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Enhancing evaluation practices for Islamic inheritance calculation systems: toward a standardized benchmark

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ABSTRACT

Accurate estate distribution is a critical aspect in Islamic law, governed by complex rules that require precise inheritance calculations. Although numerous computerized inheritance calculation systems have been developed, their reliability remains questionable due to inadequate evaluation and unclear criteria for test case selection. This study addresses this gap by introducing a structured evaluation methodology to rigorously assess the functionalities of inheritance calculation systems. A new benchmark comprising 50 test cases was developed by reviewing the functionality of existing systems, collecting prior test cases and identifying coverage gaps through a detailed gap analysis. These benchmark cases were then used to assess the performance of leading online inheritance calculators, comparing their results to expert-validated solutions. Results revealed a significant drop in performance for calculators previously reported to achieve near-perfect accuracy, with scores declining to 68% and 58% compared to earlier reports of 100% and 90%. This demonstrates the effectiveness of the proposed test cases in exposing limitations within current systems. In contrast, the Almwareeth calculator, which had not been previously evaluated, demonstrated the highest accuracy (86%) and was able to handle a wider range of cases. This study lays a critical foundation for advancing the evaluation standards of Islamic inheritance calculation systems, thereby enhancing their reliability in real-world applications.

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1. INTRODUCTION

The distribution of deceased's estate in Islam is governed by precise rules and calculations rooted in Islamic inheritance law, a significant branch of Islamic jurisprudence known as the science of *Mawaris* or *Faraid*. These rules are sophisticated and require a deep understanding of principles laid out in the Quran, the Sunnah, and the scholarly consensus to determine the rightful shares of eligible heirs.

Due to the complexity of these calculations, numerous computerized systems [1]-[15], supporting tools [16], and knowledge models [17], [18] have been developed to automate and facilitate the inheritance calculation process. The core functionality of these systems lies in recognizing eligible heirs and computing their shares with precision. Therefore, they provide invaluable assistance not only to legal professionals but also to any Muslim seeking guidance on inheritance matters.

Despite their recognized usefulness, questions remain regarding their accuracy and reliability. A recent systematic review that included studies on the development of computerized systems for inheritance calculation highlighted a significant lack of evaluation and comprehensive testing of these systems [19]. Limited number of studies [2], [12], [13], [18], [20], [21] included some form of empirical evaluation. Even among those, the evaluation contexts were insufficient and unclear, as will be elaborated in the related work section. Moreover, the evaluations were inconsistent, limited in scope, and lacking transparency regarding the selection and coverage of test cases. As a result, there is limited evidence to support the functional robustness of these systems or to enable meaningful comparisons between them. This underscores the urgent need for a more systematic and criteria-driven approach to test case selection and system evaluation.

The purpose of this study is to improve the evaluation practices of Islamic inheritance calculation systems by developing a comprehensive benchmark. Ultimately, this will improve the overall performance of such systems by addressing existing limitations in evaluation practices. To achieve this, we collected and analyzed inheritance cases previously used in system evaluations, identified areas that have been overlooked, and proposed additional test scenarios necessary for a more comprehensive assessment.

The main contributions of this study are as follows,

- Establishment of criteria for classifying and analyzing existing test cases, which serve as a foundation for identifying gaps and guiding the development of diverse and representative inheritance test cases.
- Development of a benchmark comprising diverse inheritance scenarios designed to test the full range of functionalities required in Islamic inheritance calculation systems. The benchmark serves as a reusable tool for system developers and researchers, supporting consistent, transparent, and comparable evaluations across systems and enabling the continuous improvement of inheritance calculation tools.
- Demonstration of the benchmark's utility by evaluating three online inheritance calculators. This evaluation highlights both the strengths and the limitations of calculators previously considered high performing, thereby offering practical insights for further improvement.

The remainder of this paper is structured as follows. Section 2 provides background information on Islamic inheritance law. Section 3 reviews related work, focusing on the development of inheritance calculation systems and their existing evaluation practices. Section 4 outlines the methodology used for collecting and analyzing test cases, identifying gaps, and developing a new benchmark. Section 5 presents the proposed benchmark along with the evaluation results of inheritance calculators. Finally, section 6 discusses the implications of the findings and offers directions for future research.

2. BACKGROUND

This section describes basic information related to inheritance in Islam, including types of heirs and how the estate of the deceased is distributed according to Islamic law.

The initial steps in managing a deceased person's estate include settling their debts, covering funeral expenses, and executing their will, provided that the will does not exceed one-third of the total estate. The remainder of the estate is then distributed among 1) prescribed sharers (i.e. Ashab al-furud), 2) residual sharers (i.e. Asabah), 3) distant kindred (i.e. Dhawi al-arham), and 4) the public treasury (i.e. Bayt al-mal), in cases where no eligible heirs exist.

Prescribed sharers are heirs who are specified with their allocated shares in the Holy Quran and Sunnah. Prescribed sharers include spouses, parents, daughters, sons' daughters, full sisters, paternal sisters, uterine siblings, paternal grandfathers, and grandmothers. The share allocated to each heir varies depending on the circumstances of the case. For example, a husband inherits half of the estate in the absence of descendants' children and one-fourth if they are present. Similarly, a grandfather inherits only in the absence of the father, as the presence of the father precludes the grandfather from inheritance. The shares of prescribed sharers are explained in Table 1.

Residual sharers are paternal relatives who do not have a fixed share designated for them. The inheritance rules governing residual sharers are as follows:

- Residual sharers inherit the remainder of the estate after prescribed sharers have received their designated portions.
- If the entire estate is distributed among prescribed sharers and nothing remains, residual sharers inherit nothing, except the deceased's sons and father.
- Residual sharers inherit the entire estate in the absence of prescribed sharers.
- Shares are distributed among residual sharers so that a male receive twice the share of a female.

Table 1. Prescribed sharers and their designated shares with eligibility conditions

Relative	Share	Eligibility conditions
Wife	1/4	Absence of inheriting descendant(s) (children or paternal grandchildren, and so forth down the generations)
	1/8	Presence of inheriting descendent(s)
Husband	1/2	Absence of inheriting descendent(s)
	1/4	Presence of inheriting descendent(s)
Daughter	1/2	1) Being the only daughter and 2) absence of Muasib
	2/3	1) Two or more daughters and 2) absence of Muasib
Son's daughter	1/2	1) Absence of higher-level inheriting descendant(s), 2) being the only son's daughter, and 3) absence of Muasib
	2/3	1) Absence of higher-level inheriting descendant(s), 2) two or more son's daughters, and 3) absence of Muasib
	1/6	1) Presence of a female inheriting descendant who inherits half of the estate and 2) absence of Muasib
Father	1/6	Presence of male descendant(s)
	1/6 + Reminder	1) Absence of male descendant(s) and 2) presence of female descendant(s)
Mother	1/3	1) Absence of inheriting descendant(s) and 2) absence of two or more siblings
	1/6	1) Presence of inheriting descendant(s) or 2) presence of two or more siblings
	1/3 of the remainder	In two special cases known as "Alumarytan"
Full sister	1/2	1) Absence of male inheriting ascendant(s) (father or paternal grandfather, and so forth up the generations), 2) absence of inheriting descendant(s), 3) being the only full sister, and 4) absence of Muasib
	2/3	1) Absence of male inheriting ascendant(s), 2) absence of inheriting descendant(s), 3) two or more full sisters, and 4) absence of Muasib
Paternal sister	1/2	1) Absence of male inheriting ascendant(s), 2) absence of inheriting descendant(s), 3) absence of full sibling(s), 4) being the only paternal sister, and 5) absence of Muasib
	2/3	1) Absence of male inheriting ascendant(s), 2) absence of inheriting descendant(s), 3) absence of full sibling(s), 4) two or more paternal sisters, and 5) absence of Muasib
	1/6	1) Presence of a full sister who inherits half of the estate and 2) absence of Muasib
Uterine sibling	1/3	1) Absence of male inheriting ascendant(s), 2) absence of inheriting descendant(s) and 3) two or more uterine siblings
	1/6	1) Absence of male inheriting ascendant(s), 2) absence of inheriting descendant(s) and 3) being the only uterine sibling
Grandmother	1/6	Absence of the mother
Paternal grandfather	1/6	1) Absence of the father and 2) presence of male descendant(s)
	1/6 + Reminder	1) Absence of the father, 2) absence of male descendant(s) and 2) presence of female descendant(s)

There are four types of residual sharers: 1) Asabah by cause, 2) Asabah by oneself, 3) Asabah through others, and 4) Asabah with others. Note that Asabah by cause is related to freed slaves and therefore, is irrelevant in contemporary contexts.

Asabah by themselves are categorized into four classes, ranked based on their entailment to inheritance: sonship, fatherhood, brotherhood, and cousinhood (including only paternal male cousins). When multiple individuals from different classes are present, sonship takes precedence over fatherhood, followed by brotherhood, and then cousinhood. In cases where several individuals from the same class are present, such as a son and a son's son who both belong to the sonship class, the closest relative to the deceased (e.g. the son) takes precedence over those further away (e.g. the son's son). When both the class and the closeness of the relation to the deceased are identical, preference is given to the person with stronger familial ties. For example, a full brother and a paternal brother both belong to the brotherhood class and are equally close in relation to the deceased, but the full brother takes precedence as he is related through both parents.

Asabah, through others, includes daughters through sons, sons' daughters through sons' sons (i.e. either her brother or her cousin) or son of sons' son, full sisters through full brothers, and paternal sisters through paternal brothers. Males in these cases are referred to as "Muasib".

Finally, Asabah, with others, includes cases in which full sisters or paternal sisters inherit alongside daughters or sons' daughters of the deceased who do not have male siblings (i.e. Muasib). Note that Asabah through others is considered stronger than Asabah with others in terms of inheritance entitlement.

Distant kindred are blood relatives who are neither prescribed nor residual sharers, such as aunts, maternal grandfathers, and grandchildren through daughters. There are different schools of thought regarding their eligibility to inherit and the determination of their shares. Among those who recognize their right to inherit, it is generally held that distant kindred are entitled to inherit only in the absence of both prescribed and residual heirs.

In solving inheritance cases, the first step is to determine the eligible heirs and their respective shares. An example is presented in Table 2, where a deceased woman left behind a husband, two daughters,

and a sister. In this case, the husband's share is one-quarter of the estate, the daughters' share is two-thirds of the estate, and the sister receives what remains.

After identifying the heirs and their shares, the next step is to calculate the base number of the case based on the case's properties and specific mathematical rules. This base number represents the smallest amount of which the portions (i.e. the number of segments) for each eligible heir can be calculated. In the presented example, the base number is 12.

Following this, we calculate the portion for each category of heirs, where each category includes all heirs with the same relationship to the deceased. In the example, there are three categories of heirs, with the daughter category including two individuals. These portions are then divided among the individuals in each category. As shown in Table 2, the husband's portion is 3 (12*1/4), the daughters' portion is 8 (12*2/3), and the sister's portion is 1 (12-3-8=1).

The distribution occurs without fractions (known as "Al-Inqisam cases"), as seen in the example where 8 portions can be evenly divided among the two daughters, each receiving 4 portions. Alternatively, the distribution may involve fractions (known as "Al-Inkisar cases"), such as when three daughters must share 8 portions, resulting in approximately 2.67 portions per daughter. The latter scenario requires mathematical adjustments, which involve recalculating both the base number and the portions to eliminate fractions.

There are also cases where the total of heirs' portions exceeds the base number (known as Awl), or is less than the base number (known as Radd). In both scenarios, the base number is adjusted to match the sum of the heirs' portions, and the solution is adjusted by proportionately reducing or increasing their portions.

Finally, it is important to note that there are special inheritance cases known among Islamic scholars by specific names. The solutions to these cases diverge from the general principles applied to solving inheritance cases, and there may be multiple juristic opinions regarding their resolution.

Table 2. Example of an inheritance case with a solution

Relation to the deceased	Share	Portion	Percentage (%)
Husband	1/4	3	25
2 daughters	2/3	8 (4 each)	66.67 (33.33 each)
Sister	Remainder	1	8.33

3. RELATED WORK

In this section, we discuss the different types of empirical evaluations conducted to assess systems or individual components developed for inheritance calculation under Islamic law. These evaluations were performed either by system developers during the development process or by external researchers assessing existing off-the-shelf systems.

In Khosyi'ah *et al.* [18], knowledge acquisition for Islamic inheritance law was conducted, specifying the shares allocated to specific heirs and representing this knowledge using rules, providing a foundation for future expert systems. These rules were validated through a focus group discussion with two experts in Family Law and informatics lecturers. However, no further details regarding the evaluation process are provided, and the rules have yet to be implemented and validated within the context of an actual expert system.

A web-based expert system named "E-FaraidTanah" was developed in [13] to help Muslims understand the Islamic inheritance law and calculate the distribution of the deceased's land accordingly. The evaluation of the system focused on user satisfaction, assessed through a survey conducted with 50 university students. The survey included 10 questions: five related to acceptability (e.g. "I feel comfortable using this system") and five related to usability (e.g. "It was easy to learn to use this system"). While the survey showed a positive attitude toward the system, the accuracy in solving inheritance cases was not evaluated.

Similarly, Anwar [12] designed an Android-based application "E-Mawaris" to assist students in learning and calculating estate distribution within the context of the Fiqh Mawaris course. The author claims to have evaluated the application's validity, practicality, and effectiveness with several experts. However, no details are provided about the empirical validation context, such as the number of experts involved, their qualifications, or the specific methodologies used to assess these constructs.

In Houssen [2], the design and development of a web-based expert system focused solely on prescribed sharers are described. For evaluation, test cases were created by a domain expert, and the results were validated by the same expert. Although the author states that the system underwent thorough testing, the actual number of test cases is not provided, with only three cases disclosed.

In Tabassum *et al*. [20], the "Islamic Farayez" system was designed and developed. The system was tested using ten cases, with its outputs compared to manual calculations performed by experts. However, there is no information about the criteria used for selecting these ten cases or the qualifications of the experts whose calculations served as the basis for validation.

The study reported in [22] focuses on the development and implementation of detailed architectural designs for an Islamic inheritance system. The author highlights that the developed system cannot handle special cases and only accommodates full-type relatives, such as full sisters. Due to this limitation, the system was tested using 12 cases sourced from [23], which is described next, while excluding scenarios that fall outside the system's current capabilities.

In Cheema [23], six freely accessible online inheritance calculators were evaluated based on two criteria: "accuracy score" and "efficiency grade". The accuracy score, rated out of 100, reflects how well the calculators solved 20 test cases created by the author, with each correctly solved case earning 5 points. The efficiency grade assesses the explanations and rationales provided by the calculators. Specifically, the author poses two questions "Whether the methodology adopted by the producer/developer of the calculator is intelligible to common people for whom it was created?", and "How far reliance on a calculator would relieve a lay person from seeking advice/guidance of a scholar on the distribution of one's estate?". Efficiency grades are categorized as elementary, moderate, advanced, or cannot be determined, the latter apply when the calculator does not offer justifications or provides rationales inconsistent with the settled opinions of various Sunni schools. Among the six calculators, three obtained a high accuracy score of 90, 95, and 100, while the remaining three obtained low scores of 5, 15, and 30. The calculators with high scores were assigned efficiency grades of moderate, elementary, and advanced, respectively. Among the low-scoring calculators, two were not assigned efficiency grades, while the one with a score of 30 received an elementary grade. Although the author stated that the selected test cases were drawn from books and reflect various aspects of Islamic inheritance law, no further information was provided regarding the criteria or process for selecting these cases.

Along a similar vein, Billah [24] evaluated ten free mobile applications for inheritance calculation, assessing their ability to solve five special cases. The evaluation examined the consistency of these applications in solving the cases based on a particular school of thought. Only two applications successfully solved all the cases and maintained consistency in applying the school of thought. Most applications failed to correctly identify the presented cases, primarily due to inconsistencies in their reasoning, a limited list of supported heirs, and errors in calculating the portions of heirs.

A more recent study [25] evaluated ChatGPT on 101 multiple-choice questions taken from level one of an Islamic inheritance (Faraid) course, aiming to assess its usefulness as an educational tool. The study found that ChatGPT achieved a performance accuracy of 65.35%. The study concluded that ChatGPT performed well on theoretical topics with clear rules, but showed weaknesses in topics requiring detailed analytical reasoning, such as "establishing the base number", "division of estates", and "inheritance involving a grandfather and siblings".

While we have provided an overview of various evaluation approaches, the remainder of this paper focus specifically on four studies [2], [20], [21], [23] for two reasons. Firstly, these studies evaluate the accuracy of inheritance calculation systems in solving inheritance distribution cases, which aligns with the primary focus of this paper. Secondly, they disclose the specific test cases used for evaluation, allowing for a meaningful comparison and analysis.

4. METHOD

The purpose of this study is to propose a comprehensive set of inheritance scenarios to evaluate the functionalities of inheritance calculation systems based on clearly defined criteria. The methodology consists of five main stages, detailed as follows,

4.1. Understanding the functionality of inheritance calculation systems

To establish a foundation for test case development, we began by analyzing the key functionalities of Islamic inheritance calculation systems as described in [21]. This step was guided by the findings of a recent systematic review [19], which revealed that among the studies reviewed, [21] provides the most comprehensive and detailed description of the design and development of an inheritance calculation system from a software engineering perspective.

Unlike other studies that offered only high-level descriptions or limited case demonstrations, [21] adopts a structured system analysis and design methodology, beginning with the identification of system scenarios and functional requirements through data flow diagrams. The study then presented architectural designs using structure charts to illustrate the modules of the entire system and their internal connections. The study also provided detailed designs of the system modules, including data structures and algorithms,

described using a program description language. This detailed design serves as a foundation for system implementation.

We utilized the specifications, requirements, and design reported in [21], to identify the key functionalities of inheritance calculation systems and to derive criteria for selecting test cases accordingly. However, we identified certain omissions in the system description, which we discussed and taken into consideration when designing the test cases.

4.2. Collection and review of existing test cases

We collected and reviewed the inheritance scenarios utilized to test inheritance calculation systems in previous studies identified through a systematic review [19] (refer to section 3 for details on these studies [2], [20], [21], [23]). First, all scenarios were compiled into a shared file, and both unique and overlapping inheritance scenarios were identified. Each scenario was then carefully reviewed and manually solved by the second author who is a professor in the Department of Jurisprudence with expertise in Islamic inheritance and substantial teaching experience in university-level courses on the topic, and the results were compared with the corresponding solutions provided in the original studies, where available. During this process, we documented limitations, errors, and ambiguities within the scenarios, such as missing information (e.g., unspecified number of heirs) or incorrect application of inheritance rules.

4.3. Gap analysis and identification of testing deficiencies

Building upon the collection and review of inheritance scenarios, the next step involved conducting a detailed gap analysis. This analysis aimed to identify deficiencies in existing inheritance test cases and their selection criteria by pinpointing scenarios that are underrepresented or inadequately addressed in current studies. Each scenario was evaluated against classification framework designed to capture key system functionality,

- Type of share assigned: Whether the scenario involves prescribed/fixed shares or residual shares.
- Class and type of heir involved: Such as sonhood (e.g., son, grandson) or fatherhood (e.g., father, grandfather), and the specific heir types (e.g., daughter, brother, grandmother).
- Share value allocated: Such as 1/4 or 1/2.
- Requirement for share adjustment and the method used: Whether share values required adjustment and how the adjustment was carried out.
- Involvement of *Awl* or *Radd*: Whether the scenario involves *Awl* (i.e., when the total allocated portions exceed the base number) or *Radd* (i.e., when the total is less than the base number).
- Involvement of exclusion (*Hajb*) and justification: Whether the scenario involves the exclusion of one or more heirs, along with the specific reason for their exclusion.
- Recognition of special cases: Whether the scenario represents well-known exceptional cases in Islamic inheritance law, such as *Al-Umariyatain.*
- Recognition of complex cases: Whether the scenario specifically involves fetuses in the womb, concurrent deaths, and missing or intersex individuals.
- Presence of different juristic opinions: Whether the case involves differing scholarly opinions, such as in the case of a grandfather inheriting alongside siblings.
- Presence of invalid or unexpected input: Whether the scenario is incomplete or contains logically inconsistent elements.

This framework not only reveals gaps in coverage within current test cases but also informs the design of new, more comprehensive test cases.

4.4. Development of new inheritance test cases

Building on the findings of the gap analysis, this study introduces a set of 50 new test cases designed to address the identified deficiencies. By incorporating a diverse range of cases, we ensure that all functionalities of inheritance calculation systems are thoroughly tested. This approach facilitates the detection of potential weaknesses or limitations, enabling necessary improvements. In addition, it builds confidence in the accuracy of inheritance calculation systems, making them more reliable for users who rely on them for precise estate distribution in accordance with Islamic law. All proposed cases were documented and accompanied by detailed solutions, providing a robust benchmark for system evaluation by researchers and developers.

4.5. Application of proposed test cases for system evaluation

The proposed test cases were then used to evaluate existing online systems for Islamic inheritance calculation. Systems were selected based on their performance in prior evaluation study [23]. Initially, the three top-performing calculators identified in [23] were chosen. However, the link to the calculator that

achieved a score of 95 was no longer accessible, so the evaluation proceeded with the two remaining calculators: one that scored 100, referred to as Calculator 1, and another that scored 90, referred to as Calculator 2. Additionally, we included a third calculator (Calculator 3), an advanced tool known for its ability to handle complex inheritance scenarios such as concurrent deaths, and missing people or intersex individuals. Basic information about the calculators evaluated is presented in Table 3.

Table 3. Summary of evaluated inheritance calculators

Calculator	Name	Accuracy	Link
Calculator 1	Islamic inheritance calculator	100	https://inheritance-calculator.com/
Calculator 2	Islamic inheritance calculator	95	http://inheritance.ilmsummit.org/projects/inheritance/home.aspx
Calculator 3	Almwareeth	Not previously scored	https://almwareeth.com/

The evaluation involved a comparative analysis of the calculators' performance against manual solutions provided by the second author to ensure the proposed cases validate their accuracy and identify potential shortcomings. Performance was measured using an accuracy score, calculated as the percentage of cases solved correctly. The evaluation was based on 50 newly developed test cases proposed in this study. The scoring system is defined as follows:

- Not Supported (0): The calculator does not support the specific inheritance scenario; assigned a score of 0.
- Incorrect: The calculator provided an incorrect solution; assigned a score of 0.
- Partially Correct: The calculator's solution was partially correct; assigned a score of 0.5. Criteria for partial scoring are detailed below.
- Correct: The calculator provided a fully accurate solution; assigned a score of 1.

Performance differences among the three calculators were tested using the Friedman test for related samples. When significant, Wilcoxon signed-rank tests with Bonferroni correction ($\alpha = 0.017$) were applied for pairwise comparisons to identify specific differences while controlling for multiple testing.

Both Calculator 1 and Calculator 3 offer the option to select a school of thought for calculating estate distribution. This option was left unspecified for most cases, except in two scenarios: cases involving a grandfather with siblings (Cases 32 to 39) and the case involving grandmothers from different generations (Case 12). For Cases 32 to 39, we tested the calculators across the four main Sunni schools of thought (Hanafi, Shafi'i, Maliki and Hanbali). This comparison was necessary because, under Hanafi school, siblings do not inherit in the presence of a grandfather, whereas the other schools allow for the inclusion of siblings. Our objective was to assess whether the calculators could correctly allocate shares to both the grandfather and siblings, in accordance with the juristic view that a grandfather does not necessarily exclude siblings. Notably, Calculator 1 excludes siblings by default when a grandfather is present unless a school of thought is explicitly selected. Additionally, in the case involving grandmothers from different generations (Case 12), we tested the Shafi'i, Maliki, Hanafi, and Hanbali schools. This was important because scholarly opinions are divided: according to the Hanafi and Hanbali views, the nearer grandmother excludes the more distant one, whereas in the Shafi'i and Maliki schools, both grandmothers are entitled to inherit. To accommodate differing juristic opinions, we evaluated the results under each school separately and assigned a full score when all results were correct, and a partial score if at least one result was incorrect.

5. RESULTS AND DISCUSSION

5.1. Understanding the functionality of inheritance calculation systems

The primary use case of Islamic inheritance systems is "to compute the portions of shares of deceased among the eligible heirs" [22]. According to [22], this involves the following six functions,

- Read details of the deceased: This function obtains information about heirs, including their names and numbers, and then validates this information to produce a list of valid heirs. It also outputs the total effective share, which represents the amount of the estate left after deducting loans and funeral expenses.

However, we have identified two key omissions. First, the specific validation steps are not explicitly mentioned. Ideally, the system should detect individuals who do not inherit under any circumstances according to Islamic law and handle invalid or unexpected inputs, such as having five wives as heirs, which is not permitted. Additionally, the calculation of the total effective share should involve detecting amounts related to fulfilling the deceased's wills, ensuring that it does not exceed one-third of the estate.

- Compute prescribed shares: This function performs two tasks: it verifies the eligibility of prescribed heirs and computes their respective shares.

- Compute residual shares: Similarly, this function verifies the eligibility of residual heirs and computes their shares of the remaining estate.
- Sum individual shares: This function sums individual shares and compares the sum to the total effective share. It checks whether the sum of individual shares is equal to, less than, or greater than the total effective share. If the values are equal, the shares are correct; otherwise, adjustments for Radd or Awl are required.

Although not mentioned in [22], this function should also determine the base number and assess whether shares can be distributed among heirs without fractions. If fractions are present, adjustments are also required. Given this additional responsibility, we believe a more appropriate name for this function would be "Verify individual shares", as it performs more than simply summing individual shares.

- Adjust individual shares: This function is responsible for adjusting individual shares in cases involving Radd or Awl. Additionally, it should handle cases where shares cannot be distributed without fractions, a detail omitted in [22].
- Display individual shares: The function displays the final share allocated for each heir.

In addition to these six functionalities, we believe that two main functions were overlooked in [22]. First, the system should be able to "Compute shares for distant kindred", similar to how it handles prescribed and residual heirs. This function assesses the eligibility of distant kindred and computes their shares accordingly. Second, it must be able to "Recognise special and complex cases", which is crucial because the computation of shares in special cases differs for certain heirs. This function should follow "Read details of the deceased" to inform subsequent functions responsible for computing prescribed and residual shares of the specific circumstances or conditions unique to certain heirs, necessitating different handling of shares. The aforementioned functions form the basis for selecting inheritance cases to evaluate the performance of inheritance calculation systems.

5.2. Collection and review of existing test cases

We collected 51 test cases from previous studies evaluating inheritance calculation systems. After excluding duplicates, the number of cases was reduced to 34. Among these, nine cases appeared in two studies and four in three studies, resulting in 21 unique cases. This set includes six special cases that are well-recognized in the scholarly literature on Islamic inheritance. The distribution of cases across studies is shown in Table 4, while the full list of cases is provided in Table 5.

Table 4. The distribution of cases among previous studies.

Reference	#Cases	Note
[2]	3 cases	No solutions were provided, the author mentioned testing other cases, but no details or examples beyond the three cases were provided.
[20]	10 cases	-
[23]	20 cases	-
[22]	12 cases	Cases are taken from [23]
[24]	6 cases	Al-Umariyatain, Al-Khuraqa, Akdariyah, Musyarrakah, and Al-Faridah Al-Malikiyah

A significant observation during the review process was the variation in the number of test cases provided across studies (Table 4). These discrepancies raise critical concerns about the adequacy of test coverage and highlight the absence of standardized testing practices in the field. Moreover, the overall number of test cases reported in each study appears too limited to comprehensively assess the full range of functionalities required by Islamic inheritance calculation systems.

In addition to these limitations, several quality issues were identified in the reported test cases:

- Case 1: Contains an error, stating that the deceased left both a wife and a husband. It might have been intended to test the system's error-handling ability, but we doubt this since no explanation or commentary is provided. Alternatively, the authors may have intended to state either a wife or a husband, but this distinction is as the spouse's share differs between the two scenarios.
- Case 3: Contains a typographical error, stating "congaing sister" instead of "consanguine sister".
- Case 14: The number of sons is not specified, which is necessary to determine their individual shares.
- Case 18: Similar to Case 14, the number of uterine brothers and full sisters is not specified, which is necessary to determine their individual shares.
- Case 19: Contains an error in the solution provided; the mother's share is incorrectly stated as 1/2 instead of the correct share of 1/6.

Table 5. Existing test cases. Assumed full brother when only 'brother' was specified

#	Case	Reference
1	Husband, wife, and daughter	[2]
2	Daughter of son, mother, and father	[2]
3	Full sister, uterine sister and brother, and congaing sister	[2]
4	1 Wife, 4 sons, 2 daughters, 2 sons' sons, 2 son's daughters, father, father's father, father's mother, mother, mother's mother, 2 brothers, and 2 sisters	[20]
5	2 Wives, 2 sons' sons, 2 son's daughters, father's father, mother, father's mother, and mother's mother	[20]
6	Husband, father's mother, mother's mother, and sister	[20]
7	Husband, daughter, and 2 sisters	[20]
8	3 daughters, father, father's mother, mother's mother, and 2 sisters	[20]
9	Wife, 2 sisters	[20]
10	Mother's mother, 1 son's daughter, 2 brothers, and 2 sisters	[20]
11	Father's mother, mother's mother, 3 son's daughters, and 3 brother's sons	[20]
12	Father, father's mother, mother's mother, 3 son's daughters, and 3 brother's sons	[20]
13	Father, father's mother, mother's mother, 2 son's sons, 3 son's daughters, and 2 brothers	[20]
14	Husband, father, mother, and sons	[22], [23]
15	Wife, uterine brother, and full brother	[23]
16	Father, mother, wife, and daughter	[22], [23]
17	Father, daughter, and son's daughter	[22], [23]
18	Husband, uterine brothers, and full sisters	[23]
19	Wife, mother, full sister, consanguine sister, and uterine sister	[23]
20	Father, mother, wife, daughter, and son's daughter	[22], [23]
21	Father, son, grandfather, full Brother, and uterine sister	[23]
22	Wife, daughter, full sister, full uncle's son (i.e. paternal cousin)	[22], [23]
23	Daughter, son's daughter, mother, consanguine sister, and full uncle	[23]
24	Grandmother, daughter, and son's daughter	[22], [23]
25	Husband and 2 daughters	[22], [23]
26	Husband and maternal uncle	[23]
27	Daughter's son and full sister's son	[23]
28	Wife, father, and mother (Al-Umariyatain)	[22]-[24]
29	Husband, father, and mother (Al-Umariyatain)	[22]-[24]
30	Husband, mother, and grandfather	[22], [23]
31	Husband, mother, 2 uterine brothers, and 2 full brothers (only one full brother in [24]) (Himariyya or Musyarrakah)	[23], [24]
32	Mother, full sister, and grandfather (Al-Khuraqa)	[22]-[24]
33	Husband, mother, full sister, and grandfather (Al-Akdariyah)	[22]-[24]
34	Husband, mother, paternal grandfather, 2 uterine brothers, and consanguine brother (Al-Faridah Al-Malikiyah)	[24]

5.3. Gap analysis and identification of testing deficiencies

We categorize the collected cases (Table 5) according to the core functionalities of inheritance calculation systems, which served as the basis for their selection. This allowed us to systematically assess the extent to which existing evaluations cover the expected features and behaviors of such systems. In addition, we provide observations on underrepresented or missing scenarios that limit the completeness of testing.

Reads details of the deceased. None of the reviewed cases validate the systems' ability to handle incorrect or unexpected input, except for Case 1, which appears to unintentionally include impossible scenario by stating that the deceased left both a husband and a wife. Additionally, none of the cases assess the systems' ability to recognize individuals who are ineligible to inherit under Islamic law. The absence of such test scenarios highlights a significant gap in validation and exception-handling coverage, which are fundamental principles in quality assurance. Therefore, test cases should include scenarios with invalid or unexpected inputs, as well as cases involving individuals who are not entitled to inherit under Islamic law, to address these gaps.

Computes prescribed shares to determine the eligibility of prescribed sharers, it is necessary to verify whether they satisfy specific conditions; otherwise, they must be excluded. The conditions are outlined below, and we assess whether existing test cases adequately cover them.

a) The exclusion of son's daughters due to:
   - the presence of a higher-level inheriting male descendant (case 4),
   - the presence of higher-level inheriting female descendants who inherit two-thirds (none of the cases),
   - the presence of different generations of son's daughters (none of the cases).

b) The exclusion of full sisters due to:
   - the presence of an inheriting male descendant (case 4),
   - the presence of an inheriting male ascendant (cases 8, 32, and 33).

c) The exclusion of paternal sisters due to:
   - the presence of an inheriting male descendant (none of the cases),

- the presence of an inheriting male ascendant (none of the cases),
- the presence of a full brother (none of the cases),
- the presence of full sisters who inherit two-thirds (none of the cases),
- the presence of a full sister who is entitled to a residual share "Asbah with others" (none of the cases).

d) The exclusion of uterine siblings due to:
- the presence of an inheriting male ascendant (cases 4 and 21),
- the presence of an inheriting descendant (case 21).

e) The exclusion of grandmothers due to:
- the presence of a mother (cases 4 and 5),
- the presence of a father in the case of a paternal grandmother (cases 4, 8, and 12): there are differing opinions on whether the presence of a father excludes a paternal grandmother from inheritance,
- the presence of grandmothers from different generations (none of the cases).

f) The exclusion of grandfather due to:
- the presence of a father (cases 4 and 21),
- the presence of grandfathers from different generations (none of the cases).

The analysis reveals a shortage of cases that test key exclusion scenarios. Particularly notable omissions include:

- The handling of third-generation granddaughters or grandparents, as current cases involve only two generations, making it unclear whether the systems can correctly exclude third-generation relatives in the presence of first- or second-generation relatives.
- The exclusion of a son's daughter in the presence of higher-level female descendants inheriting two-thirds,
- The exclusion of a paternal sister in the presence of a full brother, full sisters who inherit two-thirds, or in scenarios where they are "Asbah with others", as well as in the presence of an inheriting male descendant or ascendant.

Regarding the computation of prescribed shares, Table 6 presents the relevant scenarios and the existing cases that address them. As shown in Table 6, there are no test cases for a paternal sister inheriting half or two-thirds of the estate. Although this scenario appears similar to that of a full sister inheriting half or two-thirds, it differs due to an additional condition that must be verified: the absence of full siblings. Without validation, systems may incorrectly assign or deny shares to paternal sisters.

Table 6. Inheritance cases related to the computation of prescribed shares

Relative	Prescribed share	Relevant cases
Wife	1/4	9, 15, 19, 28
	1/8	4, 5, 16, 20, 22
Husband	1/2	6, 18, 26, 29, 30, 31, 33, 34
	1/4	7, 14, 25
Daughter	1/2	1, 7, 16, 17, 20, 22, 23, 24
	2/3	8, 25
Son's daughter	1/2	2, 10
	2/3	11, 12
	1/6	17, 20, 23, 24
Father	1/6	4, 13, 14, 21
	1/6 + remainder	2, 16, 17, 20
Mother	1/3	30, 32, 33
	1/6	2, 4, 5, 14, 16, 19, 20, 23, 31, 34
	1/3 of the remainder	28, 29
Full sister	1/2	3, 6, 19
	2/3	9, 18
Paternal sister	1/2	None
	2/3	None
	1/6	3, 19
Uterine sibling	1/3	3, 18, 31
	1/6	15, 19
Grandmother	1/6	6, 8, 10, 11, 12, 13, 24
Grandfather	1/6	5, 34

Computes residual shares to evaluate the systems' ability to identify eligible residual sharers, it is crucial to assess their capacity to apply precedence rules among various groups of residual heirs across

different scenarios. The following outlines these scenarios, with the corresponding test cases presented in Table 7:

a) Precedence based on heir class (sonhood, fatherhood, brotherhood, and cousinhood): These scenarios test the system's ability to differentiate class precedence among residual sharers from two or more heir classes, including:
   – Sonhood precedence: Residual sharers from the sonhood class take precedence over those from the fatherhood, brotherhood, or cousinhood classes.
   – Fatherhood precedence: Residual sharers from the fatherhood class take precedence over those from the brotherhood or cousinhood classes.
   – Brotherhood precedence: Residual sharers from the brotherhood class take precedence over those from the cousinhood class.
b) Precedence based on level within the same class: These scenarios assess whether the system correctly applies precedence rules among residual sharers from the same class, giving priority to closer relatives in generational level over more distant ones:
   – Residual sharers from the sonhood class across different levels,
   – Residual sharers from the fatherhood class across different levels,
   – Residual sharers from the brotherhood class across different levels,
   – Residual sharers from the cousinhood class across different levels.
c) Precedence based on strength of familial tie: These scenarios examine whether the system correctly prioritizes residual sharers from the same class and generation based on the closeness of their familial relationship to the deceased.
d) Precedence of Asbah through others: These cases verify whether heirs classified as Asbah through others are correctly prioritized over residual sharers from the cousinhood class.

Table 7. Inheritance cases testing residual sharer precedence

Type	Relevant cases
Precedence based on heir class:	
Precedence of sonhood over fatherhood	4, 5, 13, 14, 21
Precedence of sonhood over brotherhood	4, 13, 21
Precedence of sonhood over cousinhood	None
Precedence of fatherhood over brotherhood	12
Precedence of fatherhood over cousinhood	None
Precedence of brotherhood over cousinhood	None
Precedence based on level within the same class:	
Precedence within the sonhood class across levels	4
Precedence within the fatherhood class across levels	None
Precedence within the brotherhood class across levels	None
Precedence within the cousinhood class across levels	None
Precedence based on strength of familial tie	None
Precedence of Asbah through others	22, 23

Out of all the precedence scenarios outlined, only a limited subset is currently covered by existing test cases. Many scenarios remain untested, underscoring the need to expand the test cases with cases targeting underrepresented rules, particularly:

– Residual sharers from the sonhood, fatherhood, and brotherhood classes taking precedence over those from the cousinhood class.
– Residual sharers from the same class (fatherhood, brotherhood, and cousinhood), where the closer relative in level (i.e., closer generation) takes precedence over the more distant one.
– Residual sharers from the same class and level, where the heir with a stronger familial tie is given precedence.

To evaluate how well the systems compute shares for residual sharers, we categorized existing cases according to the different types of residual sharers, as shown in Table 8. Although most of the residual sharers are represented, several key scenarios are notably absent:

– Residual sharers from the cousinhood class,
– A son's daughter who becomes a residual sharer when accompanied by son of son's son,
– A paternal sister who becomes a residual sharer when accompanied by her brother.

Additionally, no case in the reviewed set involves a situation where all heirs are residual sharers. This is a critical testing gap because the method for determining the base number in such scenarios differs from cases that involve prescribed sharers.

Table 8. Inheritance cases related to residual sharers

Type	Relevant cases
Asabah by oneself	
Sonhood	14, 21
Fatherhood	8, 12, 28, 29, 30, 32, 33, 34
Brotherhood	11, 15, 31
Cousinhood	None
Asabah through others	
Daughter and son	4
Son's daughter and son's son	5, 13
Son's daughter and son of son's son	None
Full sister and full brother	10
Paternal sister and paternal brother	None
Asabah with others	
Full sister with daughter or son's daughters	7, 22
Paternal sister with daughter or son's daughters	23

Verify individual shares to ensure that this function can accurately determine the type of inheritance case, it is essential to have inheritance cases covering all relevant scenarios. These scenarios include:

– Cases where shares can be distributed without fractions,
– Cases where shares cannot be distributed without fractions and thus require adjustments, and
– Cases involving Awal or Radd.

The cases in previous studies were categorized as follows:

– Shares can be distributed without fractions in cases 2, 3, 15, 16, 17, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, and 34,
– Shares cannot be distributed without fractions in cases 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 25, 31, and 32,6
– Radd is present in cases 9, 24, 25, and 26,
– Awl is present in cases 18, 19, and 20.
– Cases 14 and 18 could not be classified due to the absence of information on the number of heirs, which prevents determining whether shares can be distributed with or without fractions, and whether adjustments need to be applied.

This initial classification shows that all key scenarios are represented; however, a more detailed assessment is needed for cases where shares cannot be distributed without fractions and for those involving Radd, to determine whether all applicable adjustment methods are adequately covered.

Adjust individual shares this function adjusts the shares of heirs in three scenarios: when shares cannot be distributed without fractions, or when Awl or Radd is present. There is only one method for adjustment in Awl cases, while for the other two scenarios, the adjustment methods differ based on the case characteristics.

For scenarios where shares cannot be distributed without fractions, the following adjustment strategies can be considered:

– Single group of heirs: Shares cannot be distributed without fractions among one group of heirs and are corrected by multiplying the base number and all portions by the number of heads in the group (cases 4, 5, 6, 7, 8, 9, 12, 13, 25, 32, and 33)
– Single group using common divisor: Shares cannot be distributed without fractions among one group of heirs and are corrected by multiplying the base number and all portions by the result of dividing the number of heads by the common divisor between the number of heads and the portion (cases 10 and 31)
– Multiple heir groups: Shares cannot be distributed without fractions among multiple groups of heirs (case 11)
– Regarding cases involving Radd, adjustments strategies can be categorized based on the presence of a spouse and the composition of other heirs:
– Cases with a spouse alongside one or more heirs of the same type (cases 9, 25, and 26),
– Cases with a spouse alongside multiple heirs of different types (none of the cases),
– Cases without a spouse but with one or more heirs of the same type (none of the cases),
– Cases without a spouse but with multiple heirs of different types (case 24).

Therefore, to ensure comprehensive testing coverage, additional cases should be developed to include:

– Radd in the presence of a spouse alongside multiple heirs of different types, and
– Radd in the absence of a spouse but including one or more heirs of the same type.

Compute shares for distant kindred among the 34 existing test cases, only two (cases 26 and 27) include distant kindred who are eligible to inherit. Case 26 involves both spouse and distant kindred, while case 27 consists solely of distant kindred heirs. However, to comprehensively assess the system's ability to distinguish between eligible and ineligible distant relatives, it is equally important to include test cases where

distant kindred are not entitled to inherit. Without such cases, it remains unclear whether the systems can correctly suppress distant kindred when their inheritance rights are overridden.

Recognize special and complex cases the evaluation of inheritance calculation systems includes the two cases known as Al-Umariyatain and the case known as Himariyya or Musyarrakah. Additionally, three special cases involve a grandfather along with siblings, where scholarly opinions differ. One viewpoint asserts that siblings are excluded in the presence of a grandfather, while the other does not. For those who follow the second view (not excluding siblings), different methods are used to calculate the grandfather's share. Cases involving a grandfather with prescribed sharers are solved differently from those without prescribed sharers. The three special cases currently evaluated (cases 32, 33, and 34) all involve prescribed shares but do not cover all solution variants. This highlights the need to include cases that involve prescribed shares with different solution variants, as well as cases that do not involve prescribed shares.

Finally, complex cases, such as those involving fetuses in the womb, concurrent deaths, and missing or intersex individuals, are absent from current evaluations. Including such cases is essential for a thorough and realistic assessment of the capabilities and limitations of inheritance calculation systems.

This analysis revealed significant limitations in how current inheritance calculation systems are evaluated. While prior studies provide valuable examples, our analysis identified inconsistencies in both the quantity and rigor of test cases across studies. Furthermore, it highlighted insufficient coverage of key functional scenarios. Many critical conditions that affect the eligibility and share of heirs are either underrepresented or entirely absent in existing test sets. These gaps underscore the need to design additional test cases that simulate such situations to ensure robust validation of inheritance systems. To address these gaps, we developed a comprehensive benchmark grounded in the core functionalities of inheritance calculation systems and enriched it with new test cases. This benchmark advances the field by enabling consistent validation and comparative benchmarking of inheritance calculation systems. Ultimately, this work lays a crucial foundation for the reproducible and standardized evaluation of future systems, thereby fostering the development of accurate and robust inheritance calculation systems.

5.4. Development of new inheritance test cases

Building on existing cases, this section presents new inheritance test cases designed to address previously uncovered aspects and, when combined with existing ones, enable a more comprehensive evaluation of inheritance calculation systems.

Handling errors:

1) Husband and wife.
2) 5 wives.

Recognizing relatives who are not entitled to inherit under Islamic law:

3) A stepmother.
4) Stepsiblings.

Computing prescribed shares:

5) The exclusion of a son's daughter due to the presence of higher-level inheriting female descendants who inherit two-thirds: A wife, 2 daughters, and a son's daughter.
6) The exclusion of son's daughters due to the presence of different generations of son's daughters: A daughter, a son's daughter, and a son's son's daughter.
7) The exclusion of paternal sisters due to the presence of an inheriting male descendant: A son, a paternal brother, and a paternal sister.
8) The exclusion of paternal sisters due to the presence of an inheriting male ascendant: A father, a mother, and a paternal sister.
9) The exclusion of a paternal sister due to the presence of a full brother: A daughter, a full brother, and a paternal sister.
10) The exclusion of a paternal sister due to the presence of full sister(s) who inherit two-thirds: 2 full sisters and a paternal sister.
11) The exclusion of a paternal sister due to the presence of a full sister who is entitled to a residual share "Asbah with others": A son's daughter, a full sister, and a paternal sister.
12) The exclusion of grandmothers due to the presence of grandmothers from different generations: A paternal grandmother and a maternal great-grandmother (i.e. mother of maternal grandmother).
13) The exclusion of grandfather due to the presence of grandfather from different generations: A son, a paternal grandfather, and a paternal great-grandfather.
14) A paternal sister inheriting half of the estate: A husband, a paternal sister, and 2 uterine sisters.
15) A paternal sister inheriting two-thirds of the estate: A husband, 2 paternal sisters, and 2 uterine sisters.

Computing residual shares:

16) Residual sharers from the sonhood class take precedence over those from the cousinhood classes: A son and a paternal uncle.

17) Residual sharers from the fatherhood class take precedence over those from the cousinhood classes: A father and a paternal uncle.
18) Residual sharers from the brotherhood class take precedence over those from the cousinhood class: A full brother and a son of a paternal uncle.
19) Residual sharers from the fatherhood class across different levels: A paternal grandfather and a paternal great-grandfather.
20) Residual sharers from the brotherhood class across different levels: A paternal brother's son and a son of a full brother's son.
21) Residual sharers from the cousinhood class across different levels: A paternal uncle and a son of a paternal uncle.
22) Residual sharers of the same class and level with a stronger familial tie: A full brother and a paternal brother.
23) Residual sharers from the cousinhood class: 2 daughters, 2 son's daughters, and a paternal uncle.
24) A son's daughter who becomes a residual sharer because she is accompanied by a son of a son's son: A wife, 2 daughters, a son's daughter, a son of a son's son.
25) A paternal sister who becomes a residual sharer because she is accompanied by her brother: A wife, 2 full sisters, a paternal brother, and a paternal sister.

Adjusting individual shares:

26) Radd in the presence of a spouse alongside multiple heirs of different types: A wife, a grandmother, and 2 uterine sisters.
27) Radd in the presence of a spouse alongside multiple heirs of different types: A wife, 2 daughters, and a mother.
28) Radd in the absence of a spouse but including one or more heirs of the same type (one heir): A daughter.
29) Radd in the absence of a spouse but including one or more heirs of the same type (multiple heirs): 3 full sisters.

Distant kindred:

30) Not eligible to inherit due to the presence of residual sharers: A son, a daughter, and either a maternal grandfather or the mother of the maternal grandfather.
31) Not eligible to inherit due to the presence of prescribed sharers: A mother, a full sister, and either a maternal or paternal aunt.

A grandfather with siblings in the presence of prescribed sharers:

32) A husband, 2 daughters, a mother, a grandfather, and a brother.
33) A husband, 2 daughters, a grandfather, and a brother.
34) A husband, a mother, a grandfather, and a brother.
35) A mother, a grandfather, and 3 brothers.
36) A husband, a mother, a grandfather, and 3 brothers.

A grandfather with siblings in the absence of prescribed sharers:

37) A grandfather, a full brother, and 2 full sisters.
38) A grandfather and 4 full brothers.
39) A grandfather and 3 full sisters.

Fetuses in the womb:

40) A pregnant wife, a father, and a mother.

Concurrent deaths:

41) The heirs of the second deceased are the remaining heirs of the first deceased, and their inheritance from the second is similar to their inheritance from the first: 5 sons, one of whom died before the distribution of the estate.
42) The heirs of the second deceased are the same as the heirs of the first deceased, but their relationships to each deceased differ: A mother, 2 sons, and a daughter. Then, one of the sons died, leaving behind his grandmother, full brother, and full sister.
43) The heirs of the second deceased are different from the heirs of the first deceased: A wife, a full sister, and a paternal uncle. Then, the full sister died leaving behind a husband, a daughter, and a son.
44) Some of the heirs may inherit from both sides: 3 daughters, 2 full sisters, and a full brother. Then, one of the full sisters died, leaving behind her full brother and full sister.
45) Some of the heirs may inherit from both sides): A full sister, a paternal sister, and a uterine sister. Then, the full sisters died, leaving behind 2 daughters, as well as her paternal and uterine sister.
46) Multiple deceased individuals: Two daughters and a son. Then, one daughter died, leaving behind a husband and a son. Later, the son of the first deceased also died, leaving behind a daughter and a son.

47) Multiple deceased individuals: Two full sisters, two uterine sisters, and a mother. Then, one full sister died, leaving behind the remaining full sister, the two uterine sisters, and the mother. Then, the other full sister died, leaving behind her husband, the remaining two uterine sisters, and the mother.

Missing or intersex individuals:

48) A wife, a mother, a paternal brother, and a missing full brother.
49) A son, a daughter, an intersex child whose biological sex can be identified, and an uncle.
50) A son, a daughter, an intersex child whose biological sex cannot be identified, and an uncle.

5.5. Application of proposed test cases for system evaluation

The performance of each calculator on the 50 benchmark cases is shown in Table 9. Calculator 1 correctly solved 31 cases and partially solved 6 cases, yielding an accuracy of 68%. Calculator 2 correctly solved 29 cases, achieving an overall accuracy of 58 %. Calculator 3 demonstrated the highest performance, correctly solving 41 cases and partially solving 4, resulting in an overall accuracy of 86%. A Friedman test revealed a statistically significant difference in performance among the three inheritance calculators, $\chi^2(2) = 17.40$, $p < 0.001$. Post-hoc Wilcoxon signed-rank tests with Bonferroni correction ($\alpha = 0.017$) showed that Calculator 3 outperformed both Calculator 1 ($p = 0.009$) and Calculator 2 ($p = 0.001$), whereas no significant difference was found between Calculators 1 and 2 ($p = 0.311$). These results confirm that Calculator 3 achieved significantly higher accuracy across the 50 benchmark cases.

The performance results of the three inheritance calculators across all test case categories are presented in Figure 1. Overall, Calculator 3 demonstrated the highest and most consistent performance across all categories, achieving full accuracy in most standard cases and maintaining reasonable accuracy in complex scenarios such as concurrent deaths (71%) and fetus on the womb and intersex or missing individual cases (75%). Calculator 1 performed well in basic scenarios, achieving an accuracy of 86.4% for prescribed shares and 100% for residual shares. In contrast, Calculator 2 showed lower accuracy even in these fundamental categories, with 63.6% for prescribed shares and 80% for residual shares. Both calculators 1 and 2 do not support the distribution of estates in cases involving concurrent deaths and missing or intersex individuals. Although Calculator 1 includes a text box for entering the number of missing individuals, it does not allow users to specify their relationship to the deceased and therefore cannot handle the calculation properly. As a result, this calculator is considered not to support the distribution of the estate with missing heirs. Additionally, Calculator 2 lacks support for third-generation relatives, such as grandparents or grandchildren, as well as fetuses in the womb. Calculators 2 and 3 omit distant kindred, such as a maternal or paternal aunt and the maternal grandfather, from their heir lists, and therefore cannot support cases involving these relatives.

Table 9. Performance of the online calculators on the proposed test cases. Scoring: NS = not supported; 0 = incorrect; 0.5 = partially correct; 1 = correct

Case #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Calculator 1	1	1	NS	NS	1	0	1	1	1	1	1	.5	1	1	1	1	1
Calculator 2	1	1	1	NS	1	NS	1	1	1	0	1	NS	NS	1	1	1	1
Calculator 3	1	1	NS	NS	1	1	1	1	1	1	1	1	1	1	1	1	1
Case #	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Calculator 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	.5	1	.5
Calculator 2	1	NS	1	1	1	1	NS	1	1	1	1	1	NS	NS	1	1	1
Calculator 3	1	1	1	1	1	1	1	1	1	1	1	1	NS	NS	1	1	1
Case #	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	
Calculator 1	.5	.5	1	1	1	.5	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	
Calculator 2	1	1	1	1	0	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	NS	
Calculator 3	1	1	1	1	1	0	.5	1	1	.5	.5	1	.5	1	1	1	

The error analysis for Calculator 1 identified the following issues:

- Case 6: The calculator should exclude a son's son's daughter from inheriting when female descendants (daughters and a son's daughter) are present, as they collectively inherit two-thirds of the estate. However, the calculator incorrectly assigns one-sixth to the son's son's daughter in these cases.
- Case 12: According to the Shafi'i school, the two grandmothers should jointly receive 1/6 of the estate, with the remainder going to the public treasury. However, the calculator incorrectly allocates half of the estate to each grandmother, and nothing is allocated to the public treasury.
- Cases 34, 35, and 36: All these cases involve a grandfather with siblings. In Case 34, the brother is a residual heir and should be excluded because the entire estate is distributed among the other heirs, leaving nothing for him. However, the calculator incorrectly assigned him a share. In Case 35, the grandfather's share should be one-third of what remains after the mother takes her share, with the

brother receiving what remains after that. However, the calculator incorrectly assigns the grandfather one-third of the entire estate and gives the brother what remains of the whole estate. A similar mistake occurs in Case 36, where the calculator assigns the grandfather one-third of the estate, explaining it as "The grand father has option to choose between 1/3 of the residue or to share as a male sibling with other siblings, and he choose whatever is greater, provided if they are only heirs". However, the calculator fails to account for the rule that the grandfather is entitled to the greater of either one-sixth of the estate, one-third of what remains after the prescribed heirs take their shares, or sharing as a male sibling. In this case, one-sixth is the larger share and should have been allocated to the grandfather.

– Case 40: We consider the calculator to have solved this case partially because it assumes the unborn child is a male, omits the female scenario, and fails to note that the share difference should be held until birth.

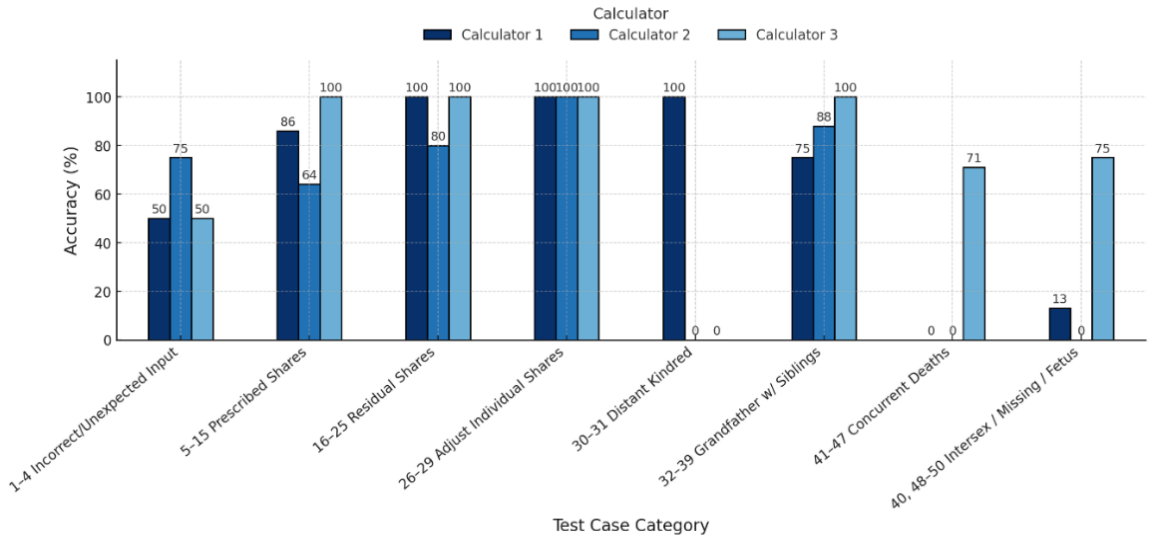

Figure 1. Accuracy of inheritance calculators by test case category

Furthermore, the error analysis of Calculator 2 revealed the following issues:

– Case 10: The paternal sister should be excluded due to the presence of full sisters who inherit two-thirds of the estate. However, the calculator incorrectly assigned a share to the paternal sister.
– Case 39: The calculator assigned one-third of the estate to the grandfather, but it should have determined the grandfather's share based on sharing as a male sibling, giving him the greater share.

Finally, the error analysis for Calculator 3 showed:

– Cases 41, 44 and 45: In scenarios involving multiple deceased individuals, the calculator fails to recognize the relationships between the heirs of successive decedents. Moreover, it does not provide users with the option to manually specify these relationships. For instance, in Case 41, there are five sons, one of whom passed away before the distribution of the estate. The surviving sons are brothers to the deceased son; however, the calculator processes each case separately, assigning portions individually in both instances without aggregating the shares. This approach leads to confusion and potential inaccuracies in the inheritance distribution. Additionally, the calculator often defaults to detailed, step-by-step solutions for each case. While this level of detail can be beneficial, it is not always necessary, as more concise methods or shortcuts can produce accurate results with less complexity. However, the calculator does not explain these alternative approaches, limiting its usefulness in educational settings where understanding multiple solution methods is essential.

6. CONCLUSION

Given the complexity and need for precise calculations in Islamic inheritance laws, various computer systems have been developed to support inheritance calculations. However, the performance of these systems cannot be fully trusted due to a lack of comprehensive evaluation and testing. This study addresses this issue by conducting methodological gap analysis and using its findings to guide the development of a comprehensive set of test cases that capture the functionality of inheritance calculation

systems, providing a rigorous benchmark for evaluating their capabilities and limitations. The subsequent evaluation of three online calculators revealed that even high-performing systems exhibited notable deficiencies, especially in handling complex cases and rare scenarios, a shortcoming that would have remained hidden without the proposed comprehensive test cases. The implications of this study are twofold. First, the proposed benchmark can serve as a standardized validation suite for the development and evaluation of Islamic inheritance systems, enabling consistent, transparent, and comparable assessments. Second, the findings highlight the critical need to enhance the robustness and reliability of these systems, particularly in handling special and complex cases. While the proposed benchmark provides valuable insights into the functionality of inheritance calculation systems and their ability to handle the distribution of estates across different scenarios, this study did not incorporate non-functional testing such as usability and performance testing. Future research should extend the evaluation of inheritance calculation systems beyond functional correctness to cover non-functional aspects such as usability, efficiency, and user satisfaction. Incorporating these dimensions will offer a more holistic understanding of system performance, supporting the development of inheritance tools that are not only accurate but also practical and user-friendly for a wide audience. Further extensions may include multilingual capabilities, school-of-thought parameterization to reflect jurisprudential diversity, and integration with legal technology platforms to enhance accessibility and real-world applicability.

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

Solutions to all existing inheritance cases used in this study, along with the proposed cases and their detailed evaluation results, are publicly available at: https://www.kaggle.com/datasets/ghaderkurdi/islamic-inheritance-system-evaluation-dataset. All other data generated or analyzed during this study, including the benchmark test cases and evaluation results, are fully presented within the paper.

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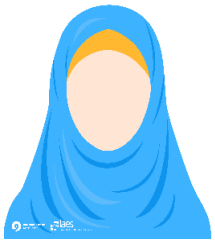

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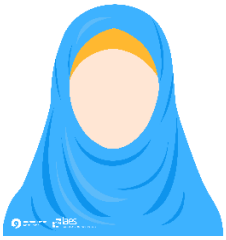

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