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A Innovation Identification Approach of Control System by Irrational (Fractional) Transfer

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Abstract

In this paper, an innovative algorithm of identification of control system, described by irrational transfer function with distributed parameter characteristics - with irrational components, is proposed. Algorithm is based on real interpolation method (RIM). Parameters of irrational transfer function can be identified by its experimental transient responses. Each of them can be represented by an analytic expression, table or graph. The proposed method is computationally efficient, simple and practical, as is illustrated by numerical examples. In the furure, the method can be used for tuning the controller and for direct application construction of adaptive controllers, working on the identification principle.

Keywords: *identification, control of distributed parameters system, distributed parameter system, irrational transfer functions and the real interpolation method.*

1. Introduction

Getting the mathematical description of automatic control systems is an important task in the study of automatic control systems. Among these tasks, identification of control distributed parameter systems (DPS) is the most difficult, and plays the most important role. Distributed parameter systems are the system, where the input, output and even parameters can vary both temporally and spatially. Complete dynamics of DPS is represented by partial differential equations (PDE). This equation system has no known analytical solution in real geometry and it has to be solved numerically [1]. Some typical examples include thermal process, fluid process, vibration, and flexible beam [2, 3]. Specialty of distributed parameter systems from the lumped parameter systems (LPS) is that their transfer functions described by partial-differential equations with boundary conditions [3].

In general, identification of DPS can be classified into two groups, identification of known structure DPS and identification of unknown structure of DPS [4]. For the known DPS, its PDE description derived from the first principle knowledge can provide a rigorous description of the system. Using proper spatial basis functions, the time-space nature can be decoupled. Then the appropriate model reduction method can approximate the infinite-dimensional system into a finite-order of ordinary differential equations (ODE), Linearization of DPS is interested in large of research papers for example: analytical method [5], linear method [6], spectral method [7]. Analytical methods for achieving this goal is not practicable. Using numerical methods have significant barriers. In the most common frequency method, the real and imaginary components should be separated. For complex transfer function, using frequency method is almost unreal.

For the unknown DPS, system identification is further developed from the modeling work of the known DPS. Extra work is needed for two following situations. One is that the PDE structure is available with only some parameters unknown, which requires parameter estimation of the DPS. Another situation is that the PDE structure is unknown, which requires structure design and parameter estimation of the DPS. Identification of DPS uses intelligent method: neural-network [8]; learning method [9, 10] or other method, using genetic algorithm.

An advantage of intelligent methods is that they can work with missing data model, or unknown structure. However these methods are at high complex and require large computing resource.

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Widely in the theory and practice of automatic control, the operation method for obtaining models of dynamic systems is to define the "input-output" relationship between two points of the system, leading to the descriptions in the form of complex transfer functions. Transfer function of DPS is complex, containing irrational and/or transcendental components. The transfer functions of these systems typically have the form such as formula (1) [11]. Because of presence of irrational and/or transcendental components in transfer function, which not allow using method and means of lumped parameters systems.

$$W(s)=W\left(e^{-\sqrt{T_1 s}},\frac{1}{\sqrt{T_2 s}},\sqrt{s},sh\sqrt{T_3 s},ch\sqrt{T_4 s},sh\sqrt{as^2+bs+c},\dots\right) \quad (1)$$

In this paper we consider the following task: identification of distributed parameter systems having transfer function with only irrational components. The fractional-order transfer function (FOTF) given by the following expression [12]:

$$G(s)=\frac{1}{a_n s^{\beta_n}+a_{n-1}s^{\beta_{n-1}}+\dots+a_1 s^{\beta_1}+a_0 s^{\beta_0}} \quad (2)$$

When β_k,(k=0,1,…,n) is an arbitrary real number, and $\beta_n>\beta_{(n-1)}>\cdots>\beta_1>\beta_0>0, a_k$,(k=0,1,…,n) is an arbitrary constant.

Finding even a few of the coefficients of (2) is always difficult, and in the most cases not possible. Therefore, nowadays an easy way is to identify - finding approximate models of the systems in the class of fractional transfer functions. In other words, a real distributed parameter system describes the model, corresponding to the lumped parameter system. Naturally, reduction method simplifies the task of identification, but immediately introduces an error to the solution of this and future solving problems [13].

In this paper the proposed approach is based on the real interpolation method (RIM) [11], which is characterized by two main features. The first feature involves the operator method, in which the problem is solved in the imaginary field, where computation has certain advantages. The second feature is that the models in the real interpolation method are a function of a real variable. Classic operator description of dynamic systems is a functions of the complex (in the case of the Laplace transform) or imaginary (in the case of the Fourier transform) variables. Transition to numerical models in these cases requires consideration of three-dimensional representations or separation of real and imaginary variables. Because of presence of irrational components, even with relatively simple expressions, generally this transition would be impossible. When using the real interpolation method barriers are removed.

2. Research Method

2.1. Real Interpolation Method in Problems of Identification of Control System, Described by Irrational Transfer Functions

Real interpolation method is one of the methods, which works on mathematical descriptions of the imaginary area. The method is based on real integral transform,

$$F(\delta)=\int_0^\infty f(t)e^{-\delta.t}dt,\delta\in[C,\infty),C\ge 0 \quad (3)$$

Which assigns to the original f(t) the image F(δ) as a function of the real variable δ. Formula of direct transform (3) can be considered as a special case of the direct Laplace transform by replacing the complex variable p = δ + jω for real δ variable. Another step towards the development of the instrumentation method - the transition from continuous functions F(δ) to their discrete form, using the computing resources and numerical methods. For these purposes, real interpolation method is represented by numerical characteristics $\{F(\delta_i)\}_\eta$. They are obtained as a set of values of the function F (δ) in the nodes δ_i = 1,2,...η, where η is the number of elements numerical characteristics, called its dimension.

Selecting of interpolation δi is a primary step in the transition to a discrete form, which has a significant impact on the numerical computing and accuracy of problem solutions. Distribution of nodes in the simplest variant is uniform. Another important advantage of real interpolation method is cross-conversion properties [11]. It dues to the fact that the behavior of the function F(δ) for large values of the argument δ is determined mainly by the behavior of the original f(t) for small values of the variable t. In the opposite case, the result is the same: the behavior of the function F(δ) for small values of the argument δ is determined mainly by the behavior of the original f(t) for large values of the variable t [11].

When considering the original f(t) of dynamic characteristics of dynamic systems, formula (3) leads to an operator model, which under certain conditions can be considered as special cases of the models based on the Laplace transform. Thus, in (3) replacing of the function f(t) by h(t)- the transient function of the dynamic system, we obtain its transfer function. From here we can find the elements of a discrete model of the system, and its transfer function by performing the discretization procedures for nodes δ_i = 1,2,...η:

$$W(\delta_i) = \delta_i \int_0^{\infty} h(t).e^{-\delta_i t} dt, i = \overline{1,\eta} \tag{4}$$

The mathematical model of the system in the form of numerical characteristics must have unique relationship to the original continuous real transfer function. This relationship can be set by a system of algebraic equations:

$$W(\delta_i) = G(\delta_i) = \frac{1}{a_n \delta_i^{\beta_n} + a_{n-1}\delta_i^{\beta_{n-1}} + \ldots + a_1 \delta_i^{\beta_1} + a_0 \delta_i^{\beta_0}},\ i = \overline{1,\eta} \tag{5}$$

This system of equations is a basis for determining the numerical values of the coefficients of the desired transfer function.

2.2. Identification Algorithm Distributed Parameters Control Systems Based on the Real Interpolation Method

The task of parametric identification of control distributed parameters system consists of determining the unknown coefficients of the transfer function with given the structure from the experimental transient characteristics h(t) to achieve a specified accuracy (or the best accuracy at a known structure of the model), according to the selected criteria. On the basis of real interpolation method, a sequence of actions are developed, that can be represented by the following algorithm.

1. Selecting the interpolation nodes δ_i = 1,2,...η_i, and defining the dimension η of the numerical characteristics.
2. Obtaining the numerical characteristics of the identified object {W (δ_i)}η.
3. Formulating and solving equations of the form (5).
4. Assessing the accuracy of problem solutions in accordance with the criteria and correction solutions, if necessary.

As already mentioned, selecting the interpolation nodes is an important step, which affects the subsequent results in terms of accuracy, the number of operations, etc. Selection begins with the defining the first node. The formula for the calculation of δ_1 is defined by the following condition: the value of the integral in (4) by the settling time ty reduces to a negligible value Δ = 0,001÷0,05, which satisfy the condition $h(t_y)\cdot e^{\wedge}(\delta_1 \cdot t_y) \le \Delta$. Hence calculated expression for the node δi can be shown below.

$$\delta_1 = \frac{-\ln\left(\frac{\Delta}{h(t_y)}\right)}{t_y} \tag{6}$$

Other nodes are determined by the condition of uniform distribution: $\delta_i = i\delta_1, i = (2,\eta)$.

The objective of the second stage consists of obtaining numerical characteristics of the formula (4). In real condition, the function h(t) is determined as a result of the experiment or presented as a graph or table. For this reason, in the formula (4) we have to move to the numerical summation by formula (7):

$$W(\delta_i)=\delta_i\sum_{j=0}^{N}h(t_j).e^{-\delta_i t_j}\Delta t_j, i=\overline{1,\eta}, j=\overline{0,N} \quad (7)$$

The third stage involves the compilation of a system of equations based on numerical characteristics $\{W(\delta_i)\}_\eta$ of transfer function (3):

$$W(\delta_i)=G(\delta_i)=\frac{1}{a_n\delta_i^{\beta_n}+a_{n-1}\delta_i^{\beta_{n-1}}+\ldots+a_1\delta_i^{\beta_1}+a_0\delta_i^{\beta_0}}, i=\overline{1,\eta} \quad (8)$$

For a small number of unknown coefficients, it is possible to find a solution of such system using standard numerical software products.

The final stage of the calculation process is aimed at verifying the accuracy of the solution. The accuracy of the solution is desirably estimated by comparing the transient characteristics – experimental transient characteristic h(t) and transient characteristic of the resulting model h.M (t) . In this case, high visibility and the possibility for improving solutions are reached. However, this attractive option is difficult to implement, because transfer function contains an irrational component that practically makes it impossible to obtain the original h.M (t). Remain option is an approximation of image G(s), using indirect methods of estimation accuracy, such as frequency characteristics, or direct comparison of coefficients if they are known. Thus as the estimation accuracy of the proposed method for identification of system with irrational transfer function is not end of the considered problem, but an instrument leads to a satisfactory solution.

3. Results and Discussion

As examples, we consider the system with fractional-odder transfer function, in which exact solution is known to estimate the result of identification task, by comparing the coefficients of exact and identified transfer function.

Example 1: In example 1 we consider the irrational transfer function in form [7].

$$W_1(s)=\frac{b_0}{\left(a_1 s^{\frac{3}{2}}+1\right)} \quad (9)$$

where b_0,a_1 are coefficient.

Exact response characteristic is shown in Figure 1, corresponding known coefficients $b_0=1$,$a_1=1$ [7].

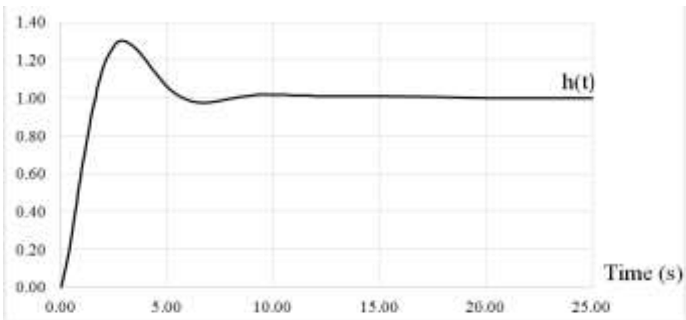

Figure 1. Response characteristic of transfer function W1(s)

In this example, the task consists of calculation the values of the parameters b_0,a_1 by h(t). By Figure1 settling time was defined t_y =10s. Indeed, on the basis of the theory on the limiting value of the image Laplace function, coefficient B can be determined by formula (3).

$$b_0 = \lim_{s\to 0} sW(s) = \lim_{t\to\infty} h(t) \tag{10}$$

Therefore b_0=1.

Dimension of the numerical characteristics determined by the number of unknown coefficients: η=1. Taking in the formula (6) Δ = 0.001, find the value of the first node, and value of the second node by assumption of a uniform grid δ_1 = 1. With the parameters of integration Δt = 0.25s and N = 40, we use the formula (7) to determine the numerical description of the object: {W (δ_1)} = {0.5033}. Now we can set up a system of equations of the form (8):

$$W(\delta_1) = \frac{b_0}{\left(a_1\delta_1^{\frac{3}{2}} + 1\right)} = 0.5033 \tag{11}$$

Replacing b_0=1 and δ_1=1 in (11), we get a_1=0.9912. Maximum relative accuracy of solution is 0.87%.

Example 2: The following example demonstrates a case when the process output is equal to the fractional semi-integral of an IIR filtered input. The actual IIR filter is, in fact, a differential compensator. Structure of transfer function is shown below.

$$W_2(s) = \left(\frac{b_1 s + b_0}{a_1 s + 1}\right)^{0.5} \tag{12}$$

Exact response characteristic is shown in Figure 2, corresponding known coefficients b_0=1,b_1=1,a_1=0.1 [7].

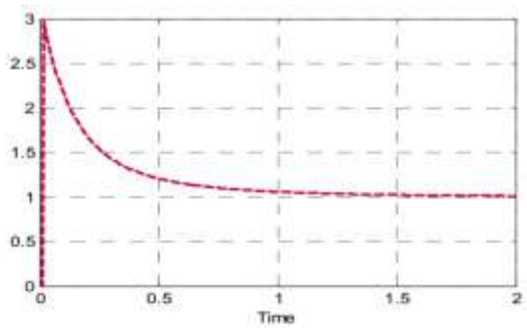

Figure 2. Response characteristic of transfer function W2(s)

Coefficient b0 can be calculated by formula (10), $b_0 = \lim_{s\to 0} sW(s) = \lim_{t\to\infty} h(t) = 1$

$$\begin{cases} W(\delta_1) = \left(\frac{b_1\delta_1 + b_0}{a_1\delta_1 + 1}\right)^{0.5} = 1.6708 \\ W(\delta_2) = \left(\frac{b_1\delta_2 + b_0}{a_1\delta_2 + 1}\right)^{0.5} = 1.9926 \end{cases} \tag{13}$$

Dimension of numerical characteristics is determined by the number of unknown coefficients: η=2. Taking in the formula (5) Δ = 0.001, with settling time t_y=1.5s, find the value of the first node, and value of the second node by assumption of a uniform grid δ_1 = 2.5 and δ_2=5. With the parameters of integration Δt = 1.5s and N = 36 we use the formula (6) to determine the numerical description of the object: $[\![\{W(\delta_i)\}_2]\!]$= {1.6708;1.9921}. Now we can set up a system of equations of the form (7).

Replacing b_0=1 and δ_1=2.5,δ_2=5, we obtain b_1=1.007,a_1=0.104. Maximum relative accuracy of solution is 3.9%.

Example 3: Consider a process with $W_3(s) = \exp(-T\sqrt{s})$, when T is a constant. When constant T=1, transient characteristic of this process is shown below [7].

Using real interpolation algorithm we could find unknown constant T=1.002 with the parameters of integration: t_y=50s;N=50; δ_1=1.

According to the numerical examples, using the real interpolation method we can determine the parameters of the irrational transfer function by the transient characteristics. The relative accuracy of identification task is in the range from 0.2% to 3.99%, and number of the integrator is about 50.

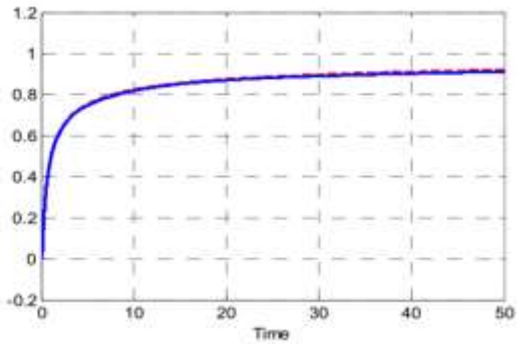

Figure 3. Response characteristic of transfer function W3(s)

4. Conclusion

In this paper we propose a method for identification of control objects, described transfer functions with distributed parameter characteristics - with irrational components. The method is based on the using computer-based method to attract numerical methods and computing facilities. The most outstanding feature of the proposed method is its computational efficiency. The method is, in fact very simple both conceptually and computationally. The obtained results are, as it can be seen from the previous example quite satisfactory. The main drawback of the proposed method is that it is not possible to guarantee selecting exact nodes of numerical characteristics.

The method is based on the application of computer-based method to attract numerical methods and digital computing facilities. The method can be used for tuning the controller. Another direct application is construction of adaptive controllers, working on the identification principle.

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