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Optimal Disturbance Rejection Control of Underactuated Autonomous Underwater in Vertical Plane

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Abstract

To realize the optimal control of underactuated autonomous underwater vehicle (AUV) in vehicle plane with external disturbances, a optimal disturbance rejection controller is proposed with respect to the quadratic performance indexes. Firstly, the depth control model of underactuated AUV system and the wave model is proposed; Then based on the theory of the quadratic optimal control and stability degree constraint, a feedforward and feedback optimal disturbance rejection control law with a higher mean-square convergence rate is derived from the Riccati equation and the Sylvester equation, which can reject the disturbance influence to AUV. Finally, the controller is applied to the dive plane control of AUV with wave force disturbances, and the results demonstrate the effectiveness and robustness of the controller.

Keywords: *underactuated AUV Systems, optimal control, disturbances, stability degree*

1. Introduction

Autonomous Underwater Vehicles (AUV) have been an area of active research for the last few decades since these vehicles have various applications in military, commercial and scientific missions. The depth-keeping control for AUV systems is a common and important navigation control problem, which can test stability of AUV, underwater depth and variable depth performance. There are many good methods to solve this problem, such as PID and improved PID method [1], sliding mode control [2-3], adaptive control [4], predictive Control [5], optimal control [6-7] and backstepping control [8], etc. In vertical plane motion, the AUV system is inevitably influenced by wind, wave, flow and other complex environmental disturbance force. Wave force is one of the main disturbances, which is treated as a disturbance for AUV, and modeled by the exosystem. The motion situation of AUV is more complex under wave force disturbances, but also which can affect its motion control accuracy, even make the control of unstable. If control failure, AUV will soon surface or depth increases sharply, so the disturbance rejection problems of AUVs have important significance in theory and practice. Another for AUV control, the convergence speed of systems states is also an important factor which can not be ignored, because of the faster decay, the better stability, so that we draw into the stability degree in design of the control law for AUV systems, based on the linear quadratic optimal control theory.

In this paper, a optimal disturbance rejection control with a higher mean-square convergence rate is proposed for underactuated AUV with respect to the quadratic performance indexes. Firstly, we introduce a model of underactuated AUV system in vertical plane, then based on the theory of linear quadratic optimal control and stability degree constraint, a feedforward and feedback optimal disturbance rejection controller with a higher mean-square convergence rate is derived from the Riccati equation and the Sylvester equation, which is robust for the disturbance influence to AUV.

The organization of the paper is as follows. Section 2 presents the AUV model and the wave force disturbances model. The optimal disturbance rejection control law is derived in Sections 3. Then simulation results are presented in Section 4.

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2. Underactuated AUV Systems and Disturbance Model

2.1. Depth Control Model and Linearization

A schematic of the AUV model with its body-fixed coordinate system is shown in Figure 1, which is a complex non-linear system, and has strong coupling between state variables, it's a very difficult problem to design an optimal control law for AUV kinematics system, so the kinematics model of AUV is transformed into a simple one from the six degrees of freedom model proposed by Fossen, and then the novel model has four degrees of freedom, and four independent input variables. In order to facilitate the analysis and synthesis of control system, the coupling effect between the roll surface movement and two case of plane motion is usually ignored, then the vehicle motion is divided into horizontal and vertical movement. In this paper, we consider the vertical movement, and assuming that the axial velocity is constant, all transverse parameter is zero, and only a AUV tail rudder propeller, so the AUV system has only one control input δ_s, and there are two degrees of freedom of motion. The kinematic and dynamic equations[9] can be expressed as follows:

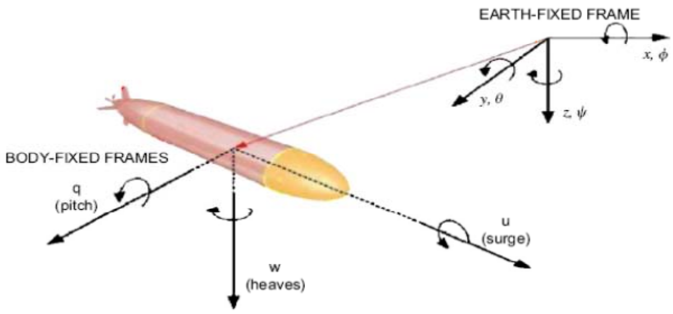

Figure 1. AUV model

$$\begin{aligned}
& m[\dot{w} - uq - x_G\dot{q} - z_G q^2] = Z_{\dot{q}}\dot{q} + Z_{\dot{w}}\dot{w} + Z_{uq}uq + \\
& Z_{uw}uw + Z_{w|w|}w\left|w\right| + Z_{q|q|}q\left|q\right| + (W - B_0)\cos\theta + u^2 Z_{uu}\delta_s \\
& I_{yy}\dot{q} + m[x_G(uq - \dot{w}) + z_G wq] = M_{\dot{q}}\dot{q} + M_{\dot{w}}\dot{w} + \\
& M_{uq}uq + M_{uw}uw + M_{w|w|}w\left|w\right| + M_{q|q|}q\left|q\right| - \\
& (x_G W - x_B B_0)\cos\theta - (z_G W - z_B B_0)\sin\theta + u^2 M_{uu}\delta_s \\
& \dot{z} = w\cos\theta - u\sin\theta \\
& \dot{\theta} = q
\end{aligned} \tag{1}$$

Where θ is the pitch angle, w is the heave velocity, δ_s is the control fin angle, I_{yy} is the moment of inertia of the vehicle about the pitch axis, u is the forward velocity, W denotes the vehicle's weight and B_0 is the vehicle buoyancy. The physical meaning of other parameters in reference [10], the nonlinear system (1) is not convenient to control system analysis and synthesis, so the model is linearzed based on the small perturbation method. Suppose the reference motion as the axial direct motion, not bow to the motion and roll motion, and second order coefficient is relatively small, which can be neglected, $(x_G, y_G, z_G) = (x_B, y_B, z_B) = 0$, then the linear equation group is available, as follow:

$$\begin{bmatrix} m - Z_{\dot{w}} & -Z_{\dot{q}} & 0 & 0 \\ -M_{\dot{w}} & I_{yy} - M_{\dot{q}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} Z_{uw} & Z_{uq} & 0 & 0 \\ M_{uw} & M_{uq}u & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -u & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \\ z \end{bmatrix} + \begin{bmatrix} Z_{uu}u^2 \\ M_{uu}u^2 \\ 0 \\ 0 \end{bmatrix} \delta_s(t) \tag{2}$$

2.2. Disturbances Model of Wave Force

The external disturbances for AUV's are complex, and wave force is one of the main disturbances. In order to study conveniently, the irregular long storm waves is simplified as point long crested waves, as follow:

$$\vartheta(t) = \sum_{j=1}^{l} \vartheta_j(t) = \sum_{j=1}^{l} L_j \cos\theta_j \tag{3}$$

Where, l is the number of component wave, $L_j = \sqrt{2S_\vartheta(\omega_j)\Delta\omega_j}$, $\theta_j = -\omega_j t + \varepsilon_j$, ε_j is a random variable, by wave theory, it is uniform distribution between $0\text{-}2\pi$, ω_j is the j component wave frequency, $S_\vartheta(\cdot)$ is the Ocean wave spectrum density function.

We construct a system model to describe the irregular wave forces for the AUV in two-dimensional horizontal plane.

Define $v_j = L_j \cos(\theta_j)$ is the horizontal velocity of water particle orbital motion. Let $v(t) = \begin{bmatrix} v_1 & \cdots & v_l \end{bmatrix}^T$, ω is the v_j frequency. By $\ddot{v}_j = -\omega_j^2 v_j$, $j = 1, 2, ..., l$, we have:

$$\ddot{v} = -\Omega v \tag{4}$$

Where $\Omega = \mathrm{diag}\{\omega_1^2, \omega_2^2, ..., \omega_l^2\}$.

Define $w(t) = \begin{bmatrix} v(t)^T, \dot{v}(t)^T \end{bmatrix}^T$, then:

$$\begin{aligned} \dot{w}(t) &= \begin{bmatrix} 0 & I \\ -\Omega & 0 \end{bmatrix} w(t) \triangleq Gw(t) \\ v(t) &= \begin{bmatrix} I & 0 \end{bmatrix} w(t) \end{aligned} \tag{5}$$

Wwhere I is the l dimensional unit matrix, and 0 is the l dimensional zero matrix.

According to the linear wave theory, the resultant force for the AUV system is $F(t) = \sum_{j=1}^{l} T_j(\omega_j) v_j(t)$, where $T_j(\omega_j)$ is the stress coefficient, which is determined by the frequency of the corresponding wave.

$$\begin{aligned} F(t) &= \begin{bmatrix} T_1(\omega) & \cdots & T_l(\omega) \end{bmatrix} v(t) \\ &= \begin{bmatrix} T_1(\omega) & \cdots & T_l(\omega) \end{bmatrix} \begin{bmatrix} I & 0 \end{bmatrix} w(t) \\ &\triangleq Hw(t) \end{aligned} \tag{6}$$

So the effect on AUV of the total wave disturbance can be described by the following system:

$$\begin{aligned} \dot{w}(t) &= Gw(t) \\ F(t) &= Hw(t) \end{aligned} \tag{7}$$

AUV in the process of operation, the wave disturbance can be directly put into the AUV dynamics mode l as external disturbing force, so we have the vertical motion model for constant speed AUV system, as follow:

$$\dot{x}(t) = Ax(t) + B\delta_s(t) + F(t) \tag{8}$$

Where,

$$x=\begin{bmatrix} w \\ q \\ \theta \\ z \end{bmatrix}, \quad A=\begin{bmatrix} m-Z_{\dot{w}} & -Z_{\dot{q}} & 0 & 0 \\ -M_{\dot{w}} & I_{yy}-M_{\dot{q}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} Z_{uw} & Z_{uq} & 0 & 0 \\ M_{uw} & M_{uq}u & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -u & 0 \end{bmatrix},$$

$$B=\begin{bmatrix} m-Z_{\dot{w}} & -Z_{\dot{q}} & 0 & 0 \\ -M_{\dot{w}} & I_{yy}-M_{\dot{q}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} Z_{uu}u^2 \\ M_{uu}u^2 \\ 0 \\ 0 \end{bmatrix}$$

3. Design Optimal Controller

3.1. Design of the Control Law

In AUV operation, the state of the system converges faster, its stability is better. In order to enhance the stability of AUV, we can choose the following quadratic performance index:

$$J=\frac{1}{2}\int_0^{\infty} e^{2\alpha t}[x(t)^T Qx(t)+\delta_s(t)^T R\delta_s(t)]dt \tag{9}$$

Where Q and R respectively is semi-definite and positive-definite matrix. $\alpha > 0$ is a known scalar function. The optimal control problem is to search the optimal control law $\delta_s^*(t)$, which makes the value of performance index (12) minimum.

Theorem 1: Consider the LQR problem of the system (8) with the performance index (9), the optimal control LQR is existent and unique, and its form as follows:

$$\begin{aligned} \delta_s^*(t) &= -R^{-1}B^T[P_\alpha x(t)+P_w w(t)] \\ &= -R^{-1}B^T[P_\alpha x(t)+P_w \hat{w}(t)] \end{aligned} \tag{10}$$

Where P_α is the unique solution the $Riccati$ matrix equation.

$$(A+\alpha I)^T P_\alpha + P_\alpha(A+\alpha I)-P_\alpha SP_\alpha + Q = 0 \tag{11}$$

P_v is the unique solution of the matrix differential equation.

$$[(A+\alpha I)^T - P_\alpha S]P_w + P_w(\alpha I + G) = -P_\alpha H \tag{12}$$

Where $S = BR^{-1}B^T$.

Proof:

Order

$$\begin{aligned} \bar{x}(t) &= e^{\alpha t}x(t) \\ \bar{u}(t) &= e^{\alpha t}\delta_s(t) \\ \bar{w}(t) &= e^{\alpha t}w(t) \end{aligned} \tag{13}$$

Taking (13) to (8) and (9), after simplification we get:

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t)+B\bar{u}(t)+H\bar{w}(t) \\ \bar{x}(0) &= x_0 \end{aligned} \tag{14}$$

Where $\overline{A}=(A+\alpha I)$.

And the new performance index as follows:

$$J=\frac{1}{2}\int_0^\infty[\overline{x}^{\mathrm{T}}(t)Q\overline{x}(t)+\overline{u}^{\mathrm{T}}(t)R\overline{u}(t)]dt \tag{15}$$

According to Pontryagin maximum principle, the optimal control problem of system (14) with the quadratic performance index (18) leads the following TPBV problems:

$$\begin{aligned}&-\dot{\lambda}(t)=Q\overline{x}(t)+\overline{A}^T\lambda(t)\\&\dot{\overline{x}}(t)=\overline{A}\overline{x}(t)-S\lambda(t)+H\overline{w}(t)\\&\overline{x}(0)=x_0,\quad \lambda(\infty)=0\end{aligned} \tag{16}$$

And the optimal control law can be expressed as:

$$\overline{u}(t)=-R^{-1}B^T\lambda(t) \tag{17}$$

In order to solve the TPBV problems (17), let:

$$\lambda(t)=P_\alpha\overline{x}(t)+P_w\overline{w}(t) \tag{18}$$

Where P_α, P_w are pending matrixes, derivate two sides of (21), and substituting the second type of (16), we get:

$$\begin{aligned}\dot{\lambda}(t)&=P_\alpha\dot{\overline{x}}(t)+P_w\dot{\overline{w}}(t)\\&=P_\alpha\{\overline{A}\overline{x}(t)-S[P_\alpha\overline{x}(t)+P_w\overline{w}(t)]\\&+H\overline{w}(t)\}+P_w[\alpha e^{\alpha t}w(t)+Ge^{\alpha t}w(t)]\\&=\left(P_\alpha\overline{A}-P_\alpha SP_\alpha\right)\overline{x}(t)-P_\alpha SP_we^{\alpha t}w(t)\\&+P_\alpha He^{\alpha t}w(t)+(\alpha P_w+P_wG)e^{\alpha t}w(t)\\&=\left(P_\alpha\overline{A}-P_\alpha SP_\alpha\right)\overline{x}(t)-P_\alpha SP_w\overline{w}(t)\\&+P_\alpha H\overline{w}(t)+P_w(\alpha I+G)\overline{w}(t)\end{aligned} \tag{19}$$

By adding (19) into the first expression of (16), it follows:

$$\begin{aligned}&(\overline{A}^TP_\alpha+P_\alpha\overline{A}-P_\alpha SP_\alpha+Q)\overline{x}(t)\\&+[(\overline{A}^T-P_\alpha S)P_w+P_w(\alpha I+G)+P_\alpha H]\overline{w}(t)=0\end{aligned} \tag{20}$$

Because of selecting either $\overline{x}(t)$, $\overline{w}(t)$ and, equation (20) is all hold, so we can get matrix differential equations of P_α, P_w.So we can get $\lambda(t)$, then from (18):

$$\overline{u}(t)=-R^{-1}B^T[P_\alpha\overline{x}(t)+P_w\overline{w}(t)] \tag{21}$$

Reference to (7) and (13), the feedforward-feedback optimal disturbance rejection control law of system (8) can be unique confirmed.

$$\begin{aligned}&\delta_s^*(t)=e^{-\alpha t}\overline{u}(t)\\&=e^{-\alpha t}*\{-R^{-1}\overline{B}^T[P_\alpha e^{\alpha t}x(t)+P_we^{\alpha t}w(t)]\}\\&=-R^{-1}\overline{B}^T[P_\alpha x(t)+P_ww(t)]\end{aligned} \tag{22}$$

Theorem 1 is proved.

Notice 1. Compared with the classical feedback optimal control law, the feedforward-feedback optimal disturbance rejection control law (22) has the feed-forward items. So for a system with a disturbance its control performance is clearly superior to the classical feedback optimal control law.

Lemma 1 If (A, B) is completely controllable, then $(\bar{A}, \bar{B})$ is completely controllable

Proof. If (A, B) is completely controllable, so:

$$rank[B, \quad AB, \quad \cdots, \quad A^{n-1}B] = n \tag{23}$$

Let,

$$\bar{A}^K = (A + \alpha I)^K = A^K + f_K(A) \tag{24}$$

Where, $f_K(A)$ is Low-level sub-K expression, so:

$$\begin{aligned} & rank\left[B, \quad \bar{A}B, \quad \bar{A}^2B, \quad \cdots, \quad \bar{A}^{n-1}B\right] \\ & = rank\left[B, \quad AB+\alpha B, \quad A^2B+f_2(A)B, \quad \cdots, \quad A^{n-1}B+f_{n-1}(A)B\right] \\ & = rank\left[B, \quad AB, \quad A^2B, \quad \cdots, \quad A^{n-1}B\right] \\ & = n \end{aligned} \tag{25}$$

The proof is complete. Similarly the observability can prove.

3.2. Design of the Disturbances Observer

In fact, $w(t)$ in (22) is unknown for it is the state vector of exosystem (2). The feedforward control term in (22) is physically unrealizable in the practical engineering. In this section, we introduce a disturbance observer to make it realizable.

Suppose that exosystem (2) is observable completely. Construct a disturbance observer as follows:

$$\begin{aligned} & \dot{\hat{w}}(t) = G\hat{w}(t) - K[F(t) - H\hat{w}(t)] \\ & \hat{w}(t_0) = \hat{w}_0 \end{aligned} \tag{26}$$

Where $\hat{w}(t)$ is the output vector of (26), K is the observer matrix of appropriate dimensions. And the observer error is denoted as:

$$\tilde{w}(t) = w(t) - \hat{w}(t) \tag{27}$$

Then we have:

$$\dot{\tilde{w}}(t) = (G - KH)\tilde{w}(t) \tag{28}$$

Because (G, H) is observable, eigenvalues of $G - KH$ can be chosen to make the observer error vector $\tilde{w}(t)$ converges to zero at an appointed speed of exponential attenuation, that is:

$$\lim_{t\to\infty} \tilde{w}(t) = \lim_{t\to\infty} \exp((G - KH)t)\tilde{w}(t_0) = 0 \tag{29}$$

So the physically realization of the control law (22) can be guaranteed and (22) is rewritten as follows:

$$\delta_s^*(t) = -R^{-1}B^T[P_\alpha x(t) + P_w \hat{w}(t)] \quad (30)$$

4. Simulation Example

The hydrodynamic coefficients of a foreign typical AUV to the nominal model [10] as follows:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -1.040 & 0.865 & -0.020 & 0 \\ 6.000 & -0.681 & 0.708 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \\ z \end{bmatrix} + \begin{bmatrix} -0.072 \\ -0.722 \\ 0 \\ 0 \end{bmatrix} \delta_s(t) \quad (31)$$

The initial state of AUV is $\begin{bmatrix} 0 & 0 & 5 & 0 \end{bmatrix}^T$, taking the axial velocity 2m/s, and the parameters of wave force disturbances as follows, $T_1(\omega) = 1.1, T_2(\omega) = 0.8, T_3(\omega) = 1.3, T_4(\omega) = 1.5$, The parameters of the quadratic performance index $Q = I, R = 1, \alpha = 0.5$.

(1) Select

$$G = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad w(0) = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$$

The wave force disturbances are sinusoidal signal. Using LQR controller and optimal disturbance rejection controller (ODRC), and the simulation comparative curves of $x(t), u(t)$, as follows:

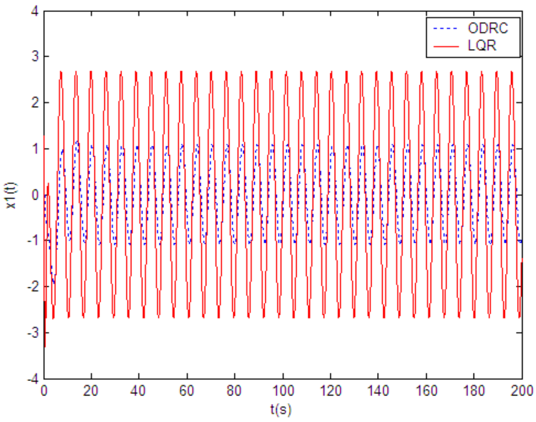

Figure 2. State vector $x_1(t)$

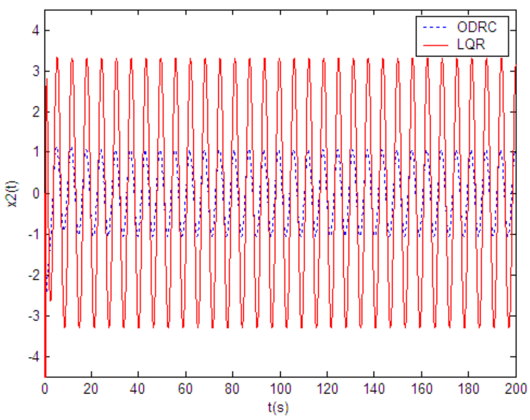

Figure 3. State vector $x_2(t)$

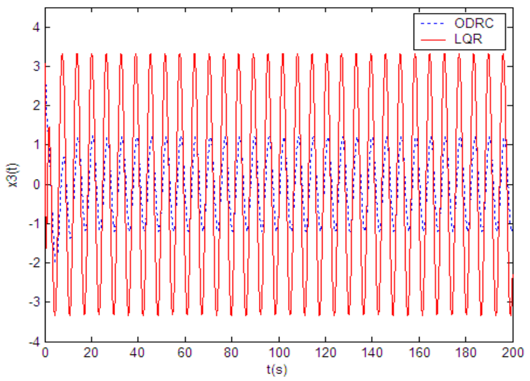

Figure 4. State vector $x_3(t)$

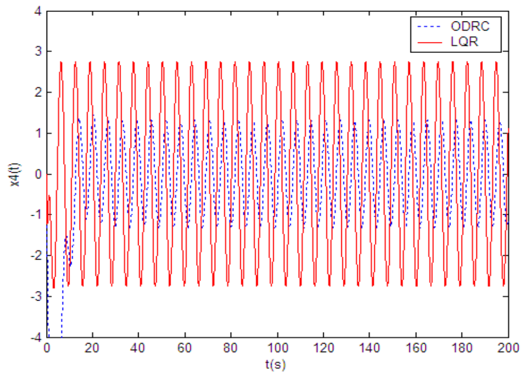

Figure 5. State vector $x_4(t)$

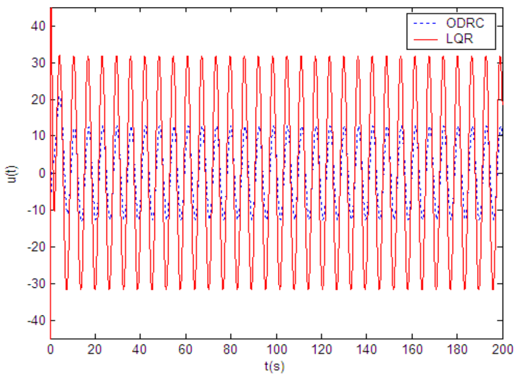

Figure 6. Control vector $u(t)$

(2) Select

$$G = \begin{bmatrix} -0.2 & 2.3 \\ -2.1 & -0.4 \end{bmatrix}, \quad w(0) = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$$

The wave force disturbances are the convergent signal, Using LQR controller and optimal disturbance rejection controller (ODRC), and the simulation comparative curves of $x(t), u(t)$, as follows:

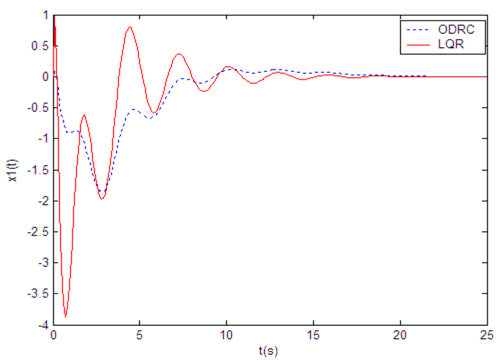

Figure 7. State vector $x_1(t)$

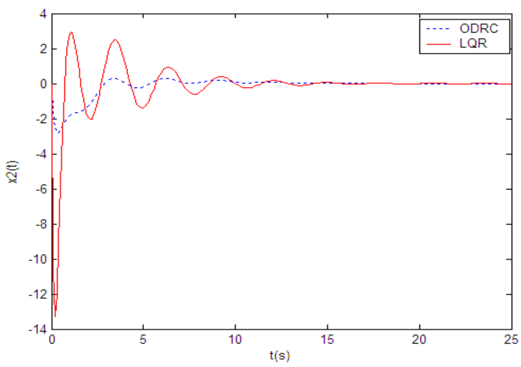

Figure 8. State vector $x_2(t)$

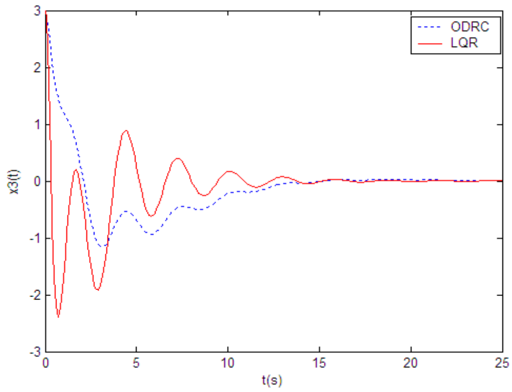

Figure 9. State vector $x_3(t)$

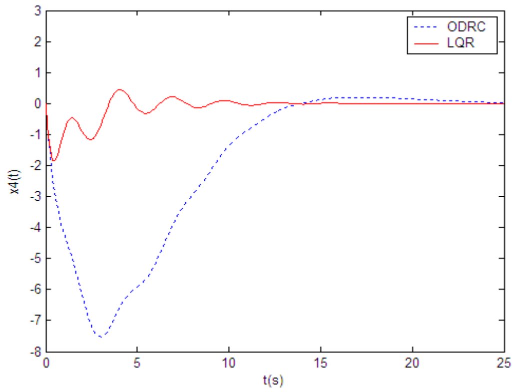

Figure 10. State vector $x_4(t)$

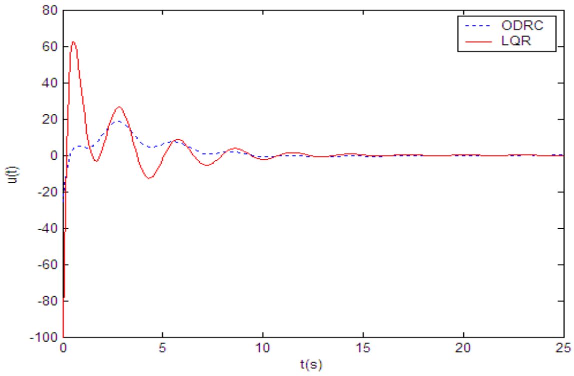

Figure 11. Control vector $u(t)$

From the simulation curves, it can be seen that the presented optimal disturbance rejection controller is effective, and it is more robust about external disturbances than LQR controller.

5. Conclusion

This paper concentrates on underactuated AUV systems control problem in vertical plane affected by the wave force disturbances, and based on the quadratic optimal control theory and stability degree constraint, the optimal disturbance rejection controller with a higher mean-square convergence rate is derived from the Riccati equation and the Sylvester equation, and we introduce a disturbance observer to make it realizable. Simulation results show that the designed control law has a good convergence effect and effectively suppress the external disturbances.

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