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Extension of Linear Channels Identification Algorithms to Non Linear Using Selected Order Cumulants

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Abstract

In this paper, we present an extension of linear communication channels identification algorithms to non linear channels using higher order cumulants (HOC). In the one hand, we develop a theoretical analysis of non linear quadratic systems using second and third order cumulants. In the other hand, the relationship linking cumulants and the coefficients of non linear channels presented in the linear case is extended to the general case of the non linear quadratic systems identification. This theoretical development is used to develop three non linear algorithms based on third and fourth order cumulants respectively. Numerical simulation results example show that the proposed methods able to estimate the impulse response parameters with different precision.

Keywords: *Higher order cumulants, Blind identification, Non linear quadratic systems.*

1. Introduction

Applications of higher order cumulants theory in the blind identification domain are widely used in various works [1]-[3], [6, 7]. Several models are identified in the literature such as the linear and non linear systems. In the part of the linear case, we have important results established that the blind identification is possible only from the second order cumulants (autocorrelation function) of the output stationary signal, but these methods is not able to identify correctly the channel models excited by non Gaussian signal and affected by Gaussian noise, because the additive Gaussian noise will be vanish in the higher order cumulants domain. The sensitivity of the second order cumulants to the additive Gaussian noise appealed to other blind identification methods exploiting the cumulants of order superieur than two [4, 5]. There are several motivations behind this interest, first the methods based on HOC are blind to any kind of a Gaussian process, whereas autocorrelation function (second order cumulants) is not. Consequently, cumulants based methods boost signal to noise ratio (SNR) when signals are corrupted by Gaussian measurement noise. Second, the HOC methods are useful in identifying non minimum phase systems and in reconstructing non minimum phase signals when the signals are non Gaussian.

The linear models are not efficient for representing and modeling all systems, because the majority of systems are represented by non linear models [7, 8]. However, when linear modeling of the channel is not adequate, the non linear modeling appeared like an alternative efficient solution in most real cases. Moreover, quadratic non linear systems are widely used in various engineering fields such as signal processing, system filtering, predicting, identification and equalization [9, 10].

In this contribution, firstly we present a theoretical development of non linear quadratic systems using higher order cumulants. Indeed, we develop the relationship linking third order cumulants and the coefficients of non linear channels, then the method developed [11] for linear channels, is extended to the general case of the non linear quadratic systems identification. Secondly, these theoretical analyses are used to develop an extension of linear algorithms based on third and fourth order cumulants proposed by safi, $et\ al.$ [5] and Abderrahim, $et\ al.$ [12] for linear

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systems to non linear algorithms for identification of quadratic systems. Numerical simulations results are given to illustrate the accuracy of the proposed methods.

2. Non linear communication channel and hypotheses

We consider a non linear channel (Fig. 1) described by the following equation:

$$y(k) = \sum_{i=0}^{q} h(i,i)x^2(k-i) + w(k), \tag{1}$$

where $y(k)$ represent the output model is generated through a quadratic non linear model driven by a stationary input sequence $x(k)$, $h(i,i)$ and q are the parameters and the order of non linear channel, respectively, $w(k)$ is additive Gaussian noise.

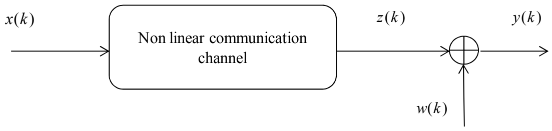

Figure 1. Non linear channel model

For this system we assume that:
H1: The order q is known;
H2: The input sequence $x(k)$ is independent and identically distributed (i.i.d) zero mean, stationary, non Gaussian and with:
$C_{k,x}(\tau_1, \tau_2, ..., \tau_{k-1}) = \begin{cases} \gamma_{k,x}, & \tau_1 = \tau_2 = ... = \tau_{k-1} = 0; \\ 0 & \text{otherwise} \end{cases}$
where $\gamma_{k,x}$ denotes the k^{th} order cumulants of the input signal $x(k)$ at origin
H3: The system is supposed causal and bounded, i.e. $h(i,i) = 0$ if $i < 0$ and $i > q$ with $h(0,0) = 1$;
H4: The measurement noise sequence $w(k)$ is assumed to be zero mean, i.i.d, Gaussian, independent of $x(k)$ with unknown variance;
H5: The system is supposed stable, i.e. $|h(i,i)| < \infty$.

3. Theoretical development for non linear communication channels using HOC

In this section, we firstly focus on additional concepts and definitions used throughout this paper. Indeed, we discuss the identifiability of the quadratic non linear model in the second and third order cumulants domain, then based an the relationship proposed by Stogioglou and McLaughlin [11] in the linear case, we develop an extension of this relation in non linear case.

3.1. Identification of quadratic non linear models in the second and third order cumulants

In this subsection, we use the Leonov Shiryaev formula [6] and the definition of the cumulants to demonstrate the equations linking the thrid order cumulants and the diagonal parameters of non linear systems.

Thus, the cumulants, order r, and the moments of the stochastic signal are linked by the following relationships of the Leonov and Shiryayev formula [6]:

$$Cum[z_1, ..., z_r] = \sum (-1)^{k-1}(k-1)! E\Big[\prod_{i\in v_1} z_i\Big].E\Big[\prod_{j\in v_2} z_j\Big]...E\Big[\prod_{k\in v_p} z_k\Big], \tag{2}$$

where the addition operation is over all the set of v_i, $1 \le i \le p \le r$ and v_i compose a partition of $1, 2, ..., r$. In Eq. (2) k is the number of elements compose a partition.
For the second order cumulants, we are $r = 2$ and $1 \le p \le 2$. The possible partitions are: (1,2) and (1)(2) thus:

$$Cum[z_1, z_2] = E[z_1 z_2] - E[z_1]E[z_2], \tag{3}$$

$$
\begin{aligned}
C_{2,z}(\tau) = Cum(z_1, z_2) &= Cum\Big[z(k), z(k+\tau)\Big] \\
&= E\Big[\sum_{i=0}^{q} h(i,i)x^2(k-i) \sum_{j=0}^{q} h(j,j)x^2(k+\tau-j)\Big] \\
&- E\Big[\sum_{i=0}^{q} h(i,i)x^2(k-i)\Big] E\Big[\sum_{j=0}^{q} h(j,j)x^2(k+\tau-j)\Big] \qquad (4)
\end{aligned}
$$

$$
\begin{aligned}
C_{2,z}(\tau) &= \sum_{i=0}^{q} h(i,i)h(i+\tau, i+\tau)E\Big[x^2(k-i)x^2(k-i)\Big] \\
&- \sum_{i=0}^{q} h(i,i)h(i+\tau, i+\tau)E\Big[x^2(k-i)\Big]E\Big[x^2(k-i)\Big] \qquad (5)
\end{aligned}
$$

Under the assumption H2 and Eq. (5), the second order cumulants becomes:

$$
C_{2,z}(\tau) = (\gamma_{4,x} - \gamma_{2,x}^2) \sum_{i=0}^{q} h(i,i)h(i+\tau, i+\tau) \qquad (6)
$$

For the third order cumulants, we are $r = 3$ and $1 \leq p \leq 3$. The possible partitions are: (1,2,3), (1)(2,3), and (1)(2)(3) so:

$$
\begin{aligned}
Cum[z_1, z_2, z_3] &= E[z_1 z_2 z_3] - E[z_1]E[z_2 z_3] - E[z_2]E[z_1 z_3] - E[z_3]E[z_1 z_2] \\
&+ 2E[z_1]E[z_2]E[z_3] \qquad (7)
\end{aligned}
$$

Thus, the third order cumulants becomes:

$$
\begin{aligned}
C_{3,z}(\tau_1, \tau_2) &= Cum\Big[z(k), z(k+\tau_1), z(k+\tau_2)\Big] \\
&= E\Big[\sum_{i=0}^{q} h(i,i)x^2(k-i) \sum_{j=0}^{q} h(j,j)x^2(k+\tau_1-j) \sum_{l=0}^{q} h(l,l)x^2(k+\tau_2-l)\Big] \\
&- [3]E\Big[\sum_{i=0}^{q} h(i,i)x^2(k-i)\Big] E\Big[\sum_{j=0}^{q} h(j,j)x^2(k+\tau_1-j) \sum_{l=0}^{q} h(l,l)x^2(k+\tau_2-l)\Big] \\
&+ [2]E\Big[\sum_{i=0}^{q} h(i,i)x^2(k-i)\Big] E\Big[\sum_{j=0}^{q} h(j,j)x^2(k+\tau_1-j)\Big] \\
&\times E\Big[\sum_{l=0}^{q} h(l,l)x^2(k+\tau_2-l)\Big] \qquad (8)
\end{aligned}
$$

Under the assumption H2 and Eq. (8), the third order cumulants becomes:

$$
C_{3,z}(\tau_1, \tau_2) = (\gamma_{6,x} - 3\gamma_{2,x}\gamma_{4,x} + 2\gamma_{2,x}^3) \sum_{i=0}^{q} h(i,i)h(i+\tau_1, i+\tau_1)h(i+\tau_2, i+\tau_2) \qquad (9)
$$

In the general case, the relationship between n^{th} order cumulants and the coefficients of non linear impulse response channel can be written under form:

$$
C_{n,z}(\tau_1, \tau_2, ..., \tau_{n-1}) = \eta_{n,x} \sum_{i=0}^{q} h(i,i)h(i+\tau_1, i+\tau_1)...h(i+\tau_{n-1}, i+\tau_{n-1}), \qquad (10)
$$

where $\eta_{n,x} = \sum_i p_i \gamma_{i,x}^{m_i}$, with $p_i \in \mathbb{Z}$ and $m_i \in \mathbb{N}$

3.2. Relationship linking cumulants and the coefficients of non linear channel

In this section, we develop a non linear extension of the Stogioglou and McLaughlin's linear relationship [11].

Proof:

Let :

$$Z_{s,n} = \sum_i \sum_j h(i,i)h(j,j)\Big[\prod_{k=1}^{s} h(i+\tau_k, i+\tau_k)\Big]\Big[\prod_{k=1}^{s} h(j+\beta_k, j+\beta_k)\Big]\Big[\prod_{k=s+1}^{n-1} h(i+j+\alpha_k, i+j+\alpha_k)\Big], \tag{11}$$

where s is an arbitrary integer number satisfying: $1 \leq s \leq n-2$.

Changing the order of summation in Eq (11), and we multiply this equation by $\eta_{n,x}$, we will obtain:

$$\eta_{n,x} Z_{s,n} = \sum_i h(i,i)\Big[\prod_{k=1}^{s} h(i+\tau_k, i+\tau_k)\Big]\eta_{n,x}\sum_j h(j,j)\Big[\prod_{k=1}^{s} h(j+\beta_k, j+\beta_k)\Big]\Big[\prod_{k=s+1}^{n-1} h(i+j+\alpha_k, i+j+\alpha_k)\Big], \tag{12}$$

where, $\eta_{n,x}$ represents the n^{th} order cumulants of the excitation signal at origin in non linear case defined by $\eta_{n,x} = \sum_i p_i \gamma_{i,x}^{m_i}$.

From Eqs (10) and (12) we obtain:

$$\eta_{n,x} Z_{s,n} = \sum_{i=0}^{q} h(i,i)\Big[\prod_{k=1}^{s} h(i+\tau_k, i+\tau_k)\Big] C_{n,y}(\beta_1, ..., \beta_s, i+\alpha_1, ..., i+\alpha_{n-s-1}) \tag{13}$$

In the same way, if we sum on i afterwards on j in (11), we will find:

$$\eta_{n,x} Z_{s,n} = \sum_j h(j,j)\Big[\prod_{k=1}^{s} h(j+\beta_k, j+\beta_k)\Big]\eta_{n,x}\sum_i h(i,i)\Big[\prod_{k=1}^{s} h(i+\tau_k, i+\tau_k)\Big]\Big[\prod_{k=s+1}^{n-1} h(i+j+\alpha_k, i+j+\alpha_k)\Big] \tag{14}$$

The same, from Eqs (10) and (14) we obtain:

$$\eta_{n,x} Z_{s,n} = \sum_{j=0}^{q} h(j,j)\Big[\prod_{k=1}^{s} h(j+\beta_k, j+\beta_k)\Big] C_{n,y}(\tau_1, ..., \tau_s, j+\alpha_1, ..., j+\alpha_{n-s-1}), \tag{15}$$

From Eqs. (13) and (15) we obtain Stogioglou-McLaughlin relation in non linear case:

$$\begin{aligned} & \sum_{j=0}^{q} h(j,j)\Big[\prod_{k=1}^{s} h(j+\tau_k, j+\tau_k)\Big] C_{n,y}(\beta_1, ..., \beta_s, j+\alpha_1, ..., j+\alpha_{n-s-1}) \\ = \quad & \sum_{i=0}^{q} h(i,i)\Big[\prod_{k=1}^{s} h(i+\beta_k, i+\beta_k)\Big] C_{n,y}(\tau_1, ..., \tau_s, i+\alpha_1, ..., i+\alpha_{n-s-1}), \end{aligned} \tag{16}$$

where $1 \leq s \leq n-2$.

4. Extension of linear algorithms to non linear algorithms

In this section, we describe an extension of linear communication channels identification algorithms to non linear using HOC. However, the linear algorithms based on third and fourth order cumulants proposed in the literature [5, 12] for linear case is extended to the non linear case for identification of quadratic systems.

4.1. First algorithm: Algcum1

The Fourier transform of Eqs. (6) and (9) gives us the bispectra and the spectrum respectively:

$$S_{3,z}(\omega_1,\omega_2) = (\gamma_{6,x} - 3\gamma_{2,x}\gamma_{4,x} + 2\gamma_{2,x}^3)H(-\omega_1-\omega_2,-\omega_1-\omega_2)H(\omega_1,\omega_1)H(\omega_2,\omega_2) \quad (17)$$

$$S_{2,z}(\omega) = (\gamma_{4,x} - \gamma_{2,x}^2)H(-\omega,-\omega)H(\omega,\omega) \quad (18)$$

If we suppose that $\omega = \omega_1 + \omega_2$, Eq. (18) becomes:

$$S_{2,z}(\omega_1+\omega_2) = (\gamma_{4,x} - \gamma_{2,x}^2)H(-\omega_1-\omega_2,-\omega_1-\omega_2)H(\omega_1+\omega_2,\omega_1+\omega_2) \quad (19)$$

Then, from Eqs. (17) and (19) we obtain the following equation:

$$S_{3,z}(\omega_1,\omega_2)H(\omega_1+\omega_2,\omega_1+\omega_2) = \frac{\gamma_{6,x} - 3\gamma_{2,x}\gamma_{4,x} + 2\gamma_{2,x}^3}{\gamma_{4,x} - \gamma_{2,x}^2}H(\omega_1,\omega_1)H(\omega_2,\omega_2)S_{2,z}(\omega_1+\omega_2,\omega_1+\omega_2) \quad (20)$$

The inverse Fourier transform of Eq. (20) demonstrates that the third order cumulants, the autocorrelation function and the impulse response channel parameters are combined by the following equation:

$$\sum_{i=0}^{q} C_{3,z}(\tau_1-i,\tau_2-i)h(i,i) = \frac{\gamma_{6,x} - 3\gamma_{2,x}\gamma_{4,x} + 2\gamma_{2,x}^3}{\gamma_{4,x} - \gamma_{2,x}^2}\sum_{i=0}^{q} h(i,i)h(\tau_2-\tau_1+i,\tau_2-\tau_1+i)C_{2,z}(\tau_1-i) \quad (21)$$

If we use the autocorrelation function property of the stationary process such as $C_{2,z}(\tau) \neq 0$ only for $-q \leq \tau \leq q$ and vanishes elsewhere if we take $\tau_1 = -q$, Eq. (21) takes the forme:

$$\sum_{i=0}^{q} C_{3,z}(-q-i,\tau_2-i)h(i,i) = \frac{\gamma_{6,x} - 3\gamma_{2,x}\gamma_{4,x} + 2\gamma_{2,x}^3}{\gamma_{4,x} - \gamma_{2,x}^2}h(0,0)h(\tau_2+q,\tau_2+q)C_{2,z}(-q), \quad (22)$$

else, if we suppose that $\tau_2 = -q$, Eq. (22) will become:

$$C_{3,z}(-q,-q)h(q,q) = \frac{\gamma_{6,x} - 3\gamma_{2,x}\gamma_{4,x} + 2\gamma_{2,x}^3}{\gamma_{4,x} - \gamma_{2,x}^2}h(0,0)C_{2,z}(-q) \quad (23)$$

Using Eqs. (22) and (23) we obtain the following relation:

$$\sum_{i=0}^{q} C_{3,z}(-q-i,\tau_2-i)h(i,i) = C_{3,z}(-q,-q)h(\tau_2+q,\tau_2+q) \quad (24)$$

The system of Eq. (24) can be written in matrix form as:

$$\begin{pmatrix} C_{3,z}(-q-1,-q-1) & \dots & C_{3,z}(-2q,-2q) \\ C_{3,z}(-q-1,-q)-\theta & \dots & C_{3,z}(-2q,-2q+1) \\ \vdots & \ddots & \vdots \\ C_{3,z}(-q-1,-1) & \dots & C_{3,z}(-2q,-q)-\theta \end{pmatrix} \times \begin{pmatrix} h(1,1) \\ \vdots \\ h(i,i) \\ \vdots \\ h(q,q) \end{pmatrix} = \begin{pmatrix} 0 \\ -C_{3,z}(-q,-q+1) \\ \vdots \\ -C_{3,z}(-q,0) \end{pmatrix}, \quad (25)$$

where $\theta = C_{3,y}(-q,-q)$.

Or in more compact form, the Eq. (25) can be written as follows:

$$Mh_s = d, \quad (26)$$

where M is the matrix of size $(q+1) \times (q)$ elements, h_s is a column vector constituted by the unknown impulse response parameters $h(i,i)$ for $i = 1, ..., q$ and d is a column vector of size $(q+1)$ as indicated in the Eq. (26).
The least squares solution of the system of Eq. (26), permits blindly identification of the parameters $h(i,i)$ and without any information of the input selective channel. Thus, the solution will be written under the following form:

$$\widehat{h}_s = (M^T M)^{-1} M^T d \tag{27}$$

4.2. Second algorithm: Algcum2

Safi, $et\ al.$ [5] use the Stogioglou-McLaughlin relation for developing an algorithm based only on fourth order cumulants in the linear case, this algorithm is extended to the non linear case. Thus, if we take $n = 4$ into (16) we obtain the following equation:

$$\begin{aligned} &\sum_{j=0}^{q} h(j,j)h(j+\tau_1, j+\tau_1)h(j+\tau_2, j+\tau_2)C_{4,y}(\beta_1, \beta_2, j+\alpha_1) \\ &= \sum_{i=0}^{q} h(i,i)h(i+\beta_1, i+\beta_1)h(i+\beta_2, i+\beta_2)C_{4,y}(\tau_1, \tau_2, i+\alpha_1) \end{aligned} \tag{28}$$

If $\tau_1 = \tau_2 = q$ et $\beta_1 = \beta_2 = 0$, (28) take the form:

$$h(0,0)h^2(q,q)C_{4,y}(0,0,\alpha_1) = \sum_{i=0}^{q} h^3(i,i)C_{4,y}(q,q,i+\alpha_1), \tag{29}$$

with

$$-q \le \alpha_1 \le q \tag{30}$$

Then, from (29) and (30) we obtain the following system of equations:

$$\begin{pmatrix} C_{4,y}(q,q,-q) & \cdots & C_{4,y}(q,q,0) \\ \vdots & \ddots & \vdots \\ C_{4,y}(q,q,0) & \cdots & C_{4,y}(q,q,q) \\ \vdots & \ddots & \vdots \\ C_{4,y}(q,q,q) & \cdots & C_{4,y}(q,q,2q) \end{pmatrix} \times \begin{pmatrix} \frac{1}{h^2(q,q)} \\ \vdots \\ \frac{h^3(i,i)}{h^2(q,q)} \\ \vdots \\ \frac{h^3(q,q)}{h^2(q,q)} \end{pmatrix} = \begin{pmatrix} C_{4,y}(0,0,-q) \\ \vdots \\ C_{4,y}(0,0,0) \\ \vdots \\ C_{4,y}(0,0,q) \end{pmatrix} \tag{31}$$

In more compact form, the system of (31) can be written in the following form:

$$M b_q = d, \tag{32}$$

where M, b_q and d are defined in the system of (32).
The least squares solution of the system of (32) is given by:

$$\widehat{b_q} = (M^T M)^{-1} M^T d \tag{33}$$

This solution give us an estimation of the quotient of the parameters $h^3(i,i)$ and $h^2(q,q)$, i.e., $b_q(i,i) = \widehat{\frac{h^3(i,i)}{h^2(q,q)}}$, $i = 1, ..., q$. Thus, in order to obtain an estimation of the parameters $\widehat{h}(i,i)$, $i = 1, ..., q$ we proceed as follows:
The parameters $h(i,i)$ for $i = 1, ..., q-1$ are estimated from the estimated values $\widehat{b}_q(i,i)$ using the following equation:

$$\widehat{h}(i,i) = sign\Big[\widehat{b}_q(i,i) \times (\widehat{b}_q(q,q))^2\Big]\Big\{abs(\widehat{b}_q(i,i)) \times (\widehat{b}_q(q,q))^2\Big\}^{1/3} \tag{34}$$

The $\widehat{h}(q,q)$ parameters is estimated as follows:

$$\widehat{h}(q,q)=\frac{1}{2}sign\Big[\widehat{b}_q(q,q)\Big]\Big\{abs(\widehat{b}_q(q,q))+\Big(\frac{1}{\widehat{b}_q(1,1)}\Big)^{1/2}\Big\} \tag{35}$$

4.3. Third algorithm: Algcum3

Abderrahim, $et\ al.$ [12] use also the relationship (16), based on fourth order cumulants, for developing an algorithm based an fourth order cumulants in linear case. This algorithm is extended to the general case of the non linear quadratic systems, with $\beta_1=\beta_2=0$, $\tau_1=q$ and $\tau_2=0$:

$$\sum_{i=1}^{q}h^3(i,i)C_{4,y}(q,0,i+\alpha_1)-h(q,q)C_{4,y}(0,0,\alpha_1)=-C_{4,y}(q,0,\alpha_1), \tag{36}$$

where $\alpha_1=-q,...,q$.
In more compact form, the system of Eq. (36) can be written in the following form:

$$M\theta=A \tag{37}$$

$\theta=[h(q,q)\quad h^3(1,1)\quad h^3(2,2)....h^3(q,q)]^T$ is a column vector of size $(q+1)$;
$A=[0...0\quad -C_{4y}(q,0,0)\quad -C_{4y}(q,0,1)....-C_{4y}(q,0,q)]^T$ is a vector of size $(2q+1)$;
The least squares solution of the system of Eq. (37), will be written under the following form:

$$\widehat{\theta}=(M^TM)^{-1}M^Td \tag{38}$$

The parameters $h(i,i)$ for $i=1,...,q$ are estimated from the estimated values $\widehat{\theta}(i,i)$ using the following equation:

$$\widehat{h}(i,i)=\sqrt[3]{\widehat{\theta}(i+1,i+1)} \tag{39}$$

5. Simulation results and Analysis

In this section, we testy the proposed approach to identify the non linear impulse response parameters of the non linear channels. For this reason, we select the model (Eq. 40) characterizing a non linear quadratic system, with known parameters and then we try to recover these parameters using proposed algorithms.

$$\begin{cases} y(k)=x^2(k)+0.15x^2(k-1)-0.35x^2(k-2)+0.90x^2(k-3)\\ \text{zeros: } z_1{=}-1.1439;\ z_2{=}0.4969+0.7348i;\ z_3{=}0.4969-0.7348i.\end{cases} \tag{40}$$

The model of the selected channel is a non minimum phase because one of its zeros are outside of the unit circle (Fig. 2).

The simulation is performed with MATLAB software and for different signal to noise ratio (SNR) defined by the following relationship:

$$SNR=10log_{10}\Big[\frac{\sigma_z^2(k)}{\sigma_n^2(k)}\Big] \tag{41}$$

To measure the accuracy of the diagonal parameter estimation with respect to the real values, we define the Normalized Mean Square Error (NMSE) for each run as:

$$NMSE=\sum_{i=0}^{q}\Big[\frac{h(i,i)-\widehat{h}(i,i)}{h(i,i)}\Big]^2 \tag{42}$$

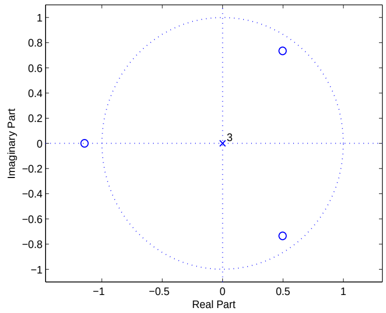

Figure 2. The zeros of Model

In order to test the robustness of the proposed algorithms we conseder a non minimum phase channel describe by (Eq. 40). The simulation results, using selected algorithms, of impulse response parameters estimation are shown in the Table 1 for different values of SNR and for N=2400.

From Table 1, we can conclude that for all values of SNR considered, the NMSE values of the

Table 1. Estimated parameters of the first channel for different SNR and excited by sample sizes $N = 2400$

SNR	$\widehat{h}(i,i)\pm std$	$ALGcum1$	$ALGcum2$	$ALGcum3$
0 dB	$\widehat{h}(1,1)\pm std$	0.1391 ± 0.0960	0.0770 ± 0.4375	0.1310 ± 0.3395
	$\widehat{h}(2,2)\pm std$	-0.4225 ± 0.0940	-0.5359 ± 0.1832	-0.5057 ± 0.1666
	$\widehat{h}(3,3)\pm std$	0.8326 ± 0.1120	0.8250 ± 0.1420	0.5371 ± 0.2283
	$NMSE$	0.0538	0.5261	0.3766
8 dB	$\widehat{h}(1,1)\pm std$	0.1539 ± 0.0622	0.1016 ± 0.3243	0.2059 ± 0.2406
	$\widehat{h}(2,2)\pm std$	-0.3747 ± 0.0527	-0.3376 ± 0.2095	-0.4090 ± 0.0708
	$\widehat{h}(3,3)\pm std$	0.7879 ± 0.0757	0.8153 ± 0.0984	0.6404 ± 0.1436
	$NMSE$	0.0212	0.1141	0.2507
16 dB	$\widehat{h}(1,1)\pm std$	0.1523 ± 0.0455	0.1366 ± 0.2959	0.1742 ± 0.2425
	$\widehat{h}(2,2)\pm std$	-0.3680 ± 0.0500	-0.2640 ± 0.2366	-0.3718 ± 0.0724
	$\widehat{h}(3,3)\pm std$	0.8155 ± 0.0589	0.8395 ± 0.0826	0.6524 ± 0.1158
	$NMSE$	0.0117	0.0729	0.1056
24 dB	$\widehat{h}(1,1)\pm std$	0.1583 ± 0.0475	0.1282 ± 0.2880	0.1522 ± 0.2609
	$\widehat{h}(2,2)\pm std$	-0.3611 ± 0.0547	-0.2983 ± 0.1876	-0.3662 ± 0.1072
	$\widehat{h}(3,3)\pm std$	0.8350 ± 0.0609	0.8234 ± 0.0812	0.6565 ± 0.1048
	$NMSE$	0.0093	0.0501	0.0755
32 dB	$\widehat{h}(1,1)\pm std$	0.1479 ± 0.0492	0.1664 ± 0.2536	0.1438 ± 0.2580
	$\widehat{h}(2,2)\pm std$	-0.3643 ± 0.0451	-0.2939 ± 0.2011	-0.3722 ± 0.1007
	$\widehat{h}(3,3)\pm std$	0.8419 ± 0.0643	0.8298 ± 0.0930	0.6723 ± 0.0955
	$NMSE$	0.0060	0.0437	0.0697
	True parameters $h(i,i)$	$h(1,1) = 0.150$	$h(2,2) = -0.350$	$h(3,3) = 0.900$

first proposed method such as (Algcum1) are lower than the other methods (Algcum2, Algcum3), this is due to the complexity of the systems of equations for each algorithm, non linear of the parameters in the (Algcum2, Algcum3) algorithms. The performance of the (Algcum2) method degrade than the (Algcum3) in very noise environment (SNR=0 dB), but it becomes more effective than (Algcum3) when the noise variance is relatively small, this is due the fact that the higher order cumulants for a Gaussian noise are not identically zero, but they have values close to zero for higher data length. This is very clear in the (Fig. 3).

In the part, of complexity of these algorithms the first proposed algorithm exploiting $(q+1)$

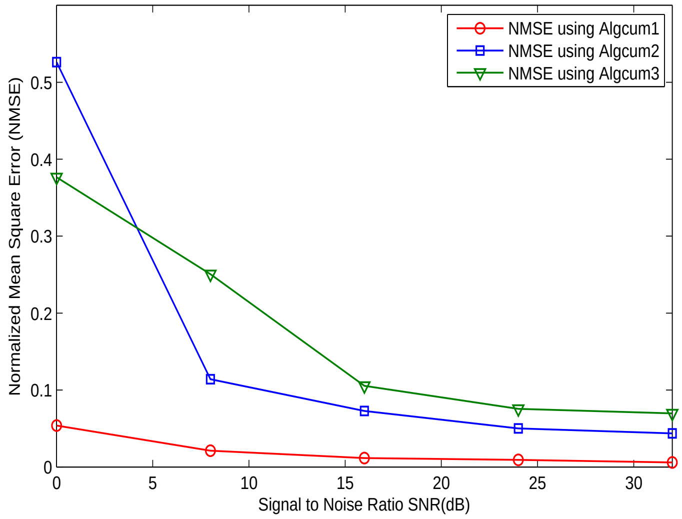

Figure 3. NMSE for each algorithm and for different SNR and for a data length N = 2400

equations, comparing de second and third proposed methods exploiting $(2q+1)$ for identify the impulse response parameters channel.

In the Fig. 4 we have presented the estimation of the magnitude and the phase of the impulse response using the proposed algorithms, for data length N = 2400 and very noise environment SNR =0 dB. From the Fig. 4 we remark that the magnitude estimation have the same appearance using two first proposed methods but using (Algcum3) algorithm we have a minor difference between the estimated and true ones. Concerning the phase estimation, we have same allure comparatively to the real model using all proposed algorithms.

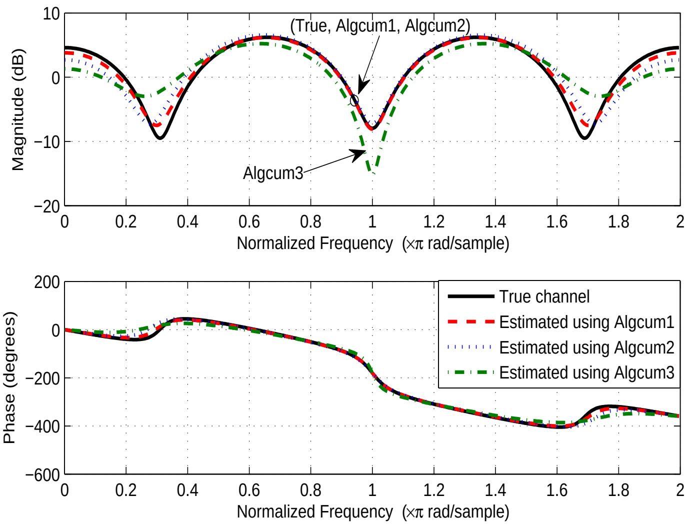

Figure 4. Estimated magnitude and phase of the non linear model channel impulse response when the data input is N=2400 and an SNR=0 dB

6. Conclusion

In this contribution, we have considered the problem of blind identification of non linear channel using selected order cumulants. We have developed a theoretical analysis for non linear quadratic systems, these tools will serve us later for proposing an extension of linear algorithms to non linear. However, we have developed three approaches based on the three and fourth order cumulants, respectively, for blind identification of diagonal parameters of quadratic systems. From simulation results and comparison between these methods one can see that the first proposed algorithm (Algcum1) can always achieve better performance than other (Algcum1, Algcum2), and is adequate for estimating diagonal quadratic systems.

The future work of this paper is the non linear Broadband Radio Access Network (BRAN) channels identification and equalization especially MC-CDMA systems using the presented methods.

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