

## Design of Adaptive Filter for Laser Gyro

Jinming Li<sup>1,2</sup>, Zeming Li<sup>\*1,2</sup>, Xiaojun Yan<sup>1,2</sup>, Yanjiao Yang<sup>1,2</sup>, Chengrui Zhai<sup>1,2</sup>

<sup>1</sup>National Key Laboratory for Electronic Measurement Technology, North University of China, Taiyuan 030051, P. R. China, Ph./Fax:+86-0351-3559258

<sup>2</sup>Key Laboratory of Instrumentation Science & Dynamic Measurement of Ministry of Education, North University of China, Taiyuan 030051, P. R. China, Ph./Fax:+86-0351-3559258

\*Corresponding author, e-mail: lizeming565@163.com

### Abstract

According to the filtering of laser gyro output signal of low precision, slow speed of dynamic response, the research and implementation of a new method of laser gyro filtering process, the scheme using LMS adaptive filtering algorithm, the dither feedback signal as the iterative filter data input, the dither signal, random noise, white noise as the reference signal filter, digital filter and the external control using FPGA, give the filter algorithm and hardware block diagram. Experimental results show that the filtering module with filter with high precision and wide dynamic response range, can meet the requirements of speed and precision of laser gyro demodulation aerospace fields.

**Keywords:** laser gyro, adaptive filter, FPGA

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### 1. Introduction

Ring laser gyro (ring laser gyro, gyroscope) with high sensitivity and precision, which is made of Sagnac effect based on optical ring road. At present it has been widely used in the strap down inertial navigation system [1]. Because the laser gyro lock-in threshold exists, when the external input speed less than lock-in threshold, laser gyro is unable to sense the external angular rate, will produce the lock-in effect, which greatly limits its application scope. In order to make the gyro has been working in the lock region, people usually use mechanical Dither. Commonly used approach is the introduction of periodic sinusoidal vibration [2]. The mechanical dither is provided by resonance of piezoelectric ceramics. In practical applications, due to small changes in the characteristics of piezoelectric ceramic instability and gyro its mechanical structure, will cause the gyro vibration period and amplitude is not a constant value. Then the gyro output data jitter stripping has certain difficulty, using conventional cycle offset conventional and notch method can not accurately stripping, which affect the accuracy of the gyroscope.

Based on the analysis on the principle of adaptive filtering algorithm, the article proposes to achieve the laser gyro output signal of high speed, high precision demodulation scheme using LMS adaptive filtering. The method can satisfy the high precision demodulation, but also has good practicability and engineering practical value.

### 2. Analysis of LMS Auto - Adapted Filter Algorithm

LMS adaptive filtering algorithm makes the smallest mean square deviation of error between the filter output and the expected output of the minimum, so it is called the least mean square (LMS) auto - adapted filter algorithm [3]. Its basic structure is as follows:

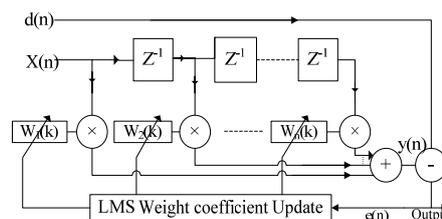


Figure 1. The Basic Model of LMS Auto-Adapted Filter

Among them,  $e(n) = d(n) - y(n)$ ,  $d(n)$  is the reference input,  $X(n)$  is the filter input. In practical applications, due to the minimum mean square are hard to calculate, we use the square of the error signal, estimated as the mean square error value [4]. In order to realize the attenuation of certain frequency reference signal,  $D(n)$ ,  $X(n)$  shall have the following mathematical relationship:

$$\begin{cases} d(n) = d1(n) + d2(n) \\ X(n) = x1(n) \end{cases} \quad (1)$$

The  $d1(n)$  and  $d2(n)$  has no correlation;  $x1(n)$  and  $d1(n)$  are most correlation, but they and  $d2(n)$  has no correlation. When the filter is stable, the components of  $d1(n)$  output in  $e(n)$  can be greatly attenuated, thereby realizing filtering.

The core of LMS adaptive filtering algorithm is to update formula weight coefficient, and what we are using is Widrow and Hoff LMS algorithm. The algorithm uses the most rapid descent method in the optimization methods and its final weight coefficient updating formula is as follows:

$$W(k) = W(k-1) + 2\mu e(k)x(k) \quad (2)$$

Among them, the  $w(k)$  represents the filter weight coefficient vector,  $x(k)$  represents the filter input data vector,  $\mu$  is the convergence factor. Only when  $\mu$  satisfies a certain range, the filter will tend to be stable after multiple iterations. To make the weight coefficient vector of the mathematical expectation can converge to the Wiener solution,  $\mu$  ranges should satisfy the following formula:

$$0 < \mu < \frac{1}{\lambda_{\max}} \quad (3)$$

$\lambda_m$  represents the maximum eigenvalue of the input data matrix. The size of the  $\mu$  is directly related to the iteration times that the filter reach a stable time. The smaller the  $T$ , iteration number is larger, but the expected output is close to the true value, and the error is smaller. So the value of  $\mu$  is often the most important factors affecting the performance of the filter. In practical application, we use the MATLAB software to calculate the appropriate value as the optimal convergence factor [5].

### 3. The Design of Adaptive Filter

#### 3.1. The Selection of the Reference Signal for the Auto - Adapted Filter

According to the principle of LMS filter, we must first select the appropriate reference input, filter input, and between them must meet certain mathematical correlation. In order to make the laser gyro work as much as possible to avoid lock region, we introduce the periodic sinusoidal vibration moreover white noise with a certain level is superimposed on the mechanical dither [6]. Then the output data of laser gyro component of three parts, including mechanical vibration, noise, the external input. They are related by the following equation:

$$N = A \sin(\omega t + \varphi) + B\Omega(t) + C\varepsilon(t) \quad (4)$$

The  $\sin(\omega t + \varphi)$  represents the mechanical dithering rate,  $\Omega(t)$  represents the input angular rate,  $\varepsilon(t)$  represents the random noise signal. We sampled the output values of laser gyro with uniform time sampling, and get down:

$$\begin{aligned} N_i &= A \sin(\omega t_i + \varphi) - A \sin(\omega t_{i-1} + \varphi) \\ &\quad + B\Omega(t)\Delta t + C\varepsilon(t)\Delta t \end{aligned} \quad (5)$$

We generally use 10KHZ as the sampling frequency, assuming that the sampling period to  $\Delta t$ , we can get the following formula using the differential principle:

$$A\sin(\omega t_i + \varphi) - A\sin(\omega t_{i-1} + \varphi) \approx A\omega \cos(\omega t_i + \varphi)\Delta t \quad (6)$$

So the Type 1.4 can be rewritten as:

$$\begin{aligned} N_i &= K\cos(\omega t_i + \varphi) + (B\Omega(t_i) + C\varepsilon(t_i))\Delta t \\ K &= A\omega\Delta t \end{aligned} \quad (7)$$

A general random noise is more than 1KHz, the external input signal is below 100Hz, the gyro dither frequency is about 300Hz, do not have the correlation between the three, and the magnitude relationship is  $K \gg C > B$ . To obtain the input angular rate, the first, the three in the type must be stripped [7]. So we will refer to  $N_i$  as the input signal of adaptive filter.

### 3.2. The Selection of the Filter Input Signal

To stripping part of the signal from the reference signal without introducing errors from the reference signal, the input signal and the dither signal, random noise signal must be relevant, and external input angular velocity is not relevant. While the dither feedback signal reflects the vibration of piezoelectric ceramics, which contains only the dither and noise of two components, its value and the external input angular rate, which can be expressed as:

$$X(t) = D\sin(\omega t + \beta) + F\varepsilon(t) \quad (8)$$

The dither feedback signal is an analog voltage signal, so we need to sample the dither feedback signal using high precision A/D. Sampling value multiplied by an appropriate coefficient, then the result can be used as the input to the filter iteration.

Based on the above analysis, we will refer to type 2.4 as a reference signal, type 2.5 as the iterative filter input signal, expressed as follows:

$$\begin{cases} N_i = K\cos(\omega t_i + \varphi) + (B\Omega(t_i) + C\varepsilon(t_i))\Delta t \\ Xi = D\sin(\omega t_i + \beta) + F\varepsilon(t_i) \end{cases} \quad (9)$$

### 3.3. The Selection of the Filter Parameters

Normalization algorithm is adopted in the LMS algorithm and implemented in MATLAB. The selection of  $\mu$  and the reference input values are not linked, so we simplify the input model, LMS filter input are as follows:

$$\begin{cases} t_i = 0 : 1999 \\ N_i = 200\cos(2 * \pi * t_i / 33 + 2) \\ Xi = 50\sin(2 * \pi * t_i / 33 + 0.5) \end{cases} \quad (10)$$

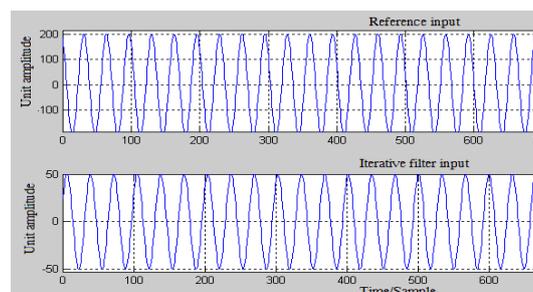


Figure 2. Filter Input Waveform in MATLAB

Iterative filter order is set to 7 bands, at different  $\mu$  values, the simulation results are as follows:

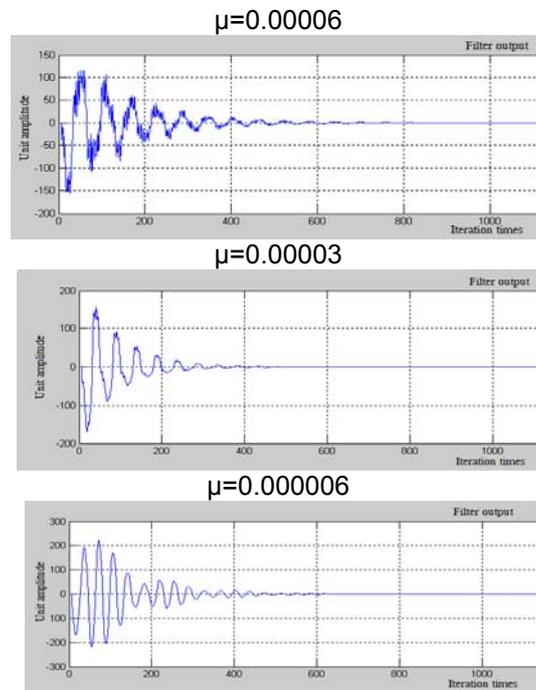


Figure 3. The Iteration Result Value with Different  $\mu$

As can be seen from the figure, the value of  $\mu$  affect the iterative step and iteration accuracy when the filter to stabilize reach steady. After analysis, we can select  $\mu=0.000006$ , this filter has higher accuracy, and at the same time, stable time is within the acceptable range.

### 3.4. Filter Simulation and Optimization

Utilization of the  $\mu$ , we modify the parameters of the input model to simulate the process of laser gyro signal. In MATLAB, the filter input model is as follows:

$$\begin{cases} t_i = 0:1999 \\ N_i = 200\cos(2 * \pi * t_i / 33 + 2) + 13.2\sin((2 * \pi / 500) * t) \\ X_i = 50\sin(2 * \pi * t_i / 33 + 0.5) \end{cases} \quad (11)$$

$N$  represents the external dynamic input frequency of gyro, typically 20 Hz. Filter output signals is as follows:

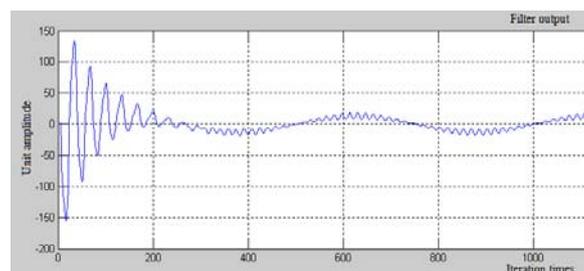


Figure 4. The Simulation Results of Basic LMS Filter Model of Signal Processing of Laser Gyro

What can be seen clearly in the figure is that filtering results has great volatility. We take the second in reference signal (outside input angular velocity) and 800 - 1000 sampling points of the filtering results, to obtain the variance:

Table 1. Comparison of Theoretical Value and Filtering Value

Signal	Variance	Sum values
Angular velocity input	13.13	-1911.711
The filtering result	19.43	-1952.031

As can be seen from the table the filtering result variance is significantly larger, hence the filtering results have greater volatility, and introduce a large error. According to the theory of adaptive filter. The final result of filter iteration filter output  $e(k)$  the minimum mean square error, and  $e(k) = d(k) - y(k)$ , so the final purpose filter iteration is the reference signal  $d(k)$  and output  $y(k)$  portion of the amplitude and phase consistency between relevance [8]. So that this output value  $e(k)$  included just a part, which is not associated with iterative filter input signal. While the basic LMS iterative formula (1.2) are directly to the  $E(k)$  into account, so we need to optimize the filter in the feedback link.

Here the reference signal of laser gyro input LMS adaptive filter, iterative filter input signal is simplified as type:

$$\begin{cases} d_n = d_1 + d_{in} \\ X_n = d_1' \end{cases} \tag{12}$$

The  $d_1$ ,  $d_{in}$  respectively represent the dithering and external input component of count signal, A/D sampling dither component values of  $d_1'$  feedback signal. Frequency range of  $d_1$ ,  $d_{in}$ ,  $d_1'$  is as shown in the following table:

Table 2. Dither Signal Frequency Components

Signal	F(Hz)
$d_1$	250-450
$d_{in}$	0-20
$d_1'$	250-450

The expression of  $e(k)$  can be expressed as follows:

$$e(k) = k_1 d_1 - k_2 d_1' + d_{in} \tag{13}$$

The two frequency has obvious segmentation, in the range of 0-20Hz, 250-450Hz. We can divided difference filter for removal of  $e(k)$  in the  $d_{in}$  signal, filtering itself has certain error, while there is relationship between error and ratio  $L$  of the signal frequency to the sampling frequency.  $L$  is small, the filtering effect is better, the higher the accuracy. Because  $f_{din}/f_{sample} \approx 10/10000$ , so in the sampling period  $S$  the value of  $d_{in}$  is seen as remaining unchanged, and  $e_1(n) = e(k_n) - e(k_{n-s}) \approx k_1 d_1 - k_2 d_1'$ . So just select the appropriate  $S$  value, reliable differential filtering can be achieved. LMS adaptive filter frame after optimization are as follows:

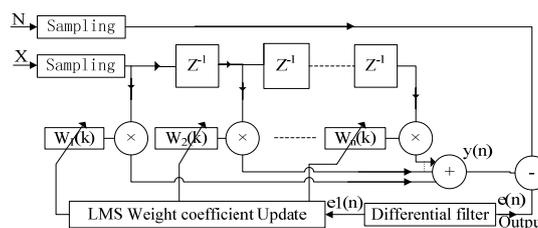


Figure 5. Optimized LMS Adaptive Filter

By choosing different  $S$  for testing, we select  $S=4$  as a differential difference value between two adjacent filter difference in the period. Then the filter output simulation results are shown in Figure 6:

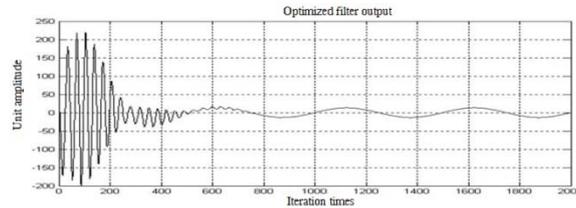


Figure 6. Optimized Filter Simulation Results

Thus, the optimization model of LMS adaptive filter, and two input selection analysis ended, completes the theory analysis and modeling of LMS adaptive filter of laser gyro.

#### 4. Circuit Implementation of the Adaptive Filter

##### 4.1. Hardware Design

According to the above principle, we realize the adaptive filtering algorithm and system control by using FPGA as the processing core, Structure diagram as follows:

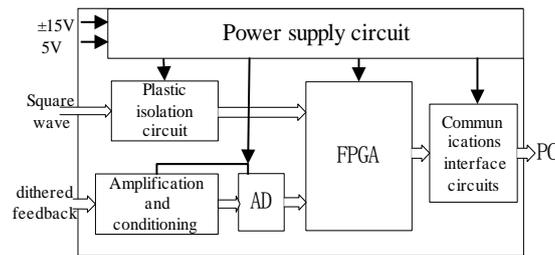


Figure 7. Adaptive Filtering Module Circuit Diagram

Five parts are included in this circuit: The power supply circuit, Plastic isolation circuit, The A/D conversion circuit, The FPGA module, communication interface circuit. square wave enter the adaptive filtering module, complete the signal shaping and photoelectric isolation first, and then the signal into the FPGA, as the input reference signal for the adaptive filter in counting sampling.

Dither feedback signal through the high precision A/D conversion, and then enter the FPGA, within the FPGA, we put the filter sampling frequency as the sampling period to sample the analog to digital conversion value, finally, multiplied by a certain coefficient as the input of iterative filter LMS adaptive filter.

##### 4.2. Software design of adaptive filter

Filter module of the FPGA program include: Phase demodulation module, An adaptive filter module, FIR filter module, The A/D acquisition module, Communication interface module. The organization of software system is illustrated on Figure 8.

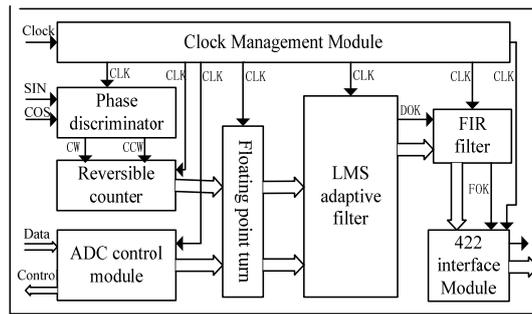


Figure 8. Adaptive Filter Module FPGA Block Diagram

Phase discrimination and reversible counting module: Sine and square wave signal into the phase demodulation module, the square wave signal and the main clock synchronization, and outputs with direction information alternating signal CW, CCW, then CW, CCW respectively into the reversible counting module, and filter sample the reversible count and clear with 10KHZ.

ADC control module: We use FPGA to control the external ADS1258, and 15MHZ ADC master clock and 7.5MHZ SPI clock and control signals is provided using the control bus, finally, we use data bus to read and write ADS1258 internal register through the SPI port;

The module of Fixed to floating: The module conversion reversible count as 32 single precision floating point format, so that the LMS adaptive filter and FIR filter to do floating point arithmetic.

Adaptive filter: Its main job is to design the iterative filter design and weight coefficient update module. Iterative filter is a FIR filter of order 7. The complex floating point addition and multiplication is designed by Weight coefficient update module. When the power filter, after 300 iterations of the iterative filter, and filter will tend to be stable [9]. FPGA adaptive filter program block diagram in Figure 9.

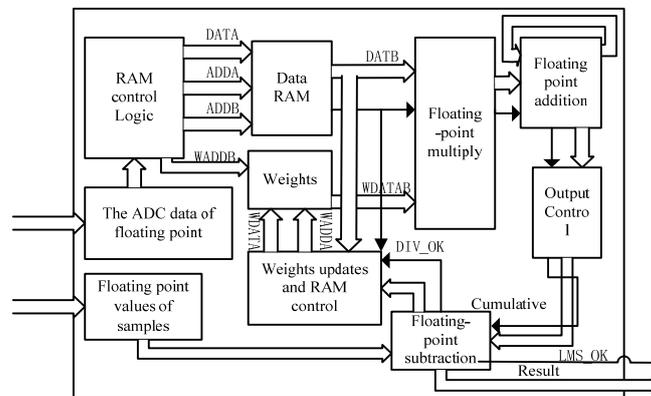


Figure 9. FPGA Block Diagram of the Adaptive Filter

### 5. Results and Analysis

The laser gyro is placed in the standard turntable, turntable in the testing process, in each rotation 3600 output fixed angle in a pulse, the pulse width is about 2us, so we modify the FPGA program, let the filtering results in a constant angle pulse does not come have been accumulated, when a fixed angle pulse arrives to send out a data, at the same time, the accumulator reset, the following test results table:

Table 3. The Test Data with Different Angular Speeds (removal of Earth's rotation effects)

Angular rate	Positive value	Negative values
±5	220753.5	-220753.5
±10	220753.3	-220753.3
±20	220753.2	-220753.2
±50	220753.4	-220753.4
±100	220753.6	-220753.6
±200	220753	-220753
±300	220754.6	-220754.6

Seen from the table in test data, in different angular rate case, turntable rotation of 3600 laser gyro output values are equal, so the adaptive filtering module also has a very high precision and wide dynamic range, and will complete the output data of gyroscope accurate demodulation. Fluctuation behind the decimal point test data is due to the process of manual calculation of rounded out by rounding.

The adaptive filter algorithm is superimposed with a 20 order FIR filter, so the filter delay block is approximately equal to the delay of low order FIR filter. Its expression is  $(F_s \cdot 20) / 2 = 1\text{ms}$ , where  $F_s$  is the sampling frequency. Compared with the original High Order Low - pass FIR filter, its processing speed is improved greatly, so the filter can meet the requirements of laser gyro demodulation speed in the fields of aerospace, weapons, etc.

## 6. Summary

The presented design of the adaptive filter has been repeatedly applied to laser gyro signal processing, filtering processing delay of the circuit is 1.1ms, much lower than the conventional delay time of 10ms, and has a high filtering accuracy and wide dynamic range, to meet the accuracy and response speed of the strict requirements of applications. there is a strong engineering use value.

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