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# A Quantum Chaos Clonal Multiobjective Evolutionary Method Reasearch

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***Abstract***

*A quantum chaos cloning multi-objective evolutionary algorithm was proposed herein based on chaos search ergodicity, quantum computing efficiency and clonal selection theory of antibodies in artificial immune system. The qubits encoded initial population is used in the new algorithm, Chaos quantum rotation gates are introduced to update individuals, crowding distance is used to keep solution population distribution and diversity. Theoretical analysis and simulation show the effectiveness of the algorithm.*

***Keywords****: multiobjective optimization, clonal select algorithm, chaos, quantum bit encoding*



## 1. Introduction

Many optimization problems can be attributed to the multi-objective optimization problem (Multi-objective Optimization Problem, MOP) in Science and Engineering Practice [1, 2], and it is very difficult to solve. At present, the existing multi-objective optimization algorithm was based on local presence convergence, diversity of the population is poor, there is high time complexity and it is sensitive to parameters and other issues. Multi-objective optimization problem is generally considered the best overall objective is to optimize multiple targets weigh an overall objective to be achieved under, and therefore need to consider the right of each sub-goal weight, and require high latitudes, large scale, so that it becomes difficult to optimize, and traditional optimization means more demanding in forming objective function. In geting better in this regard applied algorithm, there were genetic algorithms, simulated annealing, tabu search algorithm, neural network algorithm [3-5]. Feynman put forward the concept of quantum computingin the early 1980s [6], then Shor proposed factorization of large numbers matter in 1994 [7], Grover proposed quantum search algorithm of disorderly database in 1996 [8].

Therefore, the study of a more efficient multi-objective optimization algorithm is very scientific value and practical significance. Artificial Immune System (AIS) is a mimic biological immune system function in an intelligent way with a strong recognition, learning, memory and adaptability. Quantum evolutionary algorithm (QEA) have better population diversity and global optimization capability, they have smaller groups, but it does not affect the performance of the algorithm, etc., but how to set the corner is very difficult to design in the quantum evolutionary algorithm. Chaotic is with randomness, ergodicity, regularity, etc., according to its own laws, chaotic can not be repeated traverse all states within a certain range. In this paper, chaotic quantum cloning MOEA (Chaos Quantum Clonal Multiobjective Evolutionary Algorithm, CQCMEA) is proposed. The new quantum encoding method is introduced in the framework of of clonal selection algorithm in the algorithm, the the corresponding quantum chaos revolving door mutation operator is designed, the crowding distance is used to keep distribution and population diversity. Theoretical analysis and numerical simulations show CQCMEA can solve the multi-objective optimization problem, it is with a strong global search capability.

## 2. Materials and Methods

### 2.1. Multi-objective Optimization Problem

With n decision variables and m objective functions, m multi-objective optimization problem (MOP) can be expressed as Formula (1):

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$$\begin{aligned} \min \quad & \boldsymbol{y} = \boldsymbol{F}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x})) \\ \text{s.t.:} \quad & g_i(\boldsymbol{x}) \geq 0, \quad i = 1,2,\cdots,q \\ & h_j(\boldsymbol{x}) = 0, \quad j = 1,2,\cdots,p \end{aligned} \tag{1}$$

Where, $\boldsymbol{x} = (x_1, x_2, \cdots, x_n) \in \boldsymbol{X} \subset \mathbf{R}^n$ is the decision variables, $\boldsymbol{X}$ is n-dimensional decision space; $\boldsymbol{y} = (y_1, y_2, \cdots, y_n) \in \boldsymbol{Y} \subset \mathbf{R}^m$ is objective function, $\boldsymbol{Y}$ is m-dimensional target space. $\forall \boldsymbol{x}, \boldsymbol{x}^* \in \boldsymbol{X}_f$ , $\boldsymbol{x}^*$ dominates $\boldsymbol{x}$ ( it is referred to as $\boldsymbol{x}^* \succ \boldsymbol{x}$ )

$$\forall r = 1,2,\cdots,m \quad f_r(\mathbf{x}') \leq f_r(\mathbf{x}'') \wedge \exists s \in \{1,2,\cdots,m\} \quad f_s(\mathbf{x}') < f_s(\mathbf{x}'') \tag{2}$$

$\boldsymbol{x}^* \in \boldsymbol{X}_f$ is called as Pareto- the optimal solution (or non-inferior solutions), $\neg \exists \boldsymbol{x} \in \boldsymbol{X}: \ \boldsymbol{x} \succ \boldsymbol{x}^*$. Pareto- 0ptimal solution set is defined as Formula (3):

$$P_S = \left\{ \boldsymbol{x}^* \middle| \neg \exists \boldsymbol{x} \in \boldsymbol{X}: \ \boldsymbol{x} \succ \boldsymbol{x}^* \right\} \tag{3}$$

Pareto- optimal solution set is a collection of all Pareto-optimal solution, Pareto is isolution n optimal solution set, which solution set is corresponding to the composition set of the objective function value.

$$P_F = \{ \boldsymbol{F}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x})) \middle| \boldsymbol{x} \in P_S \} \tag{4}$$

It is called Pareto-front end.

### 2.2. Chaotic Quantum Cloning MOEA

#### 2.2.1. Problem Representation

1) Initialization groups: Optimization problem is corresponds to an antigen, antibody is feasible solution which is corresponding to the problem. Initial population is produced by the following method [9]:

First, initial values are given n different variables, n is the dimensional number of optimization variables. So that k = 0, using Equation (5):

$$x_{k+1}^i = \mu_i x_k^i (1 - x_k^i), i = 1,2,\cdots,n \tag{5}$$

It is first qubit on the antibody which n chaotic variable $x_1^i (i = 1,2,\cdots,n)$ initialized population is generated, where $\mu_i = 4$ $x_k^i \in (0,1)(i = 1,2,\cdots,n)$ , i was chaotic variable number. $k = 1,2,\cdots,N$ -1, N other antibodies were produced by the above method. The N antibodies are to form the initial population. For example, the k-th antibody $P_k$ is used to initialize the result of the formula (6):

$$P_k = \begin{vmatrix} \alpha_k^1 & \alpha_k^2 & \cdots & \alpha_k^n \\ \beta_k^1 & \beta_k^2 & \cdots & \beta_k^n \end{vmatrix} \tag{6}$$

Where $\alpha_k^i = \cos(2x_k^i \pi); \beta_k^i = \sin(2x_k^i \pi)$.

2) Solution space transformation: Each group antibody comprises 2n qubits' probability amplitude, linear transformation is used, this can be mapped by the 2n probability amplitude units to traverse solution space of the space optimization problem. Each probability amplitude of

antibody is corresponding to an optimized variable of solution space, i-th qubit of antibody $P_k$ is denoted as $[\alpha_k^i, \beta_k^i]^{\mathrm{T}}$ ,then the corresponding solution space variable is respectively formula (7) and (8).

$$X_{1i}^{k} = \frac{1}{2}[b_i(1+\alpha_k^i) + a_i(1-\alpha_k^i)] \tag{7}$$

$$X_{2i}^{k} = \frac{1}{2}[b_i(1+\beta_k^i) + a_i(1-\beta_k^i)] \tag{8}$$

Thus, each antibody is corresponding to the two solutions of optimization problem. Where, $[a_i, b_i]$ is the domain of variable $x_i$; probability amplitude $\alpha_k^i$ of Quantum state |0>corresponds to $X_{1i}^k$; probability amplitude $\beta_k^i$ of Quantum state |1 corresponds to $X_{2i}^k$, $i = 1,2,\cdots,n$ ; $k = 1,2,\cdots,N$.

3) Immunodominant antibody: Antibodies $P_i$ (corresponding solution is $\boldsymbol{x}^i$) is called immunodominant antibody in the antibodies' population $P = \{P_1, P_2, \cdots, P_N\}$, If and only if there is no other antibody $P_j$ ( $j = 1,2,\cdots,N \wedge j \neq i$), $P_j$ is the corresponding solution $\boldsymbol{x}^j$ in the population, so that:

$$\forall r = 1,2,\cdots,m \quad f_r(\mathbf{x}^i) \leq f_r(\mathbf{x}^j) \wedge \exists s \in \{1,2,\cdots,m\} \quad f_s(\mathbf{x}^i) < f_s(\mathbf{x}^j) \tag{9}$$

As can be seen from the above equation, immunodominant antibody is efficient solution of an antibody current populations or non-Pareto optimal solution.

### 2.2.2. Proportion Clones

After the proportion cloning is implemented to the antibody population $P = \{P_1, P_2, \cdots, P_N\}$, antibodies' population $P'$ is to obtained, it is as follows:

$$P' = \{P_1^1, P_1^2, \cdots, P_1^q\} + \cdots + \{P_N^1, P_N^2, \cdots, P_N^q\} \tag{10}$$

Wherein: $P_j^i = P_j (i = 1,2,\cdots,q,\ j = 1,2,\cdots,N)$, $q = m_c = R$ is Cloning ratio.

### 2.2.3. Revolving Door Variation of Chaotic Quantum

Chaos is the essential characteristic of nonlinear systems with a series of special properties of randomness, ergodicity and regularity. There is chaos effect in quantum systems, which is different from the traditional chaotic phenomena [10], so chaos is used to design the corner of quantum revolving door. Logistic mapping [11] is Formula (11):

$$x_{n+1} = \mu x_n (1 - x_n), \qquad n = 0,1,2,\cdots \tag{11}$$

This is a typical chaotic system, where $\mu$ is the control parameters.

1) Variation Strategy 1: The main evolution mode of quantum evolution computing is changing the phase of the antibody qubit by quantum revolving door to realize the mutation, but the quantum revolving door needs to determine the direction and size of the angle. For angular direction, now is almost all based on a lookup table, which involve multiple conditions to determine the impact the efficiency of the algorithm. To solve these problems, the introduction of quantum chaos mapping revolving door rotation angle is determined, as well as the rotation angle exhibits bidirectional design such as formula (12).

$$\Delta\theta_k^i = \begin{cases} x_{k+l}^i & if \quad rand > 0.5 \\ -x_{k+l}^i & else \end{cases} \tag{12}$$

It is determined angle. $l \in [1, m_c]$, $m_c$ is the largest numberwhich is allowed to traverse the chaos. In order to traverse the range which are presented randomness, ergodicity and uniformity, chaos variable $x_{k+1}^i$ is calculated as formula (13):

$$x_{k+1}^i = \mu_i x_k^i (1 - x_k^i) \tag{13}$$

The formula (13) are used and repeated $l$ times to obtain chaotic variables $x_{k+l}^i$.

Let the k-th variant parent $P_k'$ be Formula (14):

$$P_k' = \begin{vmatrix} \cos(\theta_k^1) & \cos(\theta_k^2) & \cdots & \cos(\theta_k^n) \\ \sin(\theta_k^1) & \sin(\theta_k^2) & \cdots & \sin(\theta_k^n) \end{vmatrix} \tag{14}$$

Where, $\theta_k^i = 2x_k^i \pi$ ($i = 1,2,\cdots,n$ ; $k = 1,2,\cdots,N \times q$). After quantum chaos revolving door is mutated, antibody is Formula (15):

$$P_k'' = \begin{vmatrix} \cos(\theta_k^1 + \Delta\theta_k^1) & \cos(\theta_k^2 + \Delta\theta_k^2) & \cdots & \cos(\theta_k^n + \Delta\theta_k^n) \\ \sin(\theta_k^1 + \Delta\theta_k^1) & \sin(\theta_k^2 + \Delta\theta_k^2) & \cdots & \sin(\theta_k^n + \Delta\theta_k^n) \end{vmatrix} \tag{15}$$

2) Variation Strategy 2: In order to prevent the quantum rotation angle is too large to miss a good solution, the traversing range of quantum revolving door corner is set as ($0.005\pi$ , $0.1\pi$)**Error! Reference source not found.** , Then according to formula (16)

$$\Delta\theta_k^i = \begin{cases} 0.005\pi + (0.1\pi - 0.005\pi) x_{k+l}^i & if \quad rand > 0.5 \\ -(0.005\pi + (0.1\pi - 0.005\pi) x_{k+l}^i) & else \end{cases} \tag{16}$$

it is used to replace mutation strategy 1 in the Formula (12), namely it is mutation strategy 2.

#### 2.2.4. Quantum Cloning Multi-Objective Evolutionary Algorithm (MOEA) Framework

**a) Algorithm 1: Chaos quantum cloning MOEA**

Input: Set antibody population size N, the immunodominant antibody population size $N_d$, cloning proportion q, chaos traverse the maximum allowable number $m_c$ of chaos traverse mutation rate $p_m$, the maximum iterative generation $Maxgen$[12].

Output: Final Pareto- approximate optimal solution set $D_{Maxgen+1}$.

Step 1: Initialization antibody populations: $P_t = \{P_1, P_2, \cdots, P_N\}$, $D_0 = \phi$, set the initial generation: $t$=0.

Step 2: solution space transform, to calculate the objective function value which is corresponding antibody populations $P_t$.

Step 3: dominant antibody $PD_t$ is obtained and denoted from the $P_t$ and $D_t$, if $|PD_t| \le N_d$, then $D_{t+1} = PD_t$; otherwise, crowding distance of all individuals is calculated in

$PD_t$, according to the crowding distance in descending order, the front $N_d$ antibodies are choosed to form immune dominant populations $D_{t+1}$.

Step 4: If $t \geq Maxgen$, the output is $D_{t+1}$, to stop running, otherwise $t = t+1$.

Step 5: new antibodies' population $P_t$ is obtained from $D_t$ , if $|D_t| > N$, crowding distance of all individuals is calculated from $D_t$, by crowding distance in descending order, the top N antibodies are selected to form a new antibody population $P_t$; if $|D_t| < N$, $P_t = D_t$ is taken directly, $N - |D_t|$ antibodies are selected randomly from contemporary disposable antibodyies to join the new antibody populations $P_t$.

Step 6: Proportion cloning operation is done to antibody population to generate antibody populations $P'$.

Step7: quantum chaos revolving door variation is implemented to the population $P_t'$, antibody populations $P_t''$ are generated, so $P_t = P_t''$, to turn Step 3.

**b) Algorithm 2 Quantum Clonal Multi-objective Evolutionary Algorithm (QCMEA)**

Step 1: Initialization evolution generation: $t$=0;

Step 2: initial population $\boldsymbol{Q}(t)$, archive collection $\boldsymbol{A}(t) = \{\ \}$;

Step 3: $\boldsymbol{P}(t)$ is generated by $Q(t)$;

Step 4: Assessment $\boldsymbol{P}(t)$;

Step 5: $\boldsymbol{A}(t)$ is obtained by the $\boldsymbol{P}(t)$;

Step 6: according to certain rules, the deletion of the archive collection $\boldsymbol{A}(t)$ reaches the set requirements;

Step 7: if the stop criterion is meet, output results, stop running; otherwise $t = t+1$, turn Step 8;

Step 8: $\boldsymbol{P}(t)$ is and generated by the selected compression $\boldsymbol{A}(t-1)$ ;

Step 9: $\boldsymbol{P}(t)$ is cloned, $\boldsymbol{Q}'(t)$ is generated by corresponding $\boldsymbol{Q}(t)$;

Step 10: $\boldsymbol{Q}''(t)$ is generated qq by $\boldsymbol{Q}'(t)$ quantum revolving door variation, make $\boldsymbol{Q}(t) = \boldsymbol{Q}''(t)$. turn Step 3;

## 3. Test and Discussion

### 3.1. Parameter Settings

In order to verify the effectiveness of the proposed algorithm, the CQCMEA is compared to the three algorithms with NSGA-II [13], PESA [14] and SPEA2 [15], for solving two well-known multi-objective optimization problem of ZDT6 [16] and DTLZ1 [17] problem. The main parameters are as follows:

For CQCMEA, the size of the population are 50 antibodies, the immunodominant antibody population size is 100, cloned ratio is 3, the maximum number of chaotic traverse is 30. For the NSGA-II, the population size is 100; for PESA, the population size of internal evolution and external archive size are set to 100, the number of super-grid cells is 10 in each dimension; for SPEA2, the population size of internal evolution the outside collection are located as 100. For the NSGA-II, PESA and SPEA2, we have adopted the Mutual simulation (simulated binary crossover, SBX) and polynomial mutation (polynomial mutation, PM) [18]. Of the four algorithms, crossover was 0.8, mutation rate is $1/n$, which $n$ represents the number of variables, the number of iterations is set to 500. Solution Set in this paper chooses two coverage [19], uniformity index [20] and the generation distance [21] for the evaluation of the four algorithms.

### 3.2. Experimental Results and Performance Analysis

The following experiments run independently 30 times for each test question. Figure 1 is comparison box diagram for CQCMEA, NSGA-II, PESA and SPEA2 based on coverage metrics [22]. Here, C represents the solution set which CQCMEA obtained; N represents the solution set which NSGA-II obtained; P represents the solution set which PESA obtained; S represents the solution set which SPEA2 obtained.

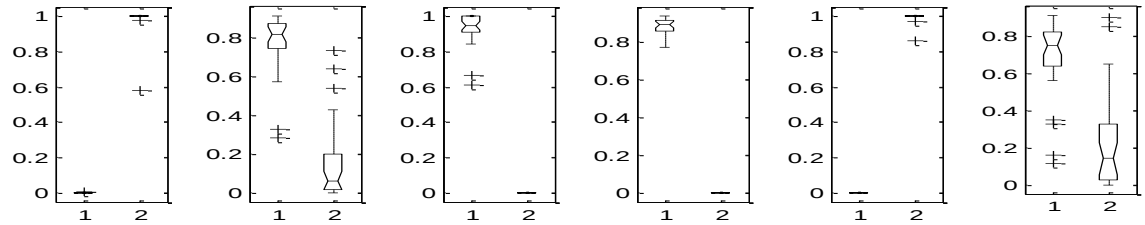


Figure 1. Cartridge Figure of CQCMEA, NSGA-II, PESA and SPEA2 Coverage Indicators of Solution Which is Obtained by Solving the Two Problem, in Each Figure, the Left Side of the Box Represent CS (C, N), CS (C, P) and CS (C, S) Distribution the Right of Box Represents CS (N, C), CS (P, C) and CS (S, C) Distribution

Two coverage comparison of CQCMEA, NSGA-II, PESA and SPEA2 Solution Set shows that, the box diagram on DTLZ3, CS (C, N), CS (C, P) and CS (C, S) is higher than the box diagram of CS (N, C), CS (P, C) and CS (S, C); and in the ZDT6, the box diagram of CS (N, C), CS (S, C) and CS (C, P) is higher than the CS (C, N), CS (C, S) and CS (P, C). So, DTLZ3 on CQCMEA is better than NSGA-II, PESA and SPEA2; ZDT6 on CQCMEA is better than PESA, but NSGA-II and SPEA2 is superior CQCMEA.

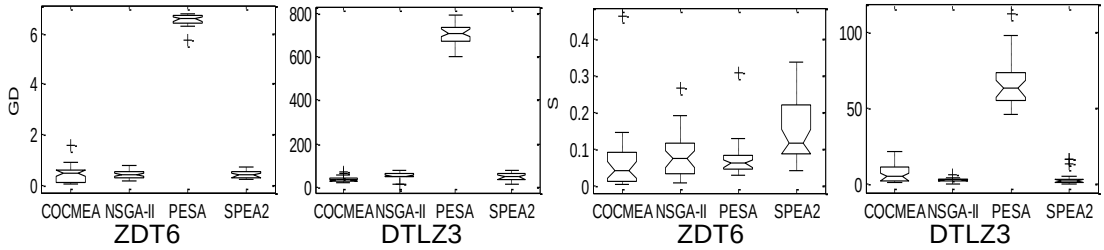


Figure 2. Box Diagram of Generations Distance and Uniformity Index which is obtained by CQCMEA, NSGA-II, PESA and SPEA2 Solving ZDT6 and DTLZ3 Problems

Figure 2 illustrates box diagram of generation distance index and uniformity index which is obtained by independent run for 30 times. Figure 2 shows that, in consideration of the convergence index case, CQCMEA is best on two issues. In consideration of the diversity index Spacing case, CQCMEA is best on ZDT6 question, and SPEA2 is best on DTLZ1 problem.

Summarize the results, the following conclusions: for ZDT6 and DTLZ3 two questions, on the convergence of indicators, CQCMEA is best on both issues; in diversity holding, CQCMEA has the best effect in the one of two issues.

### 3.3. Analysis of the Algorithm Complexity

Suppose the size of the antibodies' population for $N$ , scale of the immunodominant antibody populations is $N_d$ , chaos traversal allowed maximum number is $m_c$ , the cloning proportion (ie clone size $N_c = N \times q$ ) is $q$ , then CQCMEA complexity of once every iteration is as follows:

The time complexity of the solution space conversion is $O(N_d + N_c)$; time complexity of proportional clone is $O(N_c)$; the worst time complexity of the obtain immunodominant antibody populations is $O((N_d + N_c)^2)$; the worst time complexity of new antibody updated populations is $O(N_d \log_2 N_d)$; the worst time complexity of the updated immunodominant antibody populations is $O((N_d + N_c)\log_2(N_d + N_c))$; worst time complexity of quantum chaos revolving door variant is $O(N_c m_c)$. So the total worst-case time complexity is as the formula (17):

$$O(N_d + N_c) + O((N_d + N_c)^2) + O((N_d + N_c)\log_2(N_d + N_c)) + O(N_d \log_2 N_d) + O(N_c) + O(N_c m_c) \quad (17)$$

According to the symbol O operation rules, worst time complexity of CQCMC for each iteration can be simplified to formula (18):

$$O((N_d + N_c)^2) \quad (18)$$

### 3.4. Simulation Comparison

To test the performance of the algorithm, we use three sets of frequently used data knapsack problem. The number of experiments were chosen backpack 2, 3 and 4, the number of articles 250, 500 and 750, respectively. A similar method is found to compare the performance of the algorithm, this paper chose the non-dominated sorting genetic (NSGA-II) algorithm and Pareto Envelope-based Selection Algorithm for Multiobjective Optimization (PESA). In order to compare the fairness of the algorithm and NSGA2 and PESA algorithm uses the same set of parameters, and each set of data in the knapsack problem are the same number of iterations. Experiments run for nine knapsack problem were CQCMEA, PESA and NSGA2 100 times each, and record the value of each method and function C function S of each run. Figure 3 shows the distribution of the knapsack problem (2,750) of a particular experiment Pareto optimal front-end solution. Table 1 shows the average result of the comparison in the function S of the three algorithms were performed after 100 measurements. Table 1 shows the average result of the comparison function C on three algorithms were performed after 100 measurements.

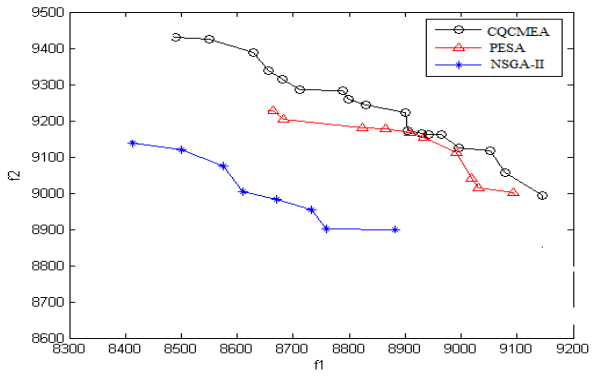


Figure 3. (2,750) Knapsack Problem

Figure 3 respectively after the distribution of the three algorithms run on behalf of the 100 non-dominated solution, it can be seen from Figure CQCMEA search space wider than PESA and NSGA2, quality of the resulting solution will be better.

Comparative results from Table 1, after each algorithm knapsack problem nine experiments each run 100 times, an average value of S CQCMEA get the maximum, which is CQCMEA algorithm Pareto optimal solution set better diversity, distribution more uniform. It is

concluded that the best performance is CQCMEA under normal circumstances, indicating CQCMEA is promising. This also proves that the revolving door adjustment strategy employed herein is successful, the algorithm can effectively avoid falling into local optimum and prevent premature. And on the question of increase in algorithm complexity, the performance is very good, so when solving complex multidimensional problem, CQCMEA more suitable.

Table 1. S Performance Comparison of Three Algorithms

| MKP | NSGA-II | PESA | CQCMEA |
|---|---|---|---|
| 2-250 | 7.211e07 | 8.254e07 | 8.665e07 |
| 2-500 | 3.243e08 | 3.334e08 | 3.543e08 |
| 2-750 | 6.921e08 | 7.376e08 | 7.465e08 |
| 3-250 | 6.881e11 | 6.854e11 | 6.865e11 |
| 3-500 | 5.119e12 | 5.343e12 | 5.423e12 |
| 3-750 | 1.703e13 | 1.765e13 | 1.854e13 |
| 4-250 | 5.143e15 | 5.165e15 | 5.243e15 |
| 4-500 | 7.211e16 | 7.233e16 | 7.365e16 |
| 4-750 | 3.633e17 | 3.565e17 | 3.922e17 |

## 4. Conclusion

Based on the new quantum bit encoding and quantum chaos revolving door variation, chaotic quantum cloning multi-objective evolutionary algorithm was proposed in this paper. Theoretical analysis and numerical simulations show that, CQCMEA can solve the multi-objective optimization problem, with a strong global search capability.

CQCMEA algorithm advantages and performance analysis:

1) New quantum encoding method is introduced, a pair of qubits can ba used to represent a component of two different solutions, so that there exists a group of qubits, any group (including qubits) are optimal solution which corresponds to the global. In the case of constant population size, the encoded qubits can be used to extend the solution space ergodic; because the real number coding and decoding, the time-consuming, low accuracy and other defects are avoided in decoding of binary solutions, thereby the efficiency of the optimization algorithm is improved.

2) Cloning operators maintain better the breadth of optimum distribution. The chaos produces initial antibodies against the population and the seed population as a whole cloning operation, it can expand the search space, it is conducive to produce new antibodies stocks and global search algorithm, so as to ensure a broad understanding of the solution distribution.

3) In chaos quantum rotation gate mutation operator, chaotic mapping is used to determine the angle of quantum rotation gate, direction is determined by random method, cumbersome of updated lookup and lookup slow are reduced in the traditional quantum revolving door, due to the ergodicity of chaos, CQCMEA algorithm can search the whole solution space, to avoid falling into local optimum.

4) The updated operator of immunodominant antibody populations maintain better the uniformity of the optimal solution. In the new algorithm, the proportion of cloning and quantum chaos revolving door are mutated every time, it can give a certain number of non-dominated solutions, when the number of non-dominated solutions exceeds the set value, according to the crowding distance, these non-dominated solutions in more crowded and its corresponding antibody solutions group of individuals are first removed, until it reaches the set value, then these more evenly distributed optimal solution is corresponding to an individual in the next generation of operations, in order to better ensure optimal solutions for uniform distribution.

The above three points is the most important measure of multi-objective optimization algorithm for optimal solution set quality, therefore, the optimal solution was obtained by the new algorithm and will be of higher quality, but also to ensure good convergence speed.

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