

Using A Fuzzy Number Error Correction Approach to Improve Algorithms in Blind Identification

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Abstract

As part of a detailed study on blind identification of Gaussian channels, the main purpose was to propose an algorithm based on cumulants and fuzzy number approach involved throughout the whole process of identification. Our objective was to compare the new design of the algorithm to the old one using the higher order cumulants, namely Alg1, Algat and the Giannakis algorithm. We were able to demonstrate that the proposed method -fuzzy number error correction- increases the performance of the algorithm by calculating the ratio of squared errors of ALGaT and AlgatF. The method can be applied to any algorithm for more improvement and efficiency.

Keywords: Blind identification, Fuzzy number, FIR channel, Gaussien channel, cumulant.

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1. Introduction

process of identification has now become crucial and is prominent in several fields, including astrophysics, geology, data transmission, radio communication, mobile radio. Thanks to Y.Sato that the issue of identification in its various aspects was raised. It has contributed to the resolution of many problems [1].

The transmission of information through a physical medium may undergo several physical alterations or modifications, affecting the nature and even the direction of the initial information. They are essentially physical phenomena whose impact is quite considerable on the authenticity of the message induced by the information. Major examples include absorption, refraction, reflection or diffusion. These cases of impact generate a signal distortion through attenuation and interference between symbols (IES). Moreover, in digital communication, such phenomena altering the amplitude and the phase can be modeled by a multi-path transmission channel infected with a white noise. Such a model uses digital Finite Impulse Response filter (FIR) infected by a Gaussian white noise [2, 3, 4, 5, 6, 7].

Identification methods allow us to determine the channel impulse response of FIR. Tackling into account on related literature, one can identify more methods and identifications that can be classified into three categories according to resolution methods [8] [9]. This involves the use of oversized linear algebraic systems, explicit solutions and solutions using cumulants which are easy to implement thoroughly. However, for the first two methods of resolution there is a clear inefficiency since minimizing functions presents many neighboring local minima whose computational complexity is big. The third method of resolution using a higher order cumulant is also unreliable and less efficient. It is for this reason that we think action should be taken to improve the reliability of the identification through the use of fuzzy number method in the implementation of higher order cumulants used in the case of non-Gaussian frequency distributions. The fuzzy number error correction method allows us to eliminate extreme values that may affect the calculation of targeted values.

Our goal is to optimize efficiency the processes of blind identification and the sensible use of higher order cumulants whose value for a Gaussian distribution is zero. Moreover, their use in the case of identification of the noisy channel by a Gaussian white noise is very frequent. Our

study will focus on the blind identification of linear non-Gaussian process adjusted average (MA) [2, 3, 4, 6, 7].

To achieve this goal, we will present a typology of order cumulants, algorithm using three and four cumulants to improve the performance of standard algorithms. Next, we will implement the assumptions related to the channel model (MA) to identify it with its useful relationships. We will move on to the presentation of two estimation methods based on cumulants including Alg1 algorithm [10] et al, the algorithm C (q, k) Giannakis [3]. Finally, we will provide a detailed presentation of our algorithm, which will be followed by a simulation to compare and assess the effectiveness of different algorithms presented in this work.

2. Model and Fundamental relationships

2.1. Hypothesis and model

We always design a model of a Single Input and Single Output (SISO) channel with a multipath phenomenon, Fig.1, by using a linear digital FIR.

The equation of the finite differences model for the FIR moving average channel (Mobile Average:

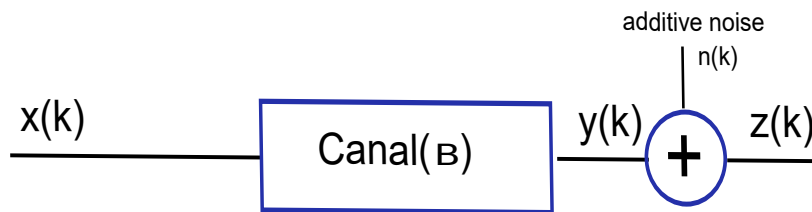


Figure 1. FIR Model channel

MA), is represented by the following [10], [11], with out noiseless:

$$Y(k) = \sum_{j=0}^q b(j).X(k - j), \quad b(0) = 1 \text{ (outnoiseless);} \tag{1}$$

and with noise:

$$Z(k) = Y(k) + n(k) \tag{2}$$

where $X(k)$ is a non-Gaussian excitation inaccessible to independent components and identically distributed (iid) with zero mean, of variance σ_x^2 , with at least one non-zero $m > 2$ order and checking $E[X^{2m}(k)] < \infty$.

$n(k)$ is a white Gaussian noise, independent of the input $X(k)$ and unknown power spectral density.

$B = [b(0), b(1), \dots, b(q)]$ represent the impulse response of FIR channel. $b(i)$ are constant for a stationary time-invariant channel. q is the order of the channel, assumed to be known [12].

2.2. Fundamental Relationships

Related literature shows that there are many important algorithms based on higher order cumulant. In this paragraph, we present the fundamental relationships using the cumulants in the case of a stationary time-invariant Gaussian noisy channel.

2.2.1. Moment and Cumulant

In this section, we present some definitions of higher order statistics, moments and cumulants. Let $x(k)$, where $1 \leq k \leq N$, is a real discrete stationary process with N length, so its moment of order m is given by [3] [4] [10] [5]

$$M_{m,x}(t_1, t_2, \dots, t_{m-1}) = E\{x(k)x(k + t_1)x(k + t_2)\dots x(k + t_{m-1})\} \tag{3}$$

Where $E\{\cdot\}$ represents the mathematical expectation.

The cumulant of order n of a non-Gaussian stationary process is given by:

$$\begin{aligned} C_{m,x}(t_1, t_2, \dots, t_{m-1}) = \\ M_{m,x}(t_1, t_2, \dots, t_{m-1}) - M_{m,G}(t_1, t_2, \dots, t_{m-1}) \end{aligned} \quad (4)$$

This relationship shows the importance of cumulants estimators relatively to the time when it comes noise Gaussian in nature.

2.2.2. Higher order cumulants

The most used moments in practice are moments of order m lower or equal to 5. In this section we give the expression cumulant based moments. The given expressions are simplified in the case of the samples adjusted to a zero mean (centered System **C.S**).

The cumulant of order m = 1 is given by:

$$C_{1,x} = M_{1,x} = E\{x(k)\}. \quad (5)$$

is equal to 0 for a zero-mean sample: centered sample. The expression (5) is equal to 0 for a zero-mean sample: centered sample (**C.S**).

The cumulant of order m = 2 is given by:

$$C_{2,x}(t_1) = M_{2,x}(t_1) - (M_{1,x})^2 \quad (6)$$

In the case of a system (**C.S**) expression (6) becomes:

$$C_{2,x}(t_1) = M_{2,x}(t_1) \quad (7)$$

The cumulant of order m = 3 is written as:

$$\begin{aligned} C_{3,x}(t_1, t_2) = M_{3,x}(t_1, t_2) - M_{1,x}(M_{2,x}(t_1) + \\ M_{2,x}(t_1 - t_2)) + 2(M_{1,x})^2 \end{aligned} \quad (8)$$

For a system (**C.S**) expression (8) becomes:

$$C_{3,x}(t_1, t_2) = M_{3,x}(t_1, t_2) \quad (9)$$

For a system (**C.S**), the cumulant of order m = 3 is written as:

$$\begin{aligned} C_{4,x}(t_1, t_2, t_3) = M_{4,x}(t_1, t_2, t_3) - M_{2,x}(t_1)M_{2,x}(t_3 - t_2) - \\ M_{2,x}(t_2)M_{2,x}(t_3 - t_1) - M_{2,x}(t_3)M_{2,x}(t_2 - t_1) \end{aligned} \quad (10)$$

2.2.3. Brillinger and Rosenblatt Equation

The common point of all conventional methods of identifying adjusted average (MA) models is the use of Brillinger and Rosenblatt formula [2] which, under the above assumptions is:

$$\begin{aligned} C_{m,Z}(\tau_1, \dots, \tau_{m-1}) = C_{m,Y}(\tau_1, \dots, \tau_{m-1}) \\ = \gamma_{m,x} \sum_{i=0}^q b(i)b(i + \tau_1) \dots b(i + \tau_{m-1}) \end{aligned} \quad (11)$$

For $m = 2$, the autocorrelation is:

$$C_{2,Z}(\tau) = C_{2,Y}(\tau) + C_{2,N}(\tau) \quad (12)$$

where $C_{2,N}(\tau)$ is the autocorrelation of the noise skewing results and $C_{2,Y}(\tau)$ is the autocorrelation of the non-noisy signal expressed by:

$$C_{2,Y}(\tau) = \gamma_{2,x} \sum_{i=0}^q b(i)b(i + \tau), \quad (\gamma_{2,x} = \sigma_x^2) \quad (13)$$

According to (11), one can easily demonstrate that the order cummulants m et n , with $(m > n)$, meet the following relationship:

$$\sum_{i=0}^q b(i)C_{m,Y}(i + \tau_1, ..i + \tau_{n-1}, \tau_n, \dots, \tau_{m-1}) = \varepsilon_{m,n} \sum_{i=0}^q b(i) \left[\prod_{j=n}^{m-1} b(i + \tau_j) \right] C_{n,Y}(i + \tau_1, ..i + \tau_{n-1}) \tag{14}$$

Where $\varepsilon_{m,n} = \frac{\gamma_{m,x}}{\gamma_{n,x}}$. This general equation establishes several basic algorithms and will also be the basis of our proposed algorithm.

2.3. Based unique order cummulants algorithms

The algorithms based only on higher order cummulants are interesting when the processed signal is contaminated by an additive Gaussian noise. Indeed cummulants of higher or equal to three orders of a Gaussian distribution is zero.

2.3.1. Algorithm Based on 4th Order Cumulant using equations 2q +1: Alg1

From equation (11) The matrix form of the algorithm is given by Alg1 [11]

$$\begin{pmatrix} 0 & \dots & 0 & C_{4,y}(q, q, 0) \\ \vdots & \ddots & & \vdots \\ 0 & & & \\ C_{4,y}(q, q, 0) & \dots & & C_{4,y}(q, q, q) \\ \vdots & \ddots & & 0 \\ C_{4,y}(q, q, q) & 0 & \dots & 0 \\ C_{4,y}(0, 0, -q) \\ \vdots \\ C_{4,y}(0, 0, 0) \\ \vdots \\ C_{4,y}(0, 0, q) \end{pmatrix} \begin{pmatrix} \frac{1}{b^2(q)} \\ \vdots \\ \frac{b^3(i)}{b^2(q)} \\ \vdots \\ \frac{b^3(q)}{b^2(q)} \end{pmatrix} = \tag{15}$$

in a more compact form, the system of equations (15) can be written as follows:

$$Mb_q = d \tag{16}$$

with M , and h_q are defined in the equation system (15). The solution in the sense of least squares, LS, of the system of equation (16) is given by:

$$\widehat{h}(q) = (M^T M)^{-1} M^T d \tag{17}$$

this solution gives us an estimate of the quotient of parameters $b^3(i)$ and $b^3(q)$, by:

$$h_q(i) = \left(\frac{\widehat{b^3(i)}}{\widehat{b^3(q)}} \right), i = 1, \dots, q. \tag{18}$$

So, to estimate the parameters $\widehat{b}(i)$, $i = 1, \dots, q$ we proceed as follows:

- The parameters $b(i)$ for $i = 1, \dots, q - 1$ are estimated from estimates of $\widehat{h}_q(i)$ values using the following equation:

$$\widehat{b}(i) = sign \left[\widehat{h}_q(i) (\widehat{h}_q(q))^2 \right] \{ abs(\widehat{h}_q(i)) (\widehat{h}_q(q))^2 \}^{1/3} \tag{19}$$

$$avec \ sign(x) = \begin{cases} 1, & if \ x > 0; \\ 0, & if \ x = 0; \\ -1, & if \ x < 0. \end{cases}$$

and $abs(x) = |x|$ indicates the absolute value of x.

- The parameter $\widehat{b}(q)$ is estimated as follows:

$$\widehat{b}(q) = \frac{1}{2} \text{sign} \left[\widehat{h}_q(q) \right] \left\{ \text{abs}(\widehat{h}_q(q)) + \left(\frac{1}{\widehat{h}_q(1)} \right)^{1/2} \right\} \quad (20)$$

2.3.2. Algorithm 'C(q,k)' of Giannakis

From (11), Giannakis showed that the coefficients (FIR) can be expressed by the following formula:

$$b(\tau) = \frac{C_{m,Y}(q, \tau, 0, \dots, 0)}{C_{m,Y}(q, 0, \dots, 0)} \quad (21)$$

with $\tau = 0, \dots, q$ and the cumulant of order m of excitation is:

$$\gamma_{m,x} = \frac{C_{m,Y}^2(q, 0, \dots, 0)}{C_{m,Y}(q, q, \dots, 0)} \quad (22)$$

For $m = 3$, we have: $b(\tau) = \frac{C_{3,Y}(q, \tau)}{C_{3,Y}(q, 0)}$ et $\gamma_{3,x} = \frac{C_{3,Y}^2(q, 0)}{C_{3,Y}(q, q)}$

2.4. Proposed Algorithm

In this section the impulse response $B = [b(0), b(1), \dots, b(q)]$ is proposed to estimate a q order RIF channel using an algorithm that combines cumulants of order 3 and 4, as a previously proposed hypothesis. It also explains the method that improves the proposed algorithm.

2.4.1. General equation

Equation (14) is transformed into an equation which links m and n such that $m = n + 1$ as following:

$$\sum_{i=0}^q b(i) C_{m,Y}(i + \tau_1, \dots, i + \tau_{n-1}, \tau_n) = \varepsilon_{m,n} \sum_{i=0}^q b(i) b(i + \tau_n) C_{n,Y}(i + \tau_1, \dots, i + \tau_{n-1}) \quad (23)$$

2.4.2. Approach combining 3 and 4 cumulants order

Especially $m = 4$ et $n = 3$, Equation (23) becomes:

$$\sum_{i=0}^q b(i) C_{4,Y}(i + \tau_1, i + \tau_2, \tau_3) = \varepsilon_{4,3} \sum_{i=0}^q b(i) b(i + \tau_3) C_{3,Y}(i + \tau_1, i + \tau_2) \quad (24)$$

We take $\tau_1 = \tau_2 = q$ et $\tau_3 = \tau$, the equation (24) becomes:

$$\sum_{i=0}^q b(i) C_{4,Y}(i + q, i + q, \tau) = \varepsilon_{4,3} \sum_{i=0}^q b(i) b(i + \tau) C_{3,Y}(i + q, i + q) \quad (25)$$

given that $C_{4,Y}(\tau_1, \tau_2, \tau_3) = C_{3,Y}(\tau_1, \tau_2) = 0$, si $\tau_i > q$; the equation (25) becomes:

$$b(0) C_{4,Y}(q, q, \tau) = \varepsilon_{4,3} b(0) b(\tau) C_{3,Y}(q, q) \quad (26)$$

We deduce:

$$b(\tau) = \frac{C_{4,Y}(q, q, \tau)}{\varepsilon_{4,3} C_{3,Y}(q, q)} \quad (27)$$

with

$$\varepsilon_{4,3} = \frac{\gamma_{4,x}}{\gamma_{3,x}} \quad (28)$$

According to equation (22), we deduce:

$$\varepsilon_{4,3} = \frac{C_{4,Y}^2(q, 0, 0) C_{3,Y}(q, q)}{C_{4,Y}(q, q, 0) C_{3,Y}^2(q, 0)} \tag{29}$$

then

$$b(\tau) = \frac{C_{4,Y}(q, q, 0) C_{3,Y}^2(q, 0) C_{4,Y}(q, q, \tau)}{C_{4,Y}^2(q, 0, 0) C_{3,Y}(q, q) C_{3,Y}(q, q)} \tag{30}$$

2.4.3. AlgatF

The reduction of numerical calculations and the performance of the used statistical estimator can be a source of some divergence of values compared to the true value. To minimize these error differences sign we will also proposes a selective choice of estimated values of impulse responses from the previous algorithms in the following format:

Since each calculated value is accompanied by an error, it is therefore considered as a fuzzy number [13] defined by an interval in the set \mathbb{R} by the following figure, Fig.2:

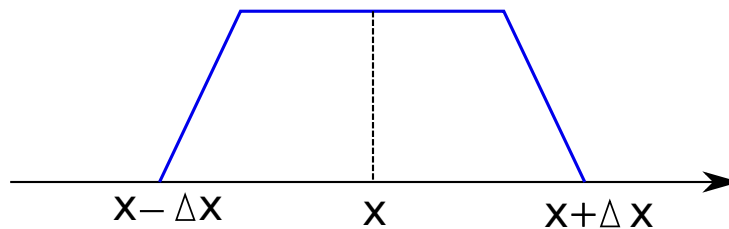


Figure 2. Fuzzy number representation

Fig.3, represents fuzzy values obtained by iterative simulation. Fuzzy values may be intersecting or not. We removed fuzzy extreme values having a zero intersection with the other fuzzy values. Indeed, these fuzzy values are far from the true value. Note that the number of fuzzy

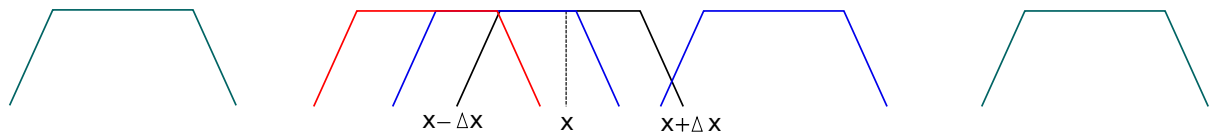


Figure 3. Representation of a fuzzy number of estimated series

values, remaining after removal of the end must be greater than at least half of the iterations.

AlgatF is the method of selection applied on ALGaT given that the fuzzy variable is selected by:

$$B = \sum_{i=0}^q b(i) \tag{31}$$

where $2x\Delta B$ is the size fuzzy interval. The sum is fed to remove the divergence due to the undesired occurrence of the minus sign in one of the component of the impulse response.

3. Simulation

In this simulation, we take 100 iterations and each time a new sample is taken by a noisy Gaussian noise with zero mean. The different algorithms provide estimates for the same samples in sizes 400, 800 and 1200 respectively. To compare the samples using the mean square error defined as follows:

$$EQM = \frac{1}{q+1} \sum_{i=0}^q (b(i) - h(i))^2 \tag{32}$$

Consider the channel, non-minimum phase (there is a zero of the transfer function outside the unit circle), figure (4) below, having the impulse response $H = [1 \ -1,083 \ -0,95 \ 0,95]$. The

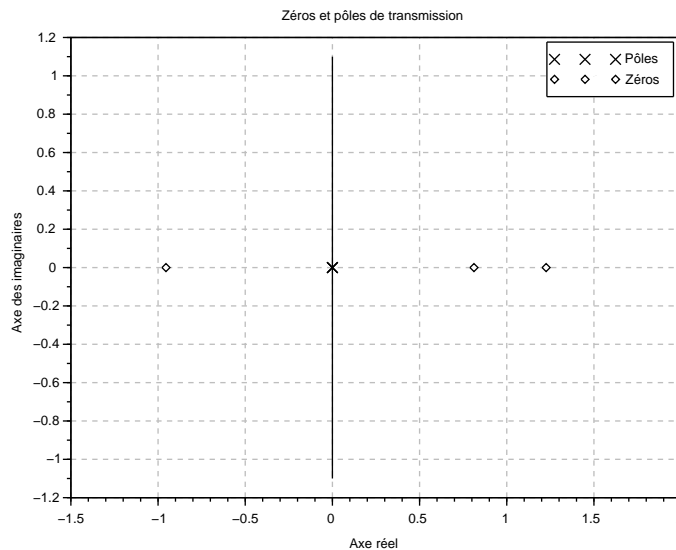


Figure 4. The zeros and poles for channel

paragraphs below summarize simulation on channel 1 for the various algorithms presented above in the case where the noise signal to noise ratio SNR equal to 10 dB and in case SNR equal to 20 dB. With

$$SNR = 10 \text{Log}_{10} \left(\frac{\sigma_y^2}{\sigma_{bruit}^2} \right) \quad (33)$$

where σ_i^2 is the standard deviation of the statistical distribution (i).

Given that $h(1) = 1$. The least precise value of $h(i)$ comes three significant digits. Our choice of the error on B is also to 3 significant figures in the following is taken into simulation $\Delta B = 0,03$.

3.1. Case SNR is 10 dB

The following table summarizes the results obtained for the proposed channel, the four algorithms namely Alg1, Alg of Gianakis, AlgaT and AlgatF. The AlgaT corrected by the proposed selection method in case $SNR = 10$ dB.

The descriptive data table (1), allows us to see a clear improvement of EQM. Indeed, for a sample size of 400, we note that the proposed method ensures amelioration, EQM by a factor of 2. In addition, the 800 sample reaches a factor of about 5.1, more than double. This factor will increase and reach about 20 in the case of the size of the sample 1200 for the same method. This increase ensures thereby a minimizing of the EQM versus other algorithm.

The improvement associated with, according to the variable fuzzifier, the method of the problem of the sign is remarkable and is also the example of the estimated h_4 AlgatF by the sample size to 800. This error sign is corrected by AlgatF.

Thus the criterion of choice is crucial and even decisive in improving the divergence of the calculation.

Note at this level the example of the estimated h_4 by AlgaT at AlgatF for 1200. According to the sample size Figure 5, we note that the curve coincides perfectly with the AlgatF ideal channel curve.

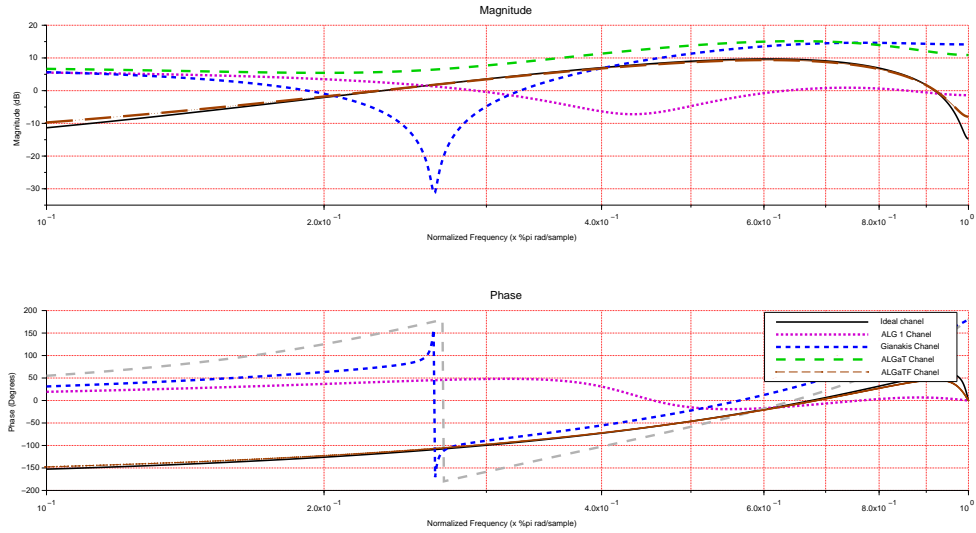


Figure 5. $N = 1200$ and $SNR = 10$ dB Magnitude and phase representation

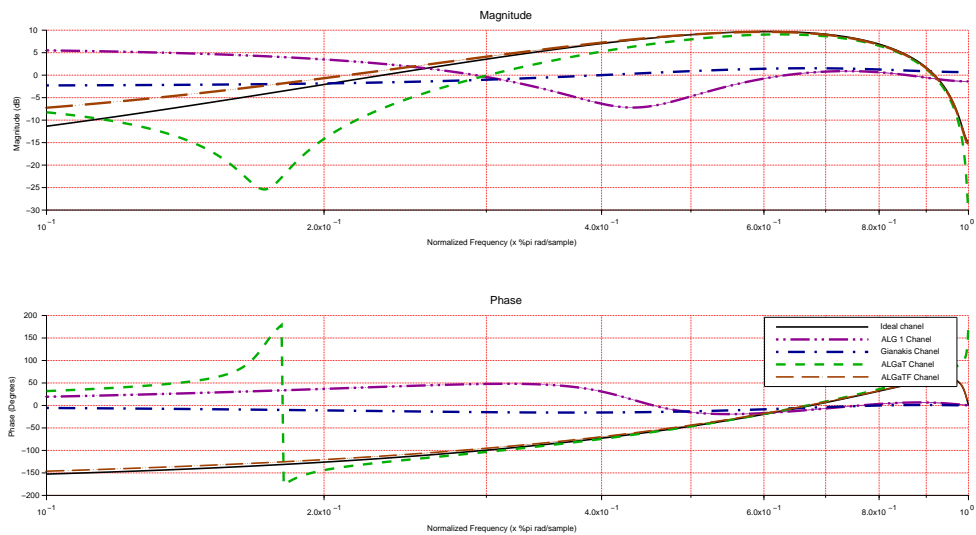


Figure 6. $N = 800$ and $SNR = 20$ dB

Table 1. Estimated values of h_i for SNR = 10 dB and N = 400,800,1200

Algorithms/Sample size	h_1	h_2	h_3	h_4	EQM
Ideal channel	1	-1,083	-0,95	0,95	0
Alg1/400	1	-0,346	-0,253	0,553	0,488
AlgGianakis/400	1	2,119	-1,724	1,984	1,544
AlgaT/400	1	-0,240	-0,494	0,220	0,539
AlgatF/400	1	-0,672	-0,592	0,699	0,268
Alg1/800	1	-0,610	-0,434	0,734	0,328
AlgGianakis/800	1	-0,555	0,155	-0,672	0,909
AlgaT/800	1	-0,227	-0,342	-0,226	0,706
AlgatF/800	1	-1,317	-0,833	1,116	0,139
Alg1/1200	1	-0,904	-0,458	0,917	0,235
AlgGianakis/1200	1	1,150	-2,407	2,539	1,388
AlgaT/1200	1	-0,615	-1,566	3,554	1,215
AlgatF/1200	1	-1.121	-0.874	0,847	0,060

3.2. Case SNR is 20 dB

The obtained results are summarized in Table 2, for channel 1, for the four ALG1 algorithms Alg of Gianakis, ALGaT and ALGatF the ALGaT corrected by the selection method proposed in case $SNR = 20$ dB.

Table 2. Estimated values of h_i for SNR = 20 dB and N = 400,800,1200

Algorithms/Sample size	h_1	h_2	h_3	h_4	EQM
Ideal channel	1	-1,083	-0,95	0,95	0
Alg1/400	1	-0,417	-0,324	0,562	0,444
AlgGianakis/400	1	1,0773	-1,846	0,467	1,068
AlgaT/400	1	-1,340	-0,970	1,734	0,369
AlgatF/400	1	-0,986	-0,966	1,005	0,051
Alg1/800	1	-0,724	-0,562	0,845	0,242
AlgGianakis/800	1	-0,206	-0,080	0,045	0,685
AlgaT/800	1	-0,751	-0,720	1,063	0,188
AlgatF/800	1	-1,085	-1,040	0,871	0,053
Alg1/1200	1	-0,874	-0,487	0,961	0,227
AlgGianakis/1200	1	-0,055	-0,233	-0,253	0,777
AlgaT/1200	1	-2,572	-1,058	2,962	1,121
AlgatF/1200	1	-1,284	-0,935	1,177	0,136

According to the descriptive data Table 2, we can also see a big improvement in the EQM for SNR = 20 dB. Indeed, for 400 the size sample, one notes that the proposed method ensures the improvement of the EQM of a factor 7.3. Furthermore, the sample reaches 800 orders of a factor 3.5, more than double.

This factor will increase and reach the 8.2 in the case of the sample size 1200. This increase also ensures minimization of the EQM versus other algorithm. The improvement associated with the so called problem of the method ensures good convergence to the true values of the impulse response. Figure 6 show that the curves of ALgatF coincides perfectly with the curve of the ideal channel.

4. Conclusion

Several blind identification algorithms based on higher order cumulants are usually used. Among them three examples Alg1, Alg of Giannakis and ALGaT, were selected. We applied the method based on the concept of fuzzy number on the latter one to obtain the corrected algorithm ALGaTF. In simulation, we considered a non-minimum phase channel and the estimated impulse response of 100 iterations for SNR of about 10 dB and 20 dB for the various algorithms. We were able to demonstrate that the proposed method increases the performance of the algorithm by calculating the ratio of squared errors of ALGaT and ALGaTF. The method can be applied to any algorithm for more improvement and efficiency. For future research, we intend to test the effect of the method on a small number of iterations so as to minimize the execution time of the algorithms.

References

- [1] Y.Sato, "A method of self-recovering equalization for multilevel amplitude-modulation systems," *IEEE transaction, comm*, vol. 23, no. 6, pp. pp 679 – 682, June 1975.
- [2] D. Brillinger and L. Rosenblatt, *Computation and interpretation of kth order spectra*. Spectral Analysis of Times Signals, New York : Wiley, 1967.
- [3] G. Giannakis and A. Swami, "Higher order statistics," *Elsevier Science Publ*, 1997.
- [4] M. I.BADI, E.ATIFY and S.SAFI, "Blind identification of transmission channel with the method of higher-order cummulants," *International Journal of Advances in Science and Technology*, vol. 6, no. 3, 2013.
- [5] E. K.Abid-Meriam and F.Loubaton, "Predection erreur method for second-ordre blind identification," *IEEE Transaction, Signal, Processing*, vol. 45, no. 3, pp. pp 694 – 705, 1997 - March.
- [6] S. Safi and A. Zeroual, "Blind parametric identification of linear stochastic non gaussian fir systems using higher order cumulants," *International Journal of Systems Sciences Taylor Francis, Signal Processing*, vol. 44, no. 15, pp. pp:855–867, 2004.
- [7] X. D. Zhang and Y. S. Zhang, "Fir system identification using higher order statistics alone," *IEEE Transactions, Signal Processing*, vol. 42, no. 12, pp. pp:2854 – 2858, 1994.
- [8] D. Dembl, "Identification du modle arma lineaires l'aide de statistiques d'ordres elevs. application l'egalisation aveugle," Ph.D. dissertation, Juillet-1995.
- [9] G.Favier, "Identification de modles paramtriques ar,ma et arma avec des statistiques d'ordres suprieur et analyse des prformances," *GRETSI*, pp. pp:137 – 140, Septembre-1993.
- [10] I. Badi, M. Boutalline, S. Safi, and B. Bouikhalene, "Blind identification and equalization of channel based on higher-order cummulants: Application of mc-cdma systems," in *Multimedia Computing and Systems (ICMCS), 2014 International Conference on*, April 2014, pp. 800–807.
- [11] A. S.Safi, "Ma system identification using higher ordre cumulants applications to modelling solar radiation," *Journal of Statistical Computation and Simulation*, vol. 72, no. 7, pp. pp 533 – 548, 2002.
- [12] A. S.Alshebeili and F.Cetin, "Cumulant based identification approaches for minimum phase fir system," *IEEE Transaction, Signal Processing*, vol. 41, no. 4, pp. pp 1576 – 1588, 1993 - Apr.
- [13] A. N. Gani and S. N. M. Assarudeen, "An algorithmic approach of solving fuzzy linear system using fourier motzkin elimination method," *Advances in Fuzzy Sets and Systems*, vol. 10, no. 2, pp. pp:95 – 109, 2011.