathrm{a}_2^1$	$\mathrm{a}_2^2 = \mathrm{a}_4^0 - \alpha_1 \mathrm{a}_4^1$	$\mathrm{a}_4^2 = \mathrm{a}_2^0 - \alpha_1 \mathrm{a}_6^1$
$\alpha_2 = \frac{a_0^1}{a_0^2}$	$\mathrm{a}_0^3 = \mathrm{a}_2^1 - \alpha_2 \mathrm{a}_2^2$	$\mathrm{a}_2^3 = \mathrm{a}_4^1 - \alpha_2 \mathrm{a}_4^2$	$\ldots$
$\alpha_3 = \frac{a_0^2}{a_0^3}$	$\mathrm{a}_0^4 = \mathrm{a}_2^2 - \alpha_3 \mathrm{a}_2^3$	$\mathrm{a}_2^4 = \mathrm{a}_4^2 - \alpha_3 \mathrm{a}_4^3$	$\ldots$
$\alpha_4 = \frac{a_0^3}{a_0^4}$	$\mathrm{a}_0^5 = \mathrm{a}_2^3 - \alpha_4 \mathrm{a}_2^4$	$\ldots$	$\ldots$
$\alpha_5 = \frac{a_0^4}{a_0^5}$	$\mathrm{a}_0^6 = \mathrm{a}_2^4 - \alpha_5 \mathrm{a}_2^5$	$\ldots$	$\ldots$
$\alpha_6 = \frac{a_0^5}{a_0^6}$	$\ldots$	$\ldots$	$\ldots$

Table 2. Beta table

I^{st} row	$b_0^1 = b_1$	$b_2^1 = b_3$	$b_4^1 = b_5$
2^{nd} row	$b_0^2 = b_2$	$b_2^2 = b_4$	$b_4^2 = b_6$
$\beta_1 = \frac{b_0^1}{a_0^1}$	$\mathrm{b}_0^3 = \mathrm{b}_2^1 - \beta_1 \mathrm{a}_2^1$	$\mathrm{b}_2^3 = \mathrm{b}_4^1 - \beta_1 \mathrm{a}_4^1$	$\ldots$
$\beta_2 = \frac{b_0^2}{a_0^2}$	$\mathrm{b}_0^4 = \mathrm{b}_2^2 - \beta_2 \mathrm{a}_2^2$	$\mathrm{b}_2^4 = \mathrm{b}_4^2 - \beta_1 \mathrm{a}_4^2$	$\ldots$
$\beta_3 = \frac{b_0^3}{a_0^3}$	$\mathrm{b}_0^5 = \mathrm{b}_2^3 - \beta_3 \mathrm{a}_2^3$	$\ldots$	$\ldots$
$\beta_4 = \frac{b_0^4}{a_0^4}$	$\mathrm{b}_0^6 = \mathrm{b}_2^4 - \beta_4 \mathrm{a}_2^4$	$\ldots$	$\ldots$
$\beta_5 = \frac{b_0^5}{a_0^5}$	$\ldots$	$\ldots$	$\ldots$

3.3. Routh Convergent

The reduced k^{th} order transfer function as $\hat{R}_k(s)$ for an original transfer function $G(s)$ is derived by truncating the $\alpha - \beta$ expansion and rational arrangement of the results. The terms appearing $\alpha_{k+1}, \ldots, \alpha_n$ and $\beta_{k+1}, \ldots, \beta_n$ are eliminated using $\alpha - \beta$ expansion. In this way the the resultant is dependent on the first k-terms [7, 25].

Assuming a set of k-functions, which are defined by $G_{i,k}$ for $i = 2, 3, \ldots, k$ and is represented as in following Eq. 13 [25].

$$G_{i,k}(s) = \cfrac{1}{\alpha_i s + \cfrac{1}{\alpha_{i+1} s + \cfrac{1}{\alpha_{i+2} s + \cdots \atop {\vdots \atop \alpha_{k-1} s + \frac{1}{\alpha_k s}}}}} \tag{13}$$

The above method possess slight modification for $i = 1$. The I^{st} term in the continued fraction expansion is $1 + \alpha_1 s$ instead of $\alpha_1 s$. In this way, the k^{th} convergent may be given by as in Eq. 14 [1, 25].

$$\begin{aligned} \hat{\mathrm{R}}_{\mathrm{k}}(\mathrm{s}) &= \begin{cases} \beta_1 \mathrm{G}_{1,\mathrm{k}}(\mathrm{s}) + \beta_2 \mathrm{G}_{1,\mathrm{k}}(\mathrm{s}) \mathrm{G}_{2,\mathrm{k}}(\mathrm{s}) + \cdots \\ + \beta_{\mathrm{k}} \mathrm{G}_{1,\mathrm{k}}(\mathrm{s}) \mathrm{G}_{2,\mathrm{k}}(\mathrm{s}) \cdots \mathrm{G}_{\mathrm{k},\mathrm{k}}(\mathrm{s}) \end{cases} \\ &= \sum_{i=1}^{k} \beta_i \prod_{i=1}^{I} G_{i,k}(s) \end{aligned} \tag{14}$$

The $A_k(s)$ is the denominator of the k^{th} convergent while $B_k(s)$ represents the numerator of it. In this way, the k^{th} convergent may be represented as in following Eq. 15 [26].

$$\begin{aligned}
&A_1(s) = \alpha_1 s + 1 \\
&B_1(s) = \beta_1 \\
&A_2(s) = \alpha_1\alpha_2 s^2 + \alpha_2 s + 1 \\
&B_2(s) = \alpha_2\beta_1 s + \beta_2 \\
&A_3(s) = \alpha_1\alpha_2\alpha_3 s^3 + \alpha_2\alpha_3 s^2 + (\alpha_1 + \alpha_3)s + 1 \\
&B_3(s) = \alpha_2\alpha_3\beta_1 s^2 + \alpha_3\beta_2 s + (\beta_1 + \beta_3) \\
&A_k(s) = \alpha_k s A_{k-1}(s) + A_{k-2}(s) \\
&B_k(s) = \alpha_k s B_{k-1}(s) + B_{k-2}(s) + \beta_k \\
&A_{-1}(s) = 1, \quad B_{-1}(s) = 0 \\
&A_0(s) = 1, \quad B_0(s) = 0
\end{aligned} \tag{15}$$

The $\hat{R}_k(s)$ represents the approximation of $\hat{G}(s)$ with preserving the frequency behaviour. The k^{th} approximate can be derived by considering the reciprocal of $\hat{R}_k(s)$ as shown in Eq. 16 [25].

$$R_k(s) = \frac{1}{s}\hat{R}_k\left(\frac{1}{s}\right) \tag{16}$$

3.4. Algorithm of Routh approximation

The following steps can be followed for determining the reduced order of a high order system.

(i) Initially determine the reciprocal ($\hat{G}(s)$) of the full order system $G(s)$

(ii) Derive the $\alpha - \beta$ elements

(iii) Determine k^{th} convergent using $\hat{R}_k(s) = \frac{B_k(s)}{A_k(s)}$

(iv) Reciprocate $\hat{R}_k(s)$ for k^{th} order Routh approximation $R_k(s)$.

4. Results and Discussions

4.1. Example-1: SISO

Considering the 8^{th} order system presented in Shamash, 1975 [8] and presented in Eq. 17.

$$G(s) = \frac{\begin{matrix}18s^7 + 514s^6 + 5982s^5 + 36380s^4 + \\ 122664s^3 + 222088s^2 + 185760s + 40320\end{matrix}}{\begin{matrix}s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 \\ +67284s^3 + 118124s^2 + 109584s + 40320\end{matrix}} \tag{17}$$

The reduced 2^{nd} order and 3^{rd} order models are presented in Eq. 18 and Eq. 19, respectively using Routh Approximation method.

$$R_2(s) = \frac{1.990s + 0.432}{s^2 + 1.174s + 0.432} \tag{18}$$

$$R_3(s) = \frac{4.968s^2 + 4.331s + 0.940}{s^3 + 2.545s^2 + 2.555s + 0.940} \tag{19}$$

The step response comparison of the original system [8] and it's reduced 2^{nd} and 3^{rd} order models are graphically compared in Fig. 1. It can be observed that the stability of the system that of with reduced models are retained except slight variation in rise time, settling time, peak value and peak time as included in Table 3. Since, the important properties of the higher order system are preserved in it's reduced (2^{nd} order) system, consequently the mathematical ease is increased greatly.

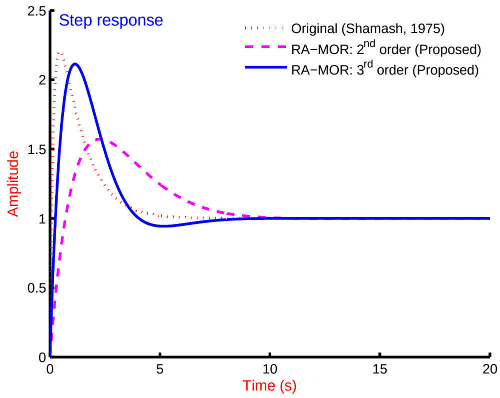

Figure 1. Step response of the original 8^{th} order system [8] and its reduced model $R_2(s)$ (Eq. 18) and $R_3(s)$ (Eq. 19) using the Routh approximation method

Table 3. Step response comparision of original system in Example-1, with reduced models using Routh Approximation

Transfer Function	Rise Time	Settling Time	Peak Value	Peak Time
Original: $G_8(s)$ [8]	0.0569	4.8201	2.2035	0.4493
MOR-RA: $R_2(s)$	0.5514	8.7327	1.5717	2.3235
MOR-RA: $R_3(s)$	0.1973	7.0765	2.1128	1.1637

4.2. Example-2: SISO

Considering the 4^{th} order system presented in Hwang,1996 [27] and presented in Eq. 20.

$$G(s) = \frac{10s^4 + 82.s^3 + 264s^2 + 396s + 156}{2s^5 + 21s^4 + 84.s^3 + 173s^2 + 148s + 40} \tag{20}$$

The reduced 2^{nd} order and 3^{rd} order models are presented in Eq. 21 and Eq. 22, respectively using Routh Approximation method.

$$R_2(s) = \frac{1.990s + 0.432}{s^2 + 1.174s + 0.432} \tag{21}$$

$$R_3(s) = \frac{4.968s^2 + 4.331s + 0.940}{s^3 + 2.545s^2 + 2.555s + 0.940} \tag{22}$$

The step response comparison of the original system [27] and it's reduced 2^{nd} and 3^{rd} order models are graphically compared in Fig. 2. It can be observed that the stability of the system that of with reduced models are retained except slight variation in rise time, settling time, peak value and peak time as included in Table 4. In this case the rise-time of the original, 2^{nd} and 3^{rd} order reduced models are 2.7456, 2.6830 and 2.5549 seconds, respectively. The difference in the rise times is minimal and is enough to prove similarity of the original and reduced models. The other step response data are enlisted in Table 4.

4.3. Example-3: SISO

Considering the 7^{th} order system presented in Jamshidi, 1983 [28] and presented in state-space form by Eq. 23 - 24 and in transfer function by Eq. 25. It represents the SMIB power

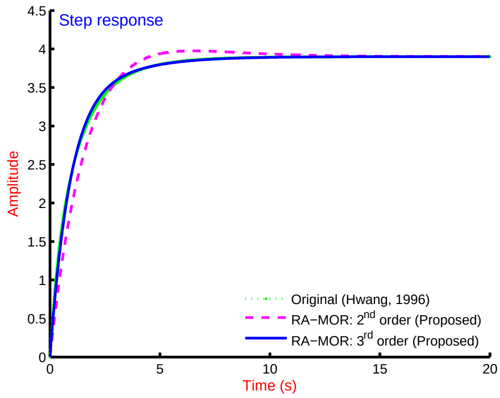

Figure 2. Step response of the original 4^{th} order system [27] and its reduced model $R_2(s)$ (Eq. 21) and $R_3(s)$ (Eq. 22) using the Routh approximation method

Table 4. Step response comparision of original system in Example-2, with reduced models using Routh Approximation

Transfer Function	Rise Time	Settling Time	Peak Value	Peak Time
Original: $G_8(s)$ [27]	2.7456	5.4346	3.8944	10.4392
MOR-RA: $R_2(s)$	2.6830	3.9639	3.9748	6.4974
MOR-RA: $R_3(s)$	2.5549	5.5932	3.8992	15.6589

system and the details are given in [29].

$$\dot{x}(t) = \begin{bmatrix} -0.58 & 0 & 0 & -0.269 & 0 & 0.2 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -5 & 2.12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 377 & 0 & 0 \\ -0.141 & 0 & 0.141 & -0.2 & -0.28 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0838 & 2 \\ -173 & 66.7 & -116 & 40.9 & 0 & -66.7 & -16.7 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} u(t) \quad (23)$$

$$y(t) = \begin{bmatrix} 1 & -1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} x(t) \quad (24)$$

$$G(s) = \frac{2s^6 + 420.4s^5 + 9435s^4 + 1.39 \times 10^5 s^3 + 4.663 \times 10^5 s^2 + 4.342 \times 10^5 + 1.877 \times 10^5}{s^7 + 23.48s^6 + 331.7s^5 + 2640s^4 + 1.757 \times 10^4 s^3 + 5.165 \times 10^4 s^2 + 3.534 \times 10^4 s + 1.729 \times 10^4} \quad (25)$$

The reduced 2^{nd} order and 3^{rd} order models are presented in Eq. 26 and Eq. 27, respectively using Routh Approximation method.

$$R_2(s) = \frac{10.085s + 4.360}{s^2 + 0.821s + 0.402} \quad (26)$$

Table 5. Step response comparision of original system in Example-3, with reduced models using Routh Approximation

Transfer Function	Rise Time	Settling Time	Peak Value	Peak Time
Original: $G_8(s)$ [28, 29]	0.1126	5.9294	16.0310	0.4307
MOR-RA: $R_2(s)$	1.1998	7.4456	13.9203	3.3225
MOR-RA: $R_3(s)$	0.5740	9.0915	13.2269	2.1099

$$R_3(s) = \frac{29.318s^2 + 27.948s + 12.081}{s^3 + 3.26s^2 + 2.275s + 1.113} \tag{27}$$

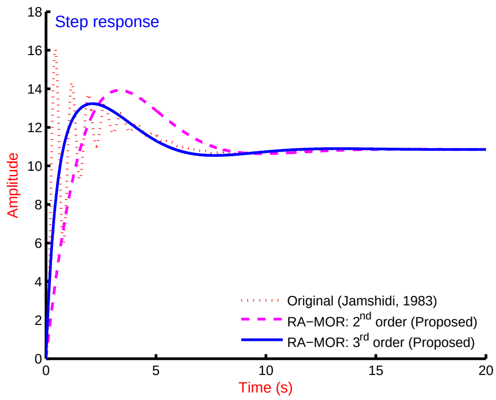

Figure 3. Step response of the original 7^{th} order system [28, 29] and its reduced model $R_2(s)$ (Eq. 26) and $R_3(s)$ (Eq. 27) using the Routh approximation method

In this example, the considered system is from the power system engineering. The original system and it's reduced 2^{nd} and 3^{rd} order models are subjected to step signal and superimposed to compare the responses in Fig. 3. It can be seen that the response due to original system is having more oscillations as compared to that of with the reduced order models. The step response information of these responses are enlisted in Table 5.

4.4. Example-4: SISO

Considering the 9^{th} order boiler system represented in transfer function form in Eq. 28 as presented in [26, 30]. The reduced 2^{nd} order and 3^{rd} order models are presented in Eq. 29 and

$$G(s) = \frac{\begin{array}{l}146.4s^8 + 9.81\times10^4 s^7 + 5.999\times10^7 s^6 + 3.206\times10^{10} s^5 + 3.582\times10^{12} s^4 \\ +1.113\times10^{14} s^3 + 1.154\times10^{15} s^2 + 3.971\times10^{15} s + 3.063\times10^{15}\end{array}}{\begin{array}{l}s^9 + 659.8s^8 + 4.136\times10^5 s^7 + 2.13\times10^8 s^6 + 2.422\times10^{10} s^5 + 8.737\times10^{11} s^4 \\ +1.523\times10^{13} s^3 + 1.221\times10^{14} s^2 + 3.636\times10^{14} s + 2.406\times10^{14}\end{array}} \tag{28}$$

Table 6. Step response comparision of original system in Example-4, with reduced models using Routh Approximation

Transfer Function	Rise Time	Settling Time	Peak Value	Peak Time
Original: $G_9(s)$ [26, 30]	0.5432	2.2753	12.6986	4.5555
MOR-RA: $R_2(s)$	0.6375	2.9668	13.2809	1.6504
MOR-RA: $R_3(s)$	0.2577	2.4749	12.6920	4.5431

Eq. 30, respectively using Routh Approximation method.

$$R_2(s) = \frac{35.448s + 27.343}{s^2 + 3.246s + 2.148} \tag{29}$$

$$R_3(s) = \frac{90.835s^2 + 319.054s + 246.1}{s^3 + 9.662s^2 + 29.214s + 19.331} \tag{30}$$

The considered 9^{th} order system is a practical boiler system as presented in [26, 30]. The

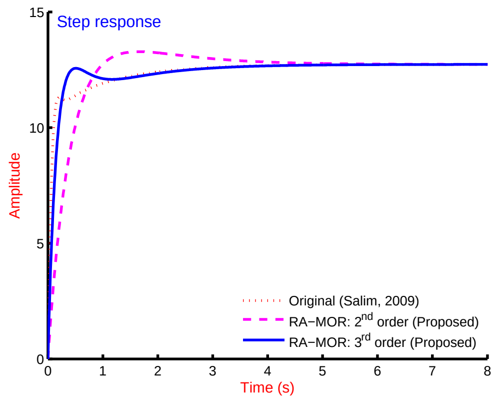

Figure 4. Step response of the original 9^{th} order system [26, 30] and its reduced model $R_2(s)$ (Eq. 29) and $R_3(s)$ (Eq. 30) using the Routh approximation method

system is reduced to 2^{nd} and 3^{rd} order models using Routh approximation method. The original and reduced models are subjected to step input and the graphical comparison is presented in Fig. 4. It can be seen that the original and main properties of the original higher order system are retained in it's reduced model responses with comparatively reduced overshoots. The step response information are included in Table 6.

4.5. Example-5: MIMO

A power plant system can be classified as a multivariable large-scale system. Numerous methods of analysis and synthesis for such processes have been developed, but the remarkable dimensions of the model structure makes their implementation very difficult. Considerable attention has therefore been devoted to the problem of deriving reduced-order models for such systems. The size and complexity of current electric power networks involves methods for studying approximated models to investigate the dynamic behaviour of such system types in a more suitable way; the methods currently used for determining reduced-order dynamic models for power systems in multi-bus, multi-machine frames are generally referred to as ”dynamic equivalents”.

An electric power system consisting of a salient-pole synchronous generator connected to an infinite bus-bar is considered. Taking into account the well known performance equations of both the machine and the transmission line, a very accurate non-linear mathematical model in the state-space form has been derived. As state variables of the electrical part of the synchronous machine, the set of winding currents of the $q-d$ equivalent circuit has been chosen. The seventh-order state vector of the original system consists of the stator currents $i_d,\ i_q$, the field circuit current i_{fd}, two damping circuit currents i_{kq}, i_{kd} and the mechanical quantities δ and ω. The input vector, in the chosen representation, consists of two quantities, the mechanical torque T_m and the voltage V_f. As output variables, the machine voltage VI and the mechanical state variables δ and ω have been chosen [3].

By considering small variations (Δ) around a steady-state operating point, a linear model has been derived. The values of the parameters, steady-state working conditions and further details on the adopted model are reported by Ramamoorty and Arumugan [31].

we indicate with:

- Change in mechanical torque (ΔT_m) as input 1
- Change in field voltage (ΔV_f) as input 2
- Change in terminal voltage (ΔV_t) as output 1
- Change in power angle ($\Delta\delta$) as output 2
- Change in speed ($\Delta\omega$) as output 3

The transfer function of multi-input multi-output (MIMO) single-machine infinite-bus (SMIB) power system can be represented as in Eqn. 31. The transfer function of the system with output ΔV_t to input ΔT_m can be represented by $G_{11}(s) = g_{11}(s)/d(s)$ and si,ilarly for others. The considered MIMO SMIB consists of six different transfer function with different sets of input and output signals. The denominator of these systems is common and represented by $d(s)$ in Eqn. 32. The polynomials presented in Eqn. 33 - Eqn. 38, are the numerators of different transfer functions due to different sets of input and output signals.

$$G(s) = \frac{\begin{bmatrix} g_{11}(s) & g_{21}(s) \\ g_{12}(s) & g_{22}(s) \\ g_{13}(s) & g_{23}(s) \end{bmatrix}}{d(s)} \tag{31}$$

$$d(s) = \begin{cases} s^7 + 258.7s^6 + 4.31\times10^5 s^5 \\ +4.835\times10^7 s^4 + 1.853\times10^9 s^3 \\ +2.54\times10^{10} s^2 + 5.973\times10^{10} s \\ +1.886\times10^{10} \end{cases} \tag{32}$$

$$g_{11}(s) = \begin{cases} -12.41s^4 + 1.213\times10^4 s^3 \\ -2.866\times10^6 s^2 - 3.325\times10^8 s \\ -6.404\times10^9 \end{cases} \tag{33}$$

$$g_{12}(s) = \begin{cases} -12.41s^5 + 1.213\times10^4 s^4 \\ -2.866\times10^6 s^3 - 3.325\times10^8 s^2 \\ -6.404\times10^9 s + 0.0006087 \end{cases} \tag{34}$$

$$g_{13}(s) = \begin{cases} 0.2005s^6 + 47.88s^5 \\ +3.928\times10^4 s^4 + 5.122\times10^6 s^3 \\ +2.288\times10^8 s^2 + 3.434\times10^9 s \\ +5.492\times10^9 \end{cases} \tag{35}$$

Table 7. Step response comparision of original system in Example-4, with reduced models using Routh Approximation

Original System	ROMs	Transfer functions
$G_{11}(s)$	$R_2(s)$	$\frac{-0.0134s-0.2581}{s^2+2.407s+0.76}$
	$R_3(s)$	$\frac{0.0055s^2-0.1914s-3.687}{s^3+14.6s^2+34.39s+10.86}$
$G_{12}(s)$	$R_2(s)$	$\frac{-0.2581s}{s^2+2.407s+0.76}$
	$R_3(s)$	$\frac{-0.1914s^2-3.687s}{s^3+14.6s^2+34.39s+10.86}$
$G_{13}(s)$	$R_2(s)$	$\frac{0.1384s+0.2213}{s^2+2.407s+0.76}$
	$R_3(s)$	$\frac{0.1256s^2+1.977s+3.162}{s^3+14.6s^2+34.39s+10.86}$
$G_{21}(s)$	$R_2(s)$	$\frac{0.8918s+0.8519}{s^2+2.407s+0.76}$
	$R_3(s)$	$\frac{0.7691s^2+12.74s+12.17}{s^3+14.6s^2+34.39s+10.86}$
$G_{22}(s)$	$R_2(s)$	$\frac{0.8519s}{s^2+2.407s+0.76}$
	$R_3(s)$	$\frac{12.74s^2+12.17s}{s^3+14.6s^2+34.39s+10.86}$
$G_{23}(s)$	$R_2(s)$	$\frac{-0.0269s-0.3653}{s^2+2.407s+0.76}$
	$R_3(s)$	$\frac{0.0006s^2-0.3842s-5.219}{s^3+14.6s^2+34.39s+10.86}$

$$g_{21}(s) = \begin{cases} 52.08s^5 + 1.076 \times 10^4 s^4 \\ +2.187 \times 10^7 s^3 + 1.377 \times 10^9 s^2 \\ +2.213 \times 10^{10} s + 2.114 \times 10^{10} \end{cases} \quad (36)$$

$$g_{22}(s) = \begin{cases} 52.08s^6 + 1.076 \times 10^4 s^5 \\ +2.187 \times 10^7 s^4 + 1.377 \times 10^9 s^3 \\ +2.213 \times 10^{10} s^2 + 2.114 \times 10^{10} s \\ +0.0009095 \end{cases} \quad (37)$$

$$g_{23}(s) = \begin{cases} 7.448s^5 + 2.701 \times 10^4 s^4 \\ +8.685 \times 10^5 s^3 - 1.664 \times 10^7 s^2 \\ -6.673 \times 10^8 s - 9.065 \times 10^9 \end{cases} \quad (38)$$

In this section a practical power system with multi-input and multi-output is considered. The system concerned is SMIB power system model with 2-inputs and 3-outputs. The system appeared as of 7^{th} order and represented by 6 different transfer functions. Each transfer function is reduced to it's 2^{nd} and 3^{rd} order models. The reduced models of the original systems are enlisted in Table 7. The respective original system and it's reduced models are subjected to step response and compared in Fig. 5 - Fig. 10. The step response information in terms of rise-time, settling-time, peak and peak-time are summarized in Table 8.

5. Conclusion

In this paper, the application of Routh Approximation is explored to obtain reduced order of SISO and MIMO systems in literature. The four examples of LTI SISO of practical importance and one on MIMO a power sytem example is considered to get 2^{nd} and 3^{rd} order reduced model. The similarity in original and reduced models are examined using step response graphical and statistical comparisons. It have been found that the reduced order models are able to retain stability of the considered system and reflects impressive degree of similarity in terms of rise time, settling time peak value and peak time. It could be easy to state that the characteristics of the reduced order models closer to original system are having more similarity as compared to

Table 8. Step response comparision of original system in Example-5, with reduced models using Routh Approximation

Systems and ROMs	Rise time (s)	Settling time (s)	Peak	Peak time (s)
$G_{11}(s)$	6.0012	10.9472	0.3393	19.8313
$R_2(s)$	6.0503	10.9570	0.3394	20.9108
$R_3(s)$	6.0014	10.9474	0.3393	19.7016
$G_{12}(s)$	-	11.8495	0.0905	0.9330
$R_2(s)$	0	12.0300	0.0867	1.0320
$R_3(s)$	0	11.8547	0.0904	0.9333
$G_{13}(s)$	5.7667	10.3078	0.2911	22.3338
$R_2(s)$	5.7608	10.2977	0.2911	22.3142
$R_3(s)$	5.7665	10.3078	0.2911	22.3338
$G_{21}(s)$	5.2590	9.7107	1.1207	22.0030
$R_2(s)$	5.2369	9.6813	1.1190	16.2674
$R_3(s)$	5.2599	9.7105	1.1189	16.1383
$G_{22}(s)$	3.0254E-15	7.6495	0.9039	0.0709
$R_2(s)$	0	12.0300	0.2861	1.0320
$R_3(s)$	0	8.0417	0.7695	0.1919
$G_{23}(s)$	6.0018	10.9253	0.4805	21.9535
$R_2(s)$	6.0483	10.9348	0.4804	20.9108
$R_3(s)$	6.0015	10.9258	0.4804	20.5690

lower order. It means the 3^{rd} reduced model is more similar to original system as compared to 2^{nd} order reduced model.

References

[1] Sambariya DK, Prasad R. Routh approximation based stable reduced model of single machine infinite bus system with power system stabilizer. *DRDO-CSIR Sponsered: IX Control Instrumentation System Conference (CISCON - 2012)*, 2012; 85–93. URL http://www.conference.bonfring.org/papers/manipal\ciscon2012/CIS-162.pdf.

[2] Sambariya DK, Prasad R. Routh stability array method based reduced model of single machine infinite bus with power system stabilizer. *International Conference on Emerging Trends in Electrical, Communication and Information Technologies (ICECIT-2012)*, 2012; 27–34, . URL http://dx.doi.org/10.13140/RG.2.1.4041.8325.

[3] Fortuna L, Nunnari G, Gallo A. *Model Order Reduction Techniques with Applications in Electrical Engineering*. Springer-Verlag London, 1992.

[4] Sambariya DK, Prasad R. Stable reduced model of a single machine infinite bus power system with power system stabilizer. *International Conference on Advances in Technology and Engineering (ICATE'13)*, 2013; 1–10, . URL http://dx.doi.org/10.1109/ICAdTE.2013.6524762.

[5] Bosley MJ, Lees FP. A survey of simple transfer-function derivations from high-order state-variable models. *Automatica* 1972; **8**(6):765–775, . URL http://dx.doi.org/10.1016/0005-1098(72)90087-8.

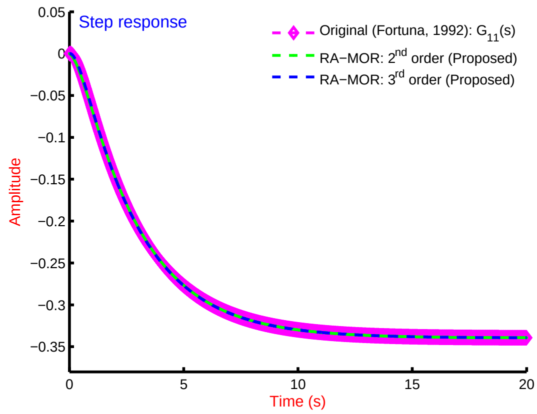

Figure 5. Response of the $G_{11}(s)$ and its $R_2(s)$ and $R_3(s)$

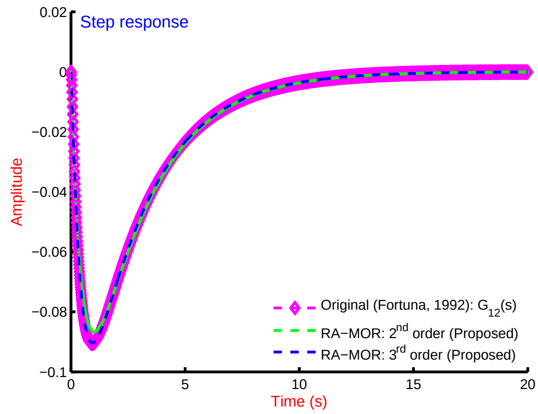

Figure 6. Response of the $G_{12}(s)$ and its $R_2(s)$ and $R_3(s)$

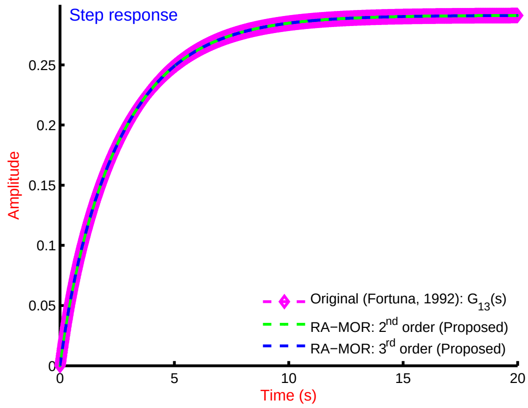

Figure 7. Response of the $G_{13}(s)$ and its $R_2(s)$ and $R_3(s)$

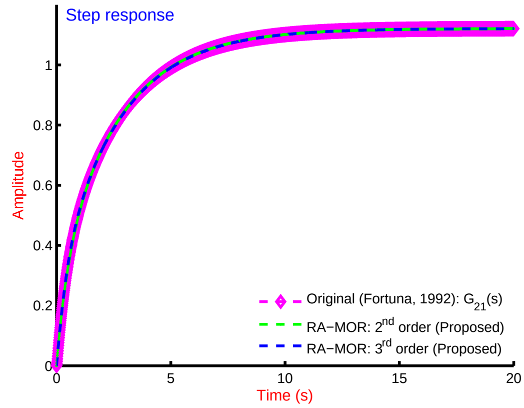

Figure 8. Response of the $G_{21}(s)$ and its $R_2(s)$ and $R_3(s)$

[6] Sambariya DK, Prasad R. Stability equation method based stable reduced model of single machine infinite bus system with power system stabilizer. *International Journal of Electronic and Electrical Engineering* 2012; **5**(2):101–106.

[7] Hutton M, Friedland B. Routh approximations for reducing order of linear, time-invariant systems. *IEEE Transactions on Automatic Control* Jun 1975; **20**(3):329–337, . URL http://dx.doi.org/10.1109/TAC.1975.1100953.

[8] Shamash Y. Model reduction using the routh stability criterion and the pad approximation technique. *International Journal of Control* 1975; **21**(3):475–484, . URL http://dx.doi.org/10.1080/00207177508922004.

[9] Gutman PO, Mannerfelt C, Molander P. Contributions to the model reduction problem. *IEEE Transactions on Automatic Control* Apr 1982; **27**(2):454–455, . URL http://dx.doi.org/10.1109/TAC.1982.1102930.

[10] Wan BW. Linear model reduction using mihailov criterion and pad approximation technique.

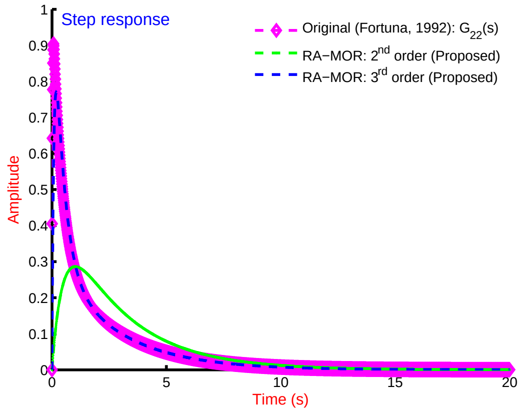

Figure 9. Response of the $G_{22}(s)$ and its $R_2(s)$ and $R_3(s)$

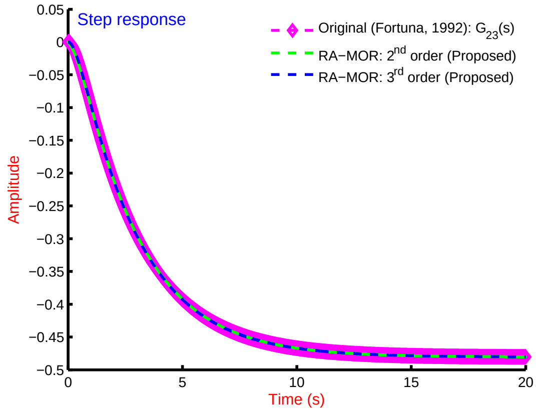

Figure 10. Response of the $G_{23}(s)$ and its $R_2(s)$ and $R_3(s)$

International Journal of Control 1981; **33**(6):1073–1089, . URL http://dx.doi.org/10.1080/00207178108922977.

[11] Chen TC, Chang CY, Han KW. Model reduction using the stability-equation method and the continued-fraction method. *International Journal of Control* 1980; **32**(1):81–94, . URL http://dx.doi.org/10.1080/00207178008922845.

[12] Sambariya DK, Arvind G. High order diminution of LTI system using stability equation method. *British Journal of Mathematics & Computer Science* 2016; **13**(5):1–15, . URL http://dx.doi.org/10.9734/BJMCS/2016/23243.

[13] Sambariya DK, Manohar H. Model order reduction by differentiation equation method using with routh array method. *IEEE Proceeding of 10^{th} International Conference on Intelligent Systems and Control (ISCO 2016)*, vol. 2, 2016; 341–346.

[14] Panda S, Yadav JS, Patidar NP, Ardil C. Evolutionary techniques for model order reduction of large scale linear systems. *International Journal of Applied Science, Engineering and Technology* 2009; **5**(1):22–28.

[15] Sambariya DK, Manohar H. Preservation of stability for reduced order model of large scale systems using differentiation method. *British Journal of Mathematics & Computer Science* 2016; **13**(6):1–17, . URL http://dx.doi.org/10.9734/BJMCS/2016/23082.

[16] Manohar H, Sambariya DK. Model order reduction of mimo system using differentiation method. *IEEE Proceeding of 10^{th} International Conference on Intelligent Systems and Control (ISCO 2016)*, vol. 2, 2016; 347–351.

[17] Sambariya DK, Prasad R. Differentiation method based stable reduced model of single machine infinite bus system with power system stabilizer. *International Journal of Applied Engineering Research* 2012; **7**(11):2116–2120. URL http://gimt.edu.in/clientFiles/FILE_REPO/2012/NOV/24/1353741189722/202.pdf.

[18] Sambariya DK, Rajawat AS. Application of routh stability array method to reduce MIMO SMIB power system. *6th IEEE International Conference on Power Systems, (ICPS-2016)*, 2016; 1–6.

[19] Sambariya DK, Rajawat AS. Model order reduction of lti system using routh stability array method. *IEEE proceedings on International Conference on Computing, Communication and Automation (ICCCA-2016)*, 2016; 1–6.

[20] Sambariya DK, Manohar H. Model order reduction by integral squared error minimization using bat algorithm. *IEEE Proceedings of 2015 RAECS UIET Panjab University Chandigarh* $21 - 22^{nd}$ *December 2015*, 2015; 1–7.

[21] Sambariya DK, Arvind G. Reduced order modelling of SMIB power system using stability equation method and firefly algorithm. *6th IEEE International Conference on Power Systems, (ICPS-2016)*, 2016; 1–6.

[22] Panda S, Tomar SK, Prasad R, Ardil C. Reduction of linear time-invariant systems using routh-approximation and pso. *International Journal of Electrical, Robotics, Electronics and Communications Engineering* 2009; **3**(9):20–27.

[23] Sambariya DK, Sharma O. Model order reduction using routh approximation and cuckoo search algorithm. *Journal of Automation and Control* 2016; **4**(1):1–9, . URL http://pubs.sciepub.com/automation/4/1/1.

[24] Sambariya DK, Arvind G. Reduced order model of single machine infinite bus power system using stability equation method and self-adaptive bat algorithm. *Universal Journal of Control and Automation* 2016; **4**(1):1–7, . URL http://dx.doi.org/10.13189/ujca.2016.040101.

[25] El-Nahas I, Sinha N, Alden R. Pade and routh approximation in the time domain. *Decision and Control, 1983. The 22nd IEEE Conference on*, 1983; 243–246, . URL http://dx.doi.org/10.1109/CDC.1983.269837.

[26] Soloklo HN, Farsangi MM. Multiobjective weighted sum approach model reduction by routh-pade approximation using harmony search. *Turk J Elec Eng & Comp Sci* 2013; **21**(0):2283 – 2293, . URL http//dx.doi.org/10.3906/elk-1112-31.

[27] Hwang C, Hwang JH. A new two-step iterative method for optimal reduction of linear siso systems. *Journal of the Franklin Institute* 1996; **333**(5):631–645, . URL http://dx.doi.org/10.1016/0016-0032(96)00049-X.

[28] Jamshidi M. *Large Scale Systems: Modeling, Control and Fuzzy Logic*. 2 edn., Printish Hall: North Holland, 1983.

[29] Bettayeb M, Salim R. Ga based h ∞ optimal model reduction: Application to power systems. *International Conference on Electric Power and Energy Conversion Systems (EPECS '09)*, 2009; 1–6.

[30] Salim R, Bettayeb M. H2 optimal model reduction of dynamic systems with time-delay using particle swarm optimization. *3rd International Conference on Complex Systems and Applications*, 2009.

[31] Ramamoorty M, Arumugam M. Design of optimal regulators for synchronous machines. *Power Apparatus and Systems, IEEE Transactions on* Jan 1973; **PAS-92**(1):269–277, . URL http://dx.doi.org/10.1109/TPAS.1973.293623.