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Direct Radio Frequency Sampling System on Software-defined Radio

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Abstract

A traditional assumption underlying most data converters is that the signal should be sampled at a rate exceeding twice the highest frequency. In this paper, we employ a method for low-rate sampling of multi- band signals via applying periodic nonuniform sampling in shift-invariant spaces generated by m kernels with period T. So, the sampling and reconstruction of signals were transformed into matrix and vector operations, the generalized inverse can be use to find the answer and an interpolator is used to insure that complete reconstruction will be achieved. Finally, we validate the method in MATLAB; the conclusion of simulation shows the frame-work presented here is feasible.

Keywords: *generalized inverse reconstruction, periodic nonuniform sampling, shift-invariant spaces, multi- band signals, interpolator*

1. Introduction

One goal in designing a software defined radio (SDR) receiver is to move the analog-to-digital converter (ADC) as close as possible to the antenna [1]. To achieve this goal, one can manage to use a wideband high-speed ADC to convert the RF signals to digital signals. With the development of wireless technology, this enables the modulation of narrow-band signals by high carrier frequencies. To demodulate the desired signals, the required sampling rate for the ADC could often be too high to be attained if the Nyquist sampling theorem is to be satisfied [2]. The uniform bandpass sampling method has been proposed to figure out the problem [3-5], and this is a promising way for multi-band radio communication. The uniform bandpass sampling is the intentional aliasing of the information bandwidth of the signal [6, 7]. The sampling frequency requirement is no longer based on the frequency of the RF carrier, but rather on the information bandwidth of the signal. Thus, the resulting processing rate can be significantly reduced. However, the uniform sampling still suffers from many constraints such problem of timing jitter in A/D conversion process [8]. For nonuniform samples, there are both iterative methods and noniterative methods to recreate the signals; these methods presuppose exact knowledge of the sample locations. This is not always the case, and there may occur situations where the location data is unavailable or partially available [9].

A signal class that plays an important role in sampling theory is signals in shift-invariant (SI) spaces [10]. A sample in shift-invariant spaces was proposed to overcome these problems. The reconstruction of sampled signals is achieved by forming linear combinations of a set of reconstruction function that span a subspace; such functions can be expressed as linear combinations of shifts of a set of generators with period T. This model encompasses many signals used in communication and signal processing. Any signal $x(t)$ in a SI space generated by m functions shifted with period T can be perfectly recovered from m sampling sequences, obtained by filtering $x(t)$ with a bank of m filters and uniformly sampling their outputs at times nT.

This paper is organized as followed. Section II sets up the sampling model. In Section III, we use generalized inverse to recover sampled signals. In section IV, we analyze the reconstruction error. Finally, section V shows simulation results.

2. Proposed Scheme

The architecture of parallel sampling system is shown in Figure 1.

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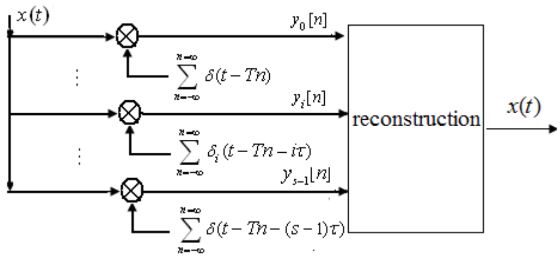

Figure 1. The Mode of the Periodic Nonuniform Sampling

The nonuniform sampling process converts a continuous analogue signal $x(t) \in L_2$-space into its discrete representation, the architecture of periodic nonuniform sampling system is shown in Figure 1.

Let $a_i(t)$ as one of s nonuniform sample sequences,

$$a_i(t) = \sum_{n=-\infty}^{+\infty} \delta(t - Tn - i\tau)(0 \le i \le s-1) \tag{1}$$

Where, T is the sampling period, τ is sequence separation.

One of s sampled functions,

$$y_i(t) = x(nT + i\tau)\sum_{n=-\infty}^{n=\infty} \delta(t - nT - i\tau) \tag{2}$$

Where, $0 \le i \le s-1$.

And the corresponding spectra is given by:

$$Y_i(\omega) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} X(\omega - 2\pi n / T)e^{-j2\pi ni\tau/T} \tag{3}$$

In order to reconstruct $x(t)$ from these samples $y[n]$($y[n]$=[$y_0[n]$, $y_1[n]$,…,$y_{s-1}[n]$]), it is assumed that $x(t)$ lies in a subspace $V(\varphi)$of L_2. In this paper, we define that the $V(\varphi)$ are generated by m space functions $\varphi(t)$.

$$V(\varphi) = \{\sum_{p=0}^{m-1}\sum_{n\in z} r_p[n]\varphi_p(t - nT) : r_p[n] \in L_2\}$$

We can represent any $x(t) \in V(\varphi)$ as follow:

$$x(t) = \sum_{p=0}^{m-1}\sum_{n\in z} r_p[n]\varphi_p(t - nT) \tag{4}$$

The only restriction on the choice of the function train{$\varphi_p(t)$} is for guaranteeing a unique stable representation of any signal in $V(\varphi)$ by sequence {$r_p[n]$}, so the generators $\varphi(t)$ must form a Riesz basis of L_2. In other words, there exist two constants $\alpha > 0$ and $\beta < \infty$, such that:

$$\alpha\left\|\mathrm{r}[n]\right\|_2^2 \le \left\|\sum_{p=0}^{m-1}\sum_{n\in z} r_p[n]\varphi_p(t - nT)\right\|_2^2 \le \beta\left\|\mathrm{r}[n]\right\|_2^2 \tag{5}$$

Where, $\left\| \mathrm{r}[n] \right\|_2^2 = \sum_{p=0}^{m-1} \sum_{n \in z} \left| r_p[n] \right|^2$, $\left\| \bullet \right\|_2$ is L_2 norm.

Proposition: if and only if $\alpha I \le W(\omega) \le \beta I$, the generator $\varphi_p(t-nT)$ form a Riesz basis. Where, *I* is the identity matrix:

$$W(\omega) = \begin{bmatrix} w_{1,1} & \cdots & w_{1,m} \\ \vdots & \ddots & \vdots \\ w_{m,1} & \cdots & w_{m,m} \end{bmatrix}$$

$$w_{a,b} = \frac{1}{T} \sum_{n \in z} \psi_a^{\ *}(\omega - 2\pi n / T) \psi_b(\omega - 2\pi n / T)$$

Here, $\psi(\omega)$ is the Fourier transform of $\varphi(t)$

Proof: (5) can be rewritten as follow:

$$\alpha \left\| \mathrm{r}[n] \right\|_2^2 \le \int_{-\infty}^{\infty} \left| \sum_{n \in z} \mathrm{r}^*[n] \varphi(t - nT) \right|^2 dt \le \beta \left\| \mathrm{r}[n] \right\|_2^2 \qquad (6)$$

From the theory of Parseval:

$$\int_{-\infty}^{\infty} \left| \sum_{n \in z} \mathrm{r}^*[n] \varphi(t - nT) \right|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \sum_{n \in z} \mathrm{R}^*(\omega) \psi(\omega) \right|^2 dt \qquad (7)$$

Where, $\mathrm{R}(\omega)$ is the discrete-time Fourier transform of $\mathrm{r}[n]$, and $\mathrm{R}(\omega)$ is 2π-periodic. Then (7) can be rewritten as:

$$\begin{aligned} &\frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \sum_{n \in z} \mathrm{R}^*(\omega) \psi(\omega - 2\pi n / T) \right|^2 dt \\ &= \frac{1}{2\pi} \sum_{n \in z} \int_0^{2\pi} \mathrm{R}^*(\omega) \psi(\omega - 2\pi n / T) \psi^*(\omega - 2\pi n / T) \mathrm{R}(\omega) d\omega \\ &= \frac{T}{2\pi} \int_0^{2\pi} \mathrm{R}^*(\omega) W(\omega) \mathrm{R}(\omega) d\omega \end{aligned} \qquad (8)$$

We can have (9) from Parseval:

$$\left\| \mathrm{r}[n] \right\|_2^2 = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{R}^*(\omega) \mathrm{R}(\omega) d\omega \qquad (9)$$

It is easy to know $W(\omega)$ is a positive self-adjoint which has real nonnegative eigenvalues. Let α is the minimal eigenvalues and β is the maximal eigenvalues. We can have:

$$AI\mathrm{R}^*(\omega)\mathrm{R}(\omega) \le \mathrm{R}^*(\omega)\mathrm{R}(\omega)W(\omega) \le BI\mathrm{R}^*(\omega)\mathrm{R}(\omega)$$

The conclusion can be obtained that $\varphi_p(t-nT)$ form a Riesz basis if and only if $\alpha I \le W(\omega) \le \beta I$.

The above-mentioned subspace $V(\varphi)$ is a single space, the more interesting aspect we are considering is that $x(t)$ lies in a union of subspaces $\bigcup V_p(\varphi)$ ($0 \le p \le m-1$).

$$x(t) \in \bigcup V_p(\varphi)$$

In Fourier domain, (4) can be represented as follow:

$$X(\omega) = \sum_{p=0}^{m-1} R_p(\omega)\psi_p(\omega) \tag{10}$$

Where, $R_P(\omega)$ is the discrete-time Fourier transform of $r_p[n]$, $\psi_p(\omega)$ is the Fourier transform of $\varphi_p(t)$.

We can obtain the DTFT of the i-th channel samples $y_i[n]$ by (3) and (5):

$$\begin{aligned} Y_i(\omega) &= \frac{1}{T}\sum_{n=-\infty}^{+\infty}\sum_{p=0}^{m-1} R_p(\omega - 2\pi n/T)\psi_p(\omega - 2\pi n/T)e^{-j2\pi ni\tau/T} \\ &= \frac{1}{T}\sum_{p=0}^{m-1} R_p(\omega)\sum_{n=-\infty}^{+\infty}\psi_p(\omega - 2\pi n/T)e^{-j2\pi ni\tau/T} \end{aligned} \tag{11}$$

Where, the fact that the $R_p(\omega)$ is 2π -periodic.

An appropriate matrix represent of (11) is given by:\

$$Y(\omega) = H(\omega)R(\omega) \tag{12}$$

Where, $Y(\omega) = (Y_0(\omega), Y_1(\omega), \cdots Y_{s-1}(\omega))'$

$$R(\omega) = (R_0(\omega), R_1(\omega), \cdots R_{m-1}(\omega))'$$

$$H(\omega) = \begin{bmatrix} h_{0,0} & h_{0,1} & \cdots & h_{0,m-1} \\ \vdots & \vdots & \ddots & \vdots \\ h_{s-1,0} & h_{s-1,1} & \cdots & h_{s-1,m-1} \end{bmatrix}$$

$$h_{i,p}(\omega) = \frac{1}{T}\sum_{n=-\infty}^{+\infty}\psi_p(\omega - 2\pi n/T)e^{-j2\pi ni\tau/T}$$

Our aim is to obtain values of $R(\omega)$. The method of reconstruction is to solve Equation (10).

3. Reconstruction Mode

The approach in this paper is to recovery sampled signals in two steps. First, we use the generalized inverse $H^-(\omega)$ to find $r_i[n]$ ($0 \le i \le m-1$); second, an interpolator is employed to achieve the complete reconstruction of sampled signal. The fundamental stages for the recovery of sampled signals are shown in Figure 2.

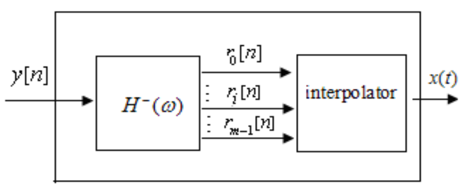

Figure 2. The Block of Reconstruction Bank

We define the function as follow:

$$J = \sum_{i=0}^{s-1} \sum_{n \in Z} \left\| y_i[n] - \hat{y}_i[n] \right\|^2 \quad (13)$$

Where, $\hat{y}_i[n]$ is coefficient that is obtained via sampling the reconstructed continuous time signal.

Again by Parseval we have:

$$\begin{aligned} J &= \sum_{i=0}^{s-1} \int_{-\pi}^{\pi} \left\| Y_i(\omega) - \hat{Y}_i(\omega) \right\|^2 d\omega \\ &= \int_{-\pi}^{\pi} (Y(\omega) - \hat{Y}(\omega))(Y(\omega) - \hat{Y}(\omega))^H d\omega \end{aligned} \quad (14)$$

Where, $Y_i(\omega)$ and $\hat{Y}_i(\omega)$ is the DTFT of $y_i[n]$ and $\hat{y}_i[n]$ respectively. $(\ )^H$ denotes the Hermitian conjugate.

$$Y(\omega) = (Y_0(\omega), Y_1(\omega), \cdots, Y_{s-1}(\omega))' \ \hat{Y}(\omega) = (\hat{Y}_0(\omega), \hat{Y}_1(\omega), \cdots, \hat{Y}_{s-1}(\omega))' .$$

We have：

$$\begin{aligned} \hat{Y}_i(\omega) &= \sum_{k=0}^{m-1} \sum_{j=0}^{s-1} Y_j(\omega) H^{-}{}_{p,j}(\omega) \\ &\bullet \sum_{n \in Z} \psi_p(\omega - 2\pi n / T) e^{-j2\pi n i \tau / T} \end{aligned} \quad (15)$$

Where, $H^{-}{}_{p,j}(\omega)$ is the *pj*th element of matrix $H^{-}(\omega)$.

A matrix represent of (8) is given by:

$$\hat{Y}(\omega) = Q(\omega) H^{-}(\omega) Y(\omega) \quad (16)$$

Where, $Q_{i,p}(\omega) = \sum_{n=-\infty}^{+\infty} \psi_p(\omega - 2\pi n / T) e^{-j2\pi n i \tau / T}$

Substitute (9) into (7), we have:

$$\begin{aligned} J &= \int_{-\pi}^{\pi} (Y(\omega) - Q(\omega) H^{-}(\omega) Y(\omega)) \\ &\bullet (Y(\omega) - Q(\omega) H^{-}(\omega) Y(\omega))^H d\omega \end{aligned} \quad (17)$$

When the value of the equation (17) is minimum, the generalized inverse can be attained by:

$$H^{-}(\omega) = Q^H(\omega) / (Q(\omega) Q^H(\omega)) \quad (18)$$

As soon as the *r*[*n*] is obtained, we can have the recovered *x*(*t*) through an interpolator.

T_N is defined as the oversampling periodic that satisfy $T_N = T / M$, we can rewrite (3) as follow:

$$x[nMT_N] = \sum_{p=0}^{m-1}\sum_{c\in z} r_p[c]\varphi_p(nT_N - cT) \tag{19}$$

Upsampling the sequence ($x[nT_N]$:$n\in Z$) by factor of M, the dth sub-sequence is given by:

$$x[nMT_N + dT_N] = \sum_{p=0}^{m-1}\sum_{c\in z} r_p[c]\varphi_p(nMT_N + dT_N - cT) \tag{20}$$

The DTFT of (14) is:

$$X_d(\omega) = \sum_{p=0}^{m-1} R_p(\omega)\psi_{p,d}(\omega) \tag{21}$$

Finally, we can have the reconstructed signals in Fourier domain:

$$\begin{aligned} x(\omega) &= \sum_{d=0}^{M-1} e^{j\omega d} x_d(M\omega) \\ &= \sum_{d=0}^{M-1} e^{j\omega d} \sum_{p=0}^{m-1} R_p(M\omega)\psi_{p,d}(M\omega) \\ &= \sum_{p=0}^{m-1} R_p(M\omega) \sum_{d=0}^{M-1} e^{j\omega d}\psi_{p,d}(M\omega) \end{aligned} \tag{22}$$

4. Error analysis

We will define an angle between two closed subspaces A and B of a Hilbert space V[11]:

$$\cos(A,B) = \inf_{f\in A,\|f\|=1} \|P_B f\| \tag{23}$$

$$\sin(A,B) = \sup_{f\in A,\|f\|=1} \|P_{B^\perp} f\| \tag{24}$$

When the reconstructed signal $\hat{x}(t)\in V$, we can conclude the sampling error $e(x(t))$ as follow:

$$\begin{aligned} \|e(x(t))\|^2 &= \|x(t) - \hat{x}(t)\|^2 \\ &= \|P_V x(t) - \hat{x}(t)\|^2 + \|P_{V^\perp} x(t)\|^2 \\ &\geq \|P_{V^\perp} x(t)\|^2 \end{aligned} \tag{25}$$

From (25), we can have:

$$P_{V^\perp} x(t) = P_{V^\perp} e(x(t)) \tag{26}$$

When $x(t)\in V\oplus W^\perp$ (W is the sampling space), we can have the Equation (27):

$$e(x(t)) = x(t) - \hat{x}(t) = E_{VW^\perp}(x(t)) \tag{27}$$

Where, $E_{VW^{\perp}}(x(t))$ is the oblique projection onto V along $W^{\perp}$.

Further, the infimum and the Supremum of sampling error can be given by Equation (28).

$$\left\|P_{V^{\perp}}(x(t))\right\|^2 \Big/ \left\|\sin(V,W^{\perp})\right\|^2 \leq \left\|e(x(t))\right\|^2 \leq \left\|P_{V^{\perp}}(x(t))\right\|^2 \Big/ \left\|\cos(V,W^{\perp})\right\|^2 \quad (28)$$

4. Simulation

In the section, we will validate the reconstruction algorithm in MATLAB. We design a sampling system that the sampling channels are *s*=2. The corresponding nonuniform sample sequences in Figure 1 are $a_0(t)=\delta(t-nT)$ and $a_1(t)=\delta(t-nT-\tau)$, we define $\tau = T/3$ that is the sequence separation between two interleaved uniform sample sequences. The generate functions $\varphi_0(t)$ and $\varphi_1(t)$ are given as follow:

$$\varphi_0(t) = \mathrm{sinc}(\frac{t}{T})e^{j2\pi\frac{2}{3T}t} \quad (29)$$

$$\varphi_1(t) = \mathrm{sinc}(\frac{t}{T})e^{-j2\pi\frac{2}{3T}t} \quad (30)$$

Where, T is the sampling period.

We suppose that the input multi-band signal:

$$x(t) = \sin(\frac{7}{3}\pi \times 10^8 t) + \sin(\pi \times 10^8 t) \quad (31)$$

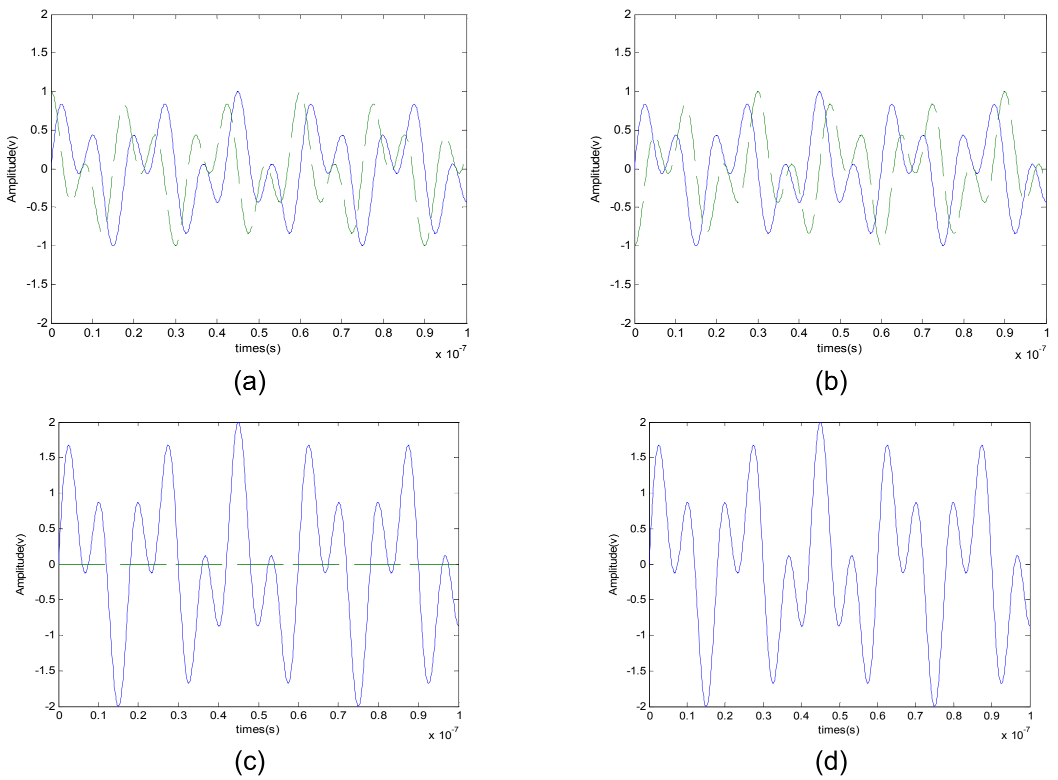

Figure 3. The Sampling System Simulation

5. Conclusion

In this paper, we use a general framework to treat sampling of multi-band signal. Our interest is that focused on how to reconstruct signal completely. The approach we chosen are that project the signal over basis functions and then sample the basis coefficients. The latter focuses on using generalized inverse to obtain $r_i[n]$ ($0 \le i \le m-1$). We showed that by using a interpolator to gain the complete multi-band signal $x(t)$ from $r[n]$. Finally, the simulation proved the method we proposed is feasible.

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References

[1] CH Tseng, SC Chou. *Direct downconversion of multiple RF signals using bandpass sampling*. Proc. ICC. 2003; 3: 2003–2007.
[2] DM Akos, M Stockmaster, JBY Tsui, J Caschera. Direct bandpass sampling of multiple distinct RF signals. *IEEE Trans. Commun.*, 1999; 47(7): 983–988.
[3] RG Vaughan, NL Scott, DR White. The theory of bandpass sampling. *IEEE Trans. Signal Process.*, 1991; 39(9): 1973–1983.
[4] Wong, TS Ng. An efficient algorithm for down-converting multiple bandpass signals using bandpass sampling. *Proc. ICC*, Jun. 2001; 3: 910–914.
[5] DM Akos, M Stockmaster, JBY Tsui, J Caschera. Direct bandpass sampling of multiple distinct RF signals. *IEEE Trans. Commun.*, 1999; 47(7): 983–988.
[6] AJ Coulson. A generalization of nonuniform bandpass sampling. *IEEE Trans. Signal Processing*. 1995; 43: 694–704.
[7] YP Lin, PP Vaidyanathan. Periodically nonuniform sampling of bandpass signals. *IEEE Trans. Circuits Syst. II*. 1998; 45; 340–351.
[8] Manel Ben-Romdhane, Chiheb Rebai, Adel Ghazel. Non-Uniform Sampling Schemes for IF Sampling Radio Receiver. SETIT 2009 5th International Conference: Sciences of Electronic, Technologies of Information and Telecommunications. 2009.
[9] DM Bechir BR. Analysis of timing jitter and dither effects on A/D converter for software radio systems. IEEE-ISIVC, Bilbao, Espagne. 2008.
[10] YC Eldar. Sampling and reconstruction in arbitrary spaces and oblique dual frame vectors. *J. Fourier Analys. Appl.*, 2003; 1(9): 77–96.
[11] WS Tang. Oblique projections, biorthogonal Riesz bases andmultiwavelets in Hilbert space. Proceedings of the American Mathematical Society. 2000; 128(2): 463-473.
[12] M Mishali, YC Eldar. Blind multi-band signal reconstruction: Compressed sensing for analog signals. *IEEE Trans. Signal Process.*, 2009; 57: 993–1009.
[13] MICHAEL UNSER, Sampling 50 Years after Shannon. *Proceedings of the IEEE*. 2000; 88(4): 569-588.
[14] O Christensen, YC Eldar. Generalized shift-invariant systems and frames for subspaces. *J. Fourier Analys. Appl.*, 2005; 11: 299–313.
[15] YM Lu, MN Do. A theory for sampling signals from a union of subspaces. *IEEE Trans. Signal Processing*. 2008; 56(6): 2334–2345.