TELKOMNIKA Indonesian Journal of Electrical Engineering
Vol. 13, No. 1, January 2015, pp. 145 ~ 150
DOI: 10.11591/telkomnika.v13i1.6712

A Breeding Estimated Particle Filter Research

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Abstract

As the normal particle filter has an expensive computation and degeneracy problem, a propagation-prediction particle filter is proposed. In this scheme, particles after transfer are propagated under the distribution of state noise, and then the produced filial particles are used to predict the corresponding parent particle referring to measurement, in which step the newest measure information is added into estimation. Therefore predicted particle would be closer to the true state, which improves the precision of particle filter. Experiment results have proved the efficiency of the algorithm and the great predominance in little particles case.

Keywords: *state estimation, particle filter, particle degeneracy, importance density function.*

1. Introduction

Particle filter method [1] the optimal Bayes filtering and Monte Carlo sampling methods evolved, the core idea is to spread in the state space using a set of random samples with associated weights to approximate the posterior probabilitydensity, the minimum variance of the sample mean instead of the integral operation, to obtain state estimates. Due to the limitations of the particle filter is not linear and noise Gaussian assumptions in the field of non-linear, non-Gaussian occasions such as target tracking, signal processing, automatic control has been widely used. However, the standard particle filter widespread large and particle degradation problem, which the particle degradation will largely impact the estimation accuracy and robustness of the particle filter. In order to solve the particle degradation problem, the posterior probability distribution closer to the importance of the probability density function. Doucet *et al* [3] extended Kalman filter (EKF) to generate the importance density function, however EKF introduces more error in the model linearization and noise Gaussian assumptions, so the improvement effect is not very satisfactory. Merwe and Doucet [4] proposed using unscented Kalman filter (UKF) instead of the EKF produces the importance density function, to obtain higher estimation accuracy, but also greatly increase the amount of computation. In addition, the method used to generate the importance density Gauss - the Hermitian filter method [5], the state parameter decomposition and annealing coefficient method [6], nonlinear interactive multi-model approach [7], the second-order central difference filtering method [8] and quadrature Kalman filter (QKF) [9]. In varying degrees, these methods improve the precision of the particle filter, but also reduce the real-time nature of the algorithm. In this paper, the problems of existing particle filtered algorithm proposed breeding to improve the estimated accuracy of the particle filter the estimated particle filter at the same time taking into account the real-time, and achieved good results.

2. Principles of Particle Filter

Consider the general nonlinear system, the state equation and observation equation is as follows:

$$\begin{cases} \mathbf{x}_k = f_k(\mathbf{x}_{k-1}, \mathbf{u}_k) \\ \mathbf{z}_k = h_k(\mathbf{x}_k, \mathbf{v}_k) \end{cases} \quad (1)$$

Received September 18, 2014; Revised October 26, 2014; Accepted November 20, 2014

Where, $\mathbf{x}_k$ is the system state vector $\mathbf{z}_k$ for the system of observation vectors, $\mathbf{u}_k$ and $\mathbf{v}_k$, respectively, for system status transfer of noise and observation noise, $f_k(\square)$ and $h_k(\square)$, respectively, for the transfer of the state of the system and observation functions.The purpose of the filtered is to estimate the state information can not be directly obtained by observing assumptions to estimate the amount of the function $g(\mathbf{x}_{0:k})$ of the system state.Due to the general nonlinear systems, the posterior probability $p(\mathbf{x}_{0:k} | \mathbf{z}_{1:k})$ is not easy to seek an easy sampling and with similar posterior probability distribution instead of $p(\mathbf{x}_{0:k} | \mathbf{z}_{1:k})$ sampling assumption $q(\mathbf{x}_{0:k} | \mathbf{z}_{1:k})$, $g(\mathbf{x}_{0:k})$ by the following equationestimate:

$$E[g(\mathbf{x}_{0:k})] = \frac{E(g(\mathbf{x}_{0:k}) w_k(\mathbf{x}_{0:k}))}{E(w_k(\mathbf{x}_{0:k}))} \tag{2}$$

$$w_k(\mathbf{x}_{0:k}) = \frac{p(\mathbf{z}_{1:k} | \mathbf{x}_{0:k}) p(\mathbf{x}_{0:k})}{q(\mathbf{x}_{0:k} | \mathbf{z}_{1:k})}$$

Estimates using the Monte Carlo method to sample from the reference distribution $q(\mathbf{x}_{0:k} | \mathbf{z}_{1:k})$, $g(\mathbf{x}_{0:k})$ mathematical expectation:

$$\overline{E(g(\mathbf{x}_{0:k}))} = \frac{\frac{1}{N}\sum_{i=1}^{N} g(\mathbf{x}_{0:k}^{(i)}) w_k(\mathbf{x}_{0:k}^{(i)})}{\frac{1}{N}\sum_{i=1}^{N} w_k(\mathbf{x}_{0:k}^{(i)})} = \sum_{i=1}^{N} g(\mathbf{x}_{0:k}^{(i)}) \tilde{w}_k(\mathbf{x}_{0:k}^{(i)}) \tag{3}$$

$$\tilde{w}_k(\mathbf{x}_{0:k}^{(i)}) = w_k(\mathbf{x}_{0:k}^{(i)}) \Big/ \sum_{i=1}^{N} w_k(\mathbf{x}_{0:k}^{(i)})$$

Where, $\mathbf{x}_{0:k}^{(i)}$ is the i-th sampling particles. If the state estimation process for optimal estimation, the reference distribution depends only on the probability density function of $\mathbf{x}_{k-1}$ and z_k , and therefore can get the right values recursive form:

$$w_k(\mathbf{x}_{0:k}^{(i)}) = w_{k-1}(\mathbf{x}_{0:k-1}^{(i)}) \frac{p(\mathbf{z}_k | \mathbf{x}_k^{(i)}) p(\mathbf{x}_k^{(i)} | \mathbf{x}_{k-1}^{(i)})}{q(\mathbf{x}_k^{(i)} | \mathbf{x}_{k-1}^{(i)}, \mathbf{z}_{1:k})} \tag{4}$$

Where, $p(\mathbf{z}_k | \mathbf{x}_k^{(i)})$ called the likelihood and characterization of the i-th particle from state $\mathbf{x}_{k-1}$ to $\mathbf{x}_k$ and the degree of similarity; systematic observation value $\mathbf{z}_k$, $p(\mathbf{x}_k^{(i)} | \mathbf{x}_{k-1}^{(i)})$ transition probability of the i-th particle state $\mathbf{x}_{k-1}$ to $\mathbf{x}_k$.

3. Particle Filter Problem

Under normal circumstances, after steps iterative recursion, most of the particles are the right weight becomes very small, and only a few particles have a greater weight. This will make a large number of calculations wasted in these small weights particles are almost zero, and its contribution to the approximation posterior probability distribution.Usually effective sample size N_{eff} measure of the degree of degradation of the algorithm.

$$N_{eff} = \frac{N}{1+\mathrm{var}(w_k^{*i})} \quad (5)$$

Where, $w_k^{*i} = p(\mathbf{x}_k^{(i)} \mid \mathbf{z}_{1:k}) / p(\mathbf{x}_k^{(i)} \mid \mathbf{x}_{k-1}^{(i)}, \mathbf{z}_k)$ is called "true weight".The effective sample size can not be strictly calculated, but its estimated value:

$$\hat{N}_{eff} = \frac{1}{\sum_{i=1}^{N} (w_k^i)^2} \quad (6)$$

Where, w_k^i is the formula (4) defines a regular weights. $\hat{N}_{eff}$ degradation of the smaller particles means more severity. [3] proved that with the passage of time, the variance of weight will increase, therefore the degradation phenomena can not be avoided.

4. The Reproductive Estimated Particle Filter

Solve the degradation problems commonly used resampling principle, the basic idea: When the particle set degradation below a certain threshold (such as $\hat{N}_{eff}$), the weight of the particles based on the importance of sampling resampling to generate a new set of support points $(\mathbf{x}_k^{i'})_{i=1}^N$, to the phase-out of the right value of low particle, the value of the right of retention of the high particle, thereby limiting the degradation phenomena. Resampling particles, however, is no longer independent, high-weight particles are copied many times, and low weight particles gradually disappear. After several iterations, all particles are collapsed to a point, resulting in a dilution of the sample.

To select appropriate close to the true distribution of the importance of the state of the system density function is another common method to solve the degradation problems. Usually using EKF and UKF be updated on the current particle, but because of its own filtering estimation linearization of nonlinear systems, and therefore unable to get rid of the limitations of linearization essentially. The same time to make use of EKF and UKF each particle, profile, thus greatly increasing the amount of calculation of the filtering process, is not conducive to real-time requirements occasions applications. Use other methods [5-9] to generate the importance density function will also appear the same problem.

To this end, this paper presents a based on the the estimated particle filter breeding methods. Proposal distribution due to the transfer of a priori as current observation information is missing, the estimated accuracy of the standard particle filter. This article will be transferred into the latest observation information get the particle reference to the observed value to reproduce estimates, the posterior probability distribution of the particle distribution to better reflect the true situation, to improve the accuracy of the estimates of the particle filter.

The new algorithm to reproduce the particles instead of using a normal Gaussian distribution, but the state transition noise distribution standard breeding particle distribution. In this manner is more in line with the actual situation, and thus also can achieve a higher accuracy than the Gaussian distribution. Particles (known as the mother particles) after transfer to reproduce, and then use the estimate, of the propagation of particles on the mother particles can be obtained a new estimate of the mother particles. These Estimate particles into the latest observation information, and therefore more accurately reflect the true state of the system. Finally, using the estimated particle estimate the state will be able to get a more accurate estimate of the state. Reproduction estimated principle is shown in Figure 1.

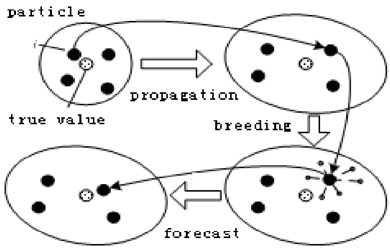

Figure 1. The i-th particle propagation forecast process

From Figure 1, we can understand the particles for reproduction, due to the different distances away from the real value of each progeny particles, so the weight is not the same. After using progeny particles weighted estimates obtained mother particles closer to the true value. The particles multiply the number does not have to generally take 10 to 20 can achieve good results. Breeding estimate particle filter algorithm steps are as follows:

Breeding estimate particle filtering algorithm:

1) System initialization. k = 0, as the initial set of particles based on the weight of the initial importance density sampling of N particles $\{\mathbf{x}_k^i,\ 1/N\}$.

2) Particle updates.Let k = k + 1, the particles according to the transfer function of the system state is updated:

$$\mathbf{x}_k^i = f_k(\mathbf{x}_{k-1}^i, \mathbf{u}_k) \tag{7}$$

3) Particle propagation.According to the system to transfer the distribution of the noise of the current particles reproduction, simultaneously calculate the weight of each progeny particles and is normalized, i.e. on $c = 1, 2, \cdots, N_c$:

$$\mathbf{x}_k^{ic} = \mathbf{x}_k^i + \mathbf{u}_k \tag{8}$$

$$w_k^{ic} = p(\mathbf{z}_k \mid \mathbf{x}_k^{ic}),\quad \tilde{w}_k^{ic} = w_k^{ic} \Big/ \sum_{c=1}^{N_c} w_k^{ic} \tag{9}$$

4) Particle estimates.Breeding progeny particles weighted estimates of current particle, the estimated value of the current particle:

$$\hat{\mathbf{x}}_k^i == \sum_{c=1}^{N_c} w_k^{ic} \mathbf{x}_k^{ic} \tag{10}$$

5) After the estimate of the likelihood value calculation according to the observed weight of each particle, and normalization:

$$w_k^i = w_{k-1}^i p(\mathbf{z}_k \mid \hat{\mathbf{x}}_k^i),\quad \tilde{w}_k^i = w_k^i \Big/ \sum_{i=1}^{N} w_k^i \tag{11}$$

6) State estimation.According to the estimates of particle weight estimation system status values:

$$\hat{\mathbf{x}}_k = \sum_{i=1}^{N} \tilde{w}_k^i \hat{\mathbf{x}}_k^i \tag{12}$$

7) Using Equation (6) $\hat{N}_{eff}$ .If $\hat{N}_{eff} < N_{thres}$, the resampling, and the weights of all the particles is reset to 1 / N.

8) Repeat steps 2 to 7.

5. Algorithm Validation

In order to verify the effectiveness of the algorithm, the standard particle filter (with re-sampling step estimate particle filter) and the breeding of this paper, a comparative analysis.References a classic example here, the state equation and observation equation, respectively:

$$x_k = 0.5x_{k-1} + \frac{25x_{k-1}}{1+x_{k-1}^{\ 2}} + 8\cos[1.2(k-1)] + u_k \qquad (13)$$

$$z_k = 0.05x_k^{\ 2} + v_k \qquad (14)$$

Among them, the state noise $u_k \sim N(0,\sigma_u^2)$,and observation noise $v_k \sim N(0,\sigma_v^2)$, $\sigma_u^2 = \sigma_v^2 = 1$. Simulation particle number 50 is selected, the number of particles of propagation 10, the 50 iterations of the two algorithms, respectively, to obtain simulation results shown in Figure 2. Figure 3 shows the error curve of the two algorithms. As can be seen, the standard particle filter due to insufficient simulation samples produced large errors, many of the state's estimated very accurate. The proposed algorithm can well estimate the state of the system, although the number of particles is very small, but still achieved a high estimate of effect.

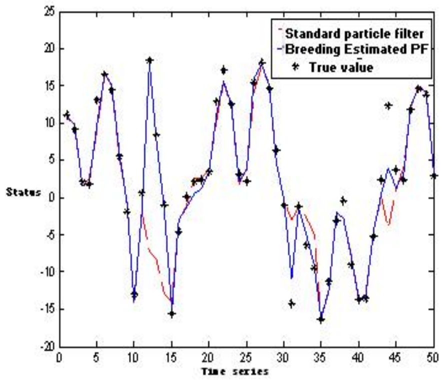

Figure 2. State estimated effect of the two algorithms

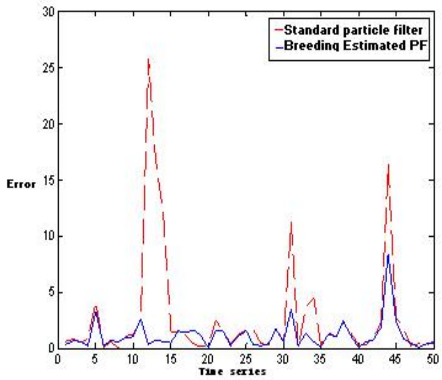

Figure 3. Comparison of two algorithms estimate error

In this paper, the filtering estimate the root mean square error (Root Mean Square Error, referred to as RMSE) to measure the accuracy of filtering its definition:

$$\mathrm{RMSE}(\hat{\mathbf{x}}_k) = \left[\frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} (\mathbf{x}_k^i - \hat{\mathbf{x}}_k^i) \right]^{1/2} \qquad (15)$$

RMSE value is lower, the higher the accuracy. The number of particles were taken 50,100 ...... 400, breeding the number of particles is still set to 10, two algorithms simulation RMSE values were calculated in a different number of particles.

As can be seen from Table 1 the the propagation estimate the running time of the particle filter (PGPF) is 2 to 3 times of the standard particle filter (PF). This is estimated propagation due to the particles, thereby increasing the amount of computation. In comparison, the EKF and UKF filtering algorithm to predict the particle (respectively referred to as the EPF and UPF) will consume more computing resources, while considerable accuracy with the proposed algorithm. Therefore, breeding estimate particle filter to improve filtering accuracy while also taking into account the real-time nature of the algorithm.

Table 1. Various particle filter algorithm running time comparing (simulation length 50)

Algorithm	Run time (unit: seconds)				
	N=50	*N*=100	*N*=200	*N*=300	*N*=400
PF	0.0196	0.0239	0.0387	0.0584	0.0851
PGPF	0.0429	0.0686	0.1240	0.1862	0.2504
EPF	0.0534	0.0947	0.1703	0.2737	0.3735
UPF	0.0938	0.1736	0.3098	0.4892	0.6103

6. Conclusions and Outlook

In this paper, the problem of the degradation of the presence of particle filter the estimated particle filter proposed breeding improved at the same time taking into account the real-time filtering accuracy. Simulation results show that the filtering performance of the algorithm when significant changes in the number of particles is always stable. The advantage of breeding estimate particle filter higher filtering accuracy and computational efficiency is also high, basically to achieve real-time requirements with fewer particles. However, with the increase in the number of state dimension in the case of reproduction of progeny particles few will affect the accuracy of the filter; this problem has not yet been verified. Breeding in the high-dimensional state the estimated particle filter will be the next step in the research direction.

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