TELKOMNIKA Indonesian Journal of Electrical Engineering
Vol. 12, No. 11, November 2014, pp. 7785 ~ 7797
DOI: 10.11591/telkomnika.v12i11.6471

Time Varying Autoregressive Model Parameters Estimation using Discrete Energy Separation Algorithm

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Abstract

Time Varying Autoregressive (TVAR) model for the Amplitude and Frequency modulated (AM-FM) signal is presented in this paper. TVAR parameters of AM-FM signal are estimated using Discrete Energy Separation (DESA) Algorithm. The performance of DESA method is shown to be comparable to the existing basis function method for AM, FM, AM-FM signal models. The proposed method is simpler to execute in hardware and consumes considerably less computational resources compared to the method using Adaptive and the Basis function methods. .It is demonstrated that the proposed technique based on DESA has certain distinct advantages over the conventional method employing basis functions. Another advantage is that the present method works well with quickly varying signals

Keywords: *amplitude and frequency modulated signal, time varying autoregressive model, basis function, discrete energy separation algorithm*

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1. Introduction

Most temporal signals encountered in real applications have time-varying statistics, which make them nonstationary [1, 2]. The problem of time dependency in nonstationary is bypassed by assuming local stationary over a relatively short time interval, in which stationary system identification and analysis techniques are applied. However, this assumption is not always valid for real life signals like speech, Electrocardiogram (ECG) and Electroencephalogram (EEG), because such signals may have time varying amplitude and frequency [3, 4]. Nonstationary signals which are a compound of constituents with time-varying amplitude and frequencies can be modeled by amplitude and frequency modulated (AM-FM) signals [5].

The best frequency resolution for stationary signals is obtained by using parametric methods. The signal is fitted in to an Autoregressive (AR) or a moving average (MA) or an Autoregressive Moving Average (ARMA) model. It is shown that the parametric method yields very high frequency resolution in the spectral estimation for even very small length of the stationary signal [6]. Spectral analysis of nonstationary signals, with high frequency-resolution is obtained by using the time varying autoregressive (TVAR) process.

In the modeling of nonstationary signals by a TVAR process, two methods may be used for estimating the TVAR parameters: the adaptive algorithm approach and the basis function approach. Adaptive Algorithms, such as the least mean square (LMS) and the recursive least square (RLS), use a dynamic model for adapting the TVAR parameters and are capable of tracking time-varying frequency, provided that the variation is slow. These methods work well with slowly varying signals but fail to track rapid variation [5]. If the coefficients change fast enough; compared to the algorithm’s Convergence time, the adaptive algorithm will not be able track the time varying parameters.

The basis function method, in which the time-variant parameters are expanded as a summation of the weighted time-functions, are capable of tracking both the fast (or) the slow time-varying frequencies. In the basis function expansion, two issues need to be resolved. First, a general class of basis functions is chosen and then, the significant basis functions need to be selected. Several classes of functions have been proposed including polynomial, wavelet and prolate spheroidal functions [15, 16]. However, no uniform rule exists to indicate which class should be adopted. Moreover, the approach of choosing the significant basis functions is based

Received July 11, 2014; Revised August 23, 2014; Accepted September 6, 2014

on trial and error [16]. The selection of the expansion dimension is questionable since there is no fundamental theorem on how to choose them. It is ideally expected that when the expansion dimension is infinite, the result of the frequencies estimation from any basis function is the same, which will exactly equal to the true frequency. But this is impractical, since the computation may require infinite memory, and infinite computational time consumption.

Modeling by a TVAR process is a general approach, and all other nonstationary models (AM, FM, AM-FM and more) can be shown to be special cases of the general approach [5]. A real AM-FM signal can be modeled using a 2-order TVAR process, and a signal composed of p real components will require a 2p- order TVAR process [14]. In the modeling by a TVAR process, the estimation of the TVAR parameters require the inversion of an covariance matrix of size [2p (q+1) x 2p (q+1)], where q is the required number of basis functions to represent each TVAR parameter.

In this paper we have established the relation between the parameters of the AM-FM signal and the parameters of the TVAR process. We have used the Discrete Energy Separation Algorithm (DESA-1) to estimate the TVAR coefficients and the modulating signals of the TVAR process [14]. The estimation technique presented here is conceptually simpler and easier to implement than the method based on basis functions.

The paper is organized as follows: In section 2, the TVAR representation of the AM-FM signal is presented. In section 3, the complete estimation procedure based DESA is provided. The review of estimation using basis functions is presented in Section 4. In section 5 experimental procedure is discussed. The experimental results for the AM, FM, and AM-FM signals with the DESA based technique and basis function technique are presented in section6. Finally, in Section 7, conclusion is provided.

2. TVAR Representation of AM-FM Signal

The AM-FM signal is given by:

$$x(n) = a(n)\cos(\varphi\,(n)) \tag{1}$$

Where $\varphi(n)$ and $a(n)$ are the phase and amplitude of the signal respectively. The AM-FM signal is given by a 2-order TVAR process [14].

$$x(n) = -a_1(n)x(n-1) - a_2(n)x(n-2) + v_n \tag{2}$$

Where v_n is the prediction error. The sequence $v_n$is assumed to be of zero mean and variance $\sigma_{\mathrm{v}}{}^2$. Let n_0 be the time instant at which the transient response has become insignificant. For $n \gg n_0$ ($\sigma_{\mathrm{w}}{}^2 \to 0$), the role of $v_n$in $x(n)$ is insignificant compared to the role of $x(n-1)$ and $x(n-2)$.

Then, the Equation (2) can be written as,

$$x(n) = -a_1(n)x(n-1) - a_2(n)x(n-2) \tag{3}$$

Or,

$$x(n) = -a_1(n)a(n-1)\cos(\varphi\,(n-1) - a_2(n)a(n-2)\cos(\varphi\,(n-2)) \tag{4}$$

Assuming $\mathrm{d}\varphi(n) = \varphi(n) - \varphi(n-1)$, and $\mathrm{d}\varphi(n-1) = \varphi(n-1) - \varphi(n-2)$,
We obtain:

$$\sin\varphi(n-1) = \frac{\cos(\varphi(n-2))}{\sin(\mathrm{d}\varphi(n-1))} - \frac{\cos(\varphi(n-1))\cos(d\varphi(n-1))}{\sin(\mathrm{d}\varphi(n-1)} \tag{5}$$

Then, $x(n)$can be written as:

$$x(n) = a(n)\cos(\varphi\,(n-1)\cos\,(\mathrm{d}\varphi(n)) - a(n)\sin(\varphi\,(n-1)\sin\,(\mathrm{d}\varphi(n)) \tag{6}$$

From Equation (5) and (6), we write:

$$x(n) = a(n)\frac{\sin(\mathrm{d}\varphi(n)+\mathrm{d}\varphi(n-1))}{\sin(\mathrm{d}\varphi(n-1))}\cos(\varphi\,(n-1) - a(n)\frac{\sin(\mathrm{d}\varphi(n))}{\sin(\mathrm{d}\varphi(n-1))}\cos(\varphi\,(n-2) \quad (7)$$

Comparing Equation (7) with (4), we get:

$$a_1(n) = -a(n)\frac{\sin(\mathrm{d}\varphi(n)+\mathrm{d}\varphi(n-1))}{\sin(\mathrm{d}\varphi(n-1))a(n-1)} \quad (8)$$

$$a_2(n) = a(n)\frac{\sin(\mathrm{d}\varphi(n))}{\sin(\mathrm{d}\varphi(n-1))a(n-2)} \quad (9)$$

Once$a_1(n) and\ a_2(n)$ are estimated, the signal $x(n)$can be reconstructed by Equation (3).

3. TVAR Parameter Estimation Using Discrete Energy Separation Algorithm

For both continuous and discrete time signals, Kaiser has defined a nonlinear energy tracking operator ψ [7].For the discrete time case, the energy operator for $x(n)$is defined as,

$$\boldsymbol{\psi}[x(n)] \triangleq x^2(n) - x(n-1)x(n+1) \quad (10)$$

For the signal,

$$x(n) = a(n)\cos(\varphi\,(n)), \quad (11)$$

We have:

$$\boldsymbol{\psi}[x(n)] \approx (a(n)\Omega(n))^2 \quad (12)$$

And,

$$\sqrt{\boldsymbol{\psi}[x(n)]} \approx |a(n)\Omega(n)| \quad (13)$$

Where,

$$\Omega(n) = \frac{\mathrm{d}\varphi(n)}{dn}\,. \quad (14)$$

When one of the variables $a(n)$ (or) $\Omega(n)$ is constant, we can get the other variable with a scaling of $\sqrt{\boldsymbol{\psi}[x(n)]}$. So, the energy operator can estimate the modulating signal, or more precisely its scaled version, when either AM(or)FM is present [7]. When both AM and FM are present simultaneously, three algorithms are described in [7] to estimate $a(n)$ and $\Omega(n)$ separately. The best among the three algorithms according to performance is the discrete energy separation algorithm1(DESA-1).The DESA-1 is defined as follows:

$$S\,(n) = x(n) - x(n-1). \quad (15)$$

$$\Omega(n) \simeq \cos^{-1}\left[1 - \frac{\boldsymbol{\psi}[s(n)]+\boldsymbol{\psi}[s(n+1)]}{4\boldsymbol{\psi}[x(n)]}\right] \quad (16)$$

$$|a(n)| \simeq \sqrt{\frac{\boldsymbol{\psi}[x(n)]}{1-\left(1-\frac{\boldsymbol{\psi}[s(n)]+\boldsymbol{\psi}[s(n+1)]}{4\boldsymbol{\psi}[x(n)]}\right)^2}} \quad (17)$$

Thus, $\Omega(n) and\ a(n)$can be estimated using the Equation (16) and (17), and the TVAR coefficients are estimated with the following equation.

$$a_1(n) = -a(n)\frac{\sin(\Omega(n)+\Omega(n-1))}{\sin(\Omega(n-1))a(n-1)} \quad (18)$$

$$a_2(n) = a(n)\frac{\sin(\Omega(n))}{\sin(\Omega(n-1))a(n-2)} \quad (19)$$

The coefficients thus estimated exhibit ripples and therefore require smoothing using a filter [8]. Ripples can be reduced by using the smoothing. Smoothing can be done using binomial filter with filter coefficients (1,6,15,20,15,6,1).The signal $x(n)$ can be reconstructed by equation(3)using the estimated values of $a_1(n) and\ a_2(n)$.

4. TVAR Parameters Estimation Using Basis Functions

The coefficients a_k(n) of TVAR model in Equation (2) are assumed to be smooth in the sense that if the first derivative of each coefficient may be arbitrarily large, the higher order derivatives necessarily vanish. So, the coefficients a_k(n) can be approximated by a set of basis functions.

The non stationary discrete-time stochastic process x_n is represented by p[th] order TVAR model as:

$$x_n = -\sum_{k=1}^{p} a_{k,n}\, x_{n-k} + v_n \quad (20)$$

Here $a_{k,n}$ are time-varying coefficients and v_n is a stationary white noise process and whose mean is zero and variance is ${\sigma_v}^2$. According to the time-varying coefficients evolution, TVAR is likely to be categorized in to two group's i.e. adaptive method and basis function approach.

TVAR model based on the basis function technique is able to trace a strong non-stationary signal. In this technique, each of its time-varying coefficients are modeled as linear combination of a set of basis functions [15].

The purpose of the basis is to permit fast and smooth time variation of the coefficients. If we denote $u_{m,n}$as the basis function and consider a set of (q + 1) function for a given model, we can state the TVAR coefficients in general as:

$$a_{k,n} = \sum_{m=0}^{q} a_{km} u_{m,n} \quad (21)$$

From (21) we examine that, we have to calculate the set of parameters a_{km} for {k=1,2,........,p; m=0,1,2,............,q; a_{0m}=1} in order to compute the TVAR coefficients $a_{k,n}$,and the TVAR model is absolutely specified by this set.

The TVAR coefficients are designed as follows, we consider single realization of the process x_n.For a given realization of x_n we can analyze (20) as a time-varying linear prediction error filter and consider v_n to be the prediction error.

$$v_n = x_n - \hat{x}_n \quad (22)$$

Where,

$$\hat{x}_n \triangleq -\sum_{k=1}^{p} a_{k,n}\, x_{n-k} \quad (23)$$

The total squared prediction error, which is as well as the error in modeling x_n, is now specified by:

$$\epsilon_p = \sum_{\tau} |v_n|^2$$

Substitute (21) in (23) and the prediction error v_n can be written as:

$$v_n = x_n + \sum_{k=1}^{p}\sum_{m=0}^{q} a_{km} u_{m,n} x_{n-k} \quad (24)$$

The total squared prediction error can be formulated as:

$$\epsilon_p = \sum_{\tau} \left| x_n + \sum_{k=1}^{p}\sum_{m=0}^{q} a_{km} u_{m,n} x_{n-k} \right|^2 \quad (25)$$

For modeling the non stationary stochastic process x_n , $using$ covariance technique, we make no assumptions on the data outside [0, N-1]. In equation (25) τ is the interval over which the summation is performed and set $\tau = [p, N-1]$. By minimizing the mean squared prediction error in (25) we can estimate the time-varying parameters a_{km} [16]. We can minimize the mean squared prediction error in (25) by means of setting the gradient of ϵ_p with respect to a_{lg}^* zero.

$$\frac{\partial \epsilon_p}{\partial a_{lg}^*} = \sum_\tau \frac{\partial v_n v_n^*}{\partial a_{lg}^*} = \sum_\tau v_n \frac{\partial v_n^*}{\partial a_{lg}^*} = 0 \quad (26)$$
$$\{l = 1,2,\cdots,p; g = 0,1,\cdots,q\}$$

Where,

$$v_n^* = x_n^* + \sum_{l=1}^{p} \sum_{g=0}^{q} a_{lg}^* u_{g,n}^* x_{n-l}^*$$

And the derivative of v_n^* with respect to a_{lg}^*

$$\frac{\partial v_n^*}{\partial a_{lg}^*} = u_{g,n}^* x_{n-l}^*$$

Consequently (26) becomes,

$$\sum_\tau v_n u_{g,n}^* x_{n-l}^* = 0 \quad (27)$$

The above mentioned condition is similar to the orthogonality law encountered in stationary signal modeling. Substitute (24) in (27) we have:

$$\sum_\tau \left(x_n + \sum_{k=1}^{p} \sum_{m=0}^{q} a_{km} u_{m,n} x_{n-k}\right) u_{g,n}^* x_{n-l}^* = 0 \quad (28)$$

Now we define a function $c_{mg}(l,k)$ as shown below,

$$c_{mg}(l,k) \triangleq \sum_\tau u_{m,n} x_{n-k} u_{g,n}^* x_{n-l}^* \quad (29)$$

Using the above definition in (28) we have,

$$\sum_{k=1}^{p} \sum_{m=0}^{q} a_{km} c_{mg}(l,k) = -c_{0g}(l,0) \quad (30)$$

The above equation represents a system of p(q+1) linear equations. The above system of linear equations can be efficiently represented in matrix form as follows.

Define a column vector a_m as follows:

$$a_m = \begin{bmatrix} a_{1m} & a_{2m} & \cdots & a_{pm} \end{bmatrix}^T, \quad (31)$$
$$where\ m = 0,1,\cdots,q$$

We can use the function (10) to find the following matrix for $0 \le (m,g) \le q$

$$C_{mg} = \begin{bmatrix} c_{mg}(1,1) & c_{mg}(1,2) & \cdots & c_{mg}(1,p) \\ c_{mg}(2,1) & c_{mg}(2,2) & \cdots & c_{mg}(2,p) \\ \vdots & & \ddots & \vdots \\ c_{mg}(p,1) & c_{mg}(p,2) & \cdots & c_{mg}(p,p) \end{bmatrix} \quad (32)$$

The above matrix is of size pxp and all the different values for m and g resulting in (q+1)x(q+1) such matrices, by means of these matrices, we can now describe a block matrix as shown below,

$$C = \begin{bmatrix} C_{00} & C_{01} & \cdots & C_{0q} \\ C_{10} & C_{11} & \ddots & C_{1q} \\ \vdots & \vdots & \vdots & \vdots \\ . & . & . & . \\ C_{q0} & C_{q1} & \cdots & C_{qq} \end{bmatrix} \quad (33)$$

The above Block matrix C has (q+1)x(q+1) elements and each element is a matrix of size pxp, which implies the Block matrix C of size p(q+1)x p(q+1).
Now we describe a column vector d_m as shown below:

$$d_m = [c_{0m}(1,0) \quad c_{0m}(2,0) \quad \cdots \quad c_{0m}(p,0)]^T \quad (34)$$
$$where\ m = 0,1,\cdots,q$$

By using the definitions from (31)-(34) we can represent the system of linear equations in (30) in a compact matrix form as follows:

$$\underbrace{\begin{bmatrix} C_{00} & \cdots & C_{0q} \\ \vdots & \ddots & \vdots \\ C_{q0} & \cdots & C_{qq} \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} a_0 \\ \vdots \\ a_q \end{bmatrix}}_{a} = -\underbrace{\begin{bmatrix} d_0 \\ \vdots \\ d_q \end{bmatrix}}_{d} \quad (35)$$
$$Ca = -d$$

By solving the above matrix equation, we can obtain the set of TVAR parameters a_{km} (elements of a), the predictor coefficients $a_{k,n}$ can now be calculated using (21).The matrix C is of size p(q+1)xp(q+1),to solve the above system of linear equations we requires O(p^3 $(q+1)3)$computations.

In the basis function expansion, two issues need to be resolved. First a general class of basis functions is to be chosen, which can suitably capture the time variation, and then, the significant number of basis functions need to be selected. Several classes of functions have been proposed in the literature such as time basis functions, Legendre polynomial, Chebyshev polynomial, Discrete prolate spheroidal (DPSS) sequence, Fourier basis, discrete cosine basis, Walsh basis, Multi wavelet basis functions. However, no uniform rule exists to indicate which class should be adopted. The approach of choosing the significant number of basis functions (order selection) is based on trial and error [15]: Moreover, the expansion of the TVAR parameters into the basis sequences substantially increases the number of model parameters that is to be estimated. To compare the performance of DESA with Basis function method we use discrete cosine basis function.

4.1. Discrete Cosine Basis Function

$$u_{m,n} = \alpha(m)\cos\left(\frac{\pi m(2n+1)}{2N}\right)$$

Where,

$$\alpha(m) = \begin{cases} \sqrt{\frac{1}{N}} & m = 0 \\ \sqrt{\frac{2}{N}} & m = 0,1,2 \ldots\ldots q \end{cases} \quad (36)$$
n=1,2.....N

For all the above mentioned problems, the TVAR model parameter estimation using basis function approach requires high computational complexity. In this paper we have established the relation between the parameters of the AM-FM signal and the parameters of the TVAR process. We have used the Discrete Energy Separation Algorithm1(DESA-1) to estimate the TVAR coefficients and the modulating signals of the TVAR process [11]. The DESA-1 based TVAR parameter estimation is conceptually simpler and easier to implement than the method based on basis functions. TVAR parameter estimation using DESA requires O (p $(q+1)^2$) computations whereas Basis function method requires O($p^3\ (q+1)^3$) computations.

5. Experimental Procedure

Step 1: Calculate the TVAR parameters a_{km} using Equations (18), (19) and form the coefficients $a_{k,n}$ using (21).

Step 2: Solve the roots of the time-varying auto regressive polynomial formed by TVAR linear prediction filter A(z; n)= $1+\sum_{k=1}^{p} a_{k,n} z^{-k}$ at each instant n to find the time-varying poles: $P_{i,n}$, i=1, 2.....p.

Step 3: The instantaneous frequency of the non stationary signal for each sample instant n can be estimated from the instantaneous angles of the poles using the formula $f_{i,n}=\frac{\arg[P_{i,n}]}{2\pi}$ for $|P_{i,n}|\simeq 1$.

Step 4: From time varying parameters $a_{k,n}$we can predict non stationary signal using (23) with initial

Conditions x_n; n=0,1,...p where p is the TVAR model order

Step 5: The time varying power spectral density can be estimated from time varying parameter $a_{k,n}$ as follows:

$$P(f;n) = \frac{\sigma^2}{\left|1+\sum_{k=1}^{p} a_{k,n} e^{-j2\pi f k}\right|^2} \tag{36}$$

Where $a_{k,n}$are TVAR coefficients and σ^2 is:

$$\sigma^2 = \frac{1}{N-p}\sum_{n=p+1}^{N-1}\left|x_n+\sum_{k=1}^{p} a_{k,n} x_{n-k}\right|^2 \tag{37}$$

6. Simulation Results

For simulation, we have considered three modulated signal models, the Discrete Amplitude Modulated (AM) signal, Discrete Frequency Modulated (FM) signal and Discrete Amplitude and Frequency modulated (AM-FM) signals.

6.1. Discrete AM Signal

Consider the discrete Amplitude modulated (AM) signal,

$$y(n) = (1+k\cos(\omega_a n))\cos(\theta n) \tag{38}$$

For n=1,2…..N, where k=0.8, $\omega_a=\frac{\pi}{128}$, $\theta=\frac{\pi}{6}$ and N = 512.

The IF law of the above signal is given by:

$$f_{in} = \frac{\theta}{2\pi} \tag{39}$$

The coefficients of the TVAR process $a_1(n)$ and $a_2(n)$ of the discrete AM Signal are estimated using (18), (19) and Equation (21) and shown in Figure 1. When the TVAR coefficients are estimated using basis functions, p=2 and q=8 discrete cosine basis functions are found to give best results. Figure 1 shows the TVAR parameters estimated by the DESA-1 and using basis method. TVAR parameter estimation using basis function approach requires O (p^3 $(q+1)^3$) computations, where as DESA based approach requires O (p $(q+1)^2$) computations.

Using step 2 in experimental procedure we can compute time varying poles, the time varying poles are plotted in Figure (2). From Figure (2) we observe that the poles are close to the unit circle. For every sample instant n, we now come across the angles of the poles and divide by 2π to find the IF estimate of the AM component. The true IF & estimated IF of the AM component are shown in Figure (3). From Figure (3) we observe that the TVAR based technique has resulted in really nice IF estimation. The mean square error (MSE) among the true IF $f_{i,n}$and estimated IF $\hat{f}_{i,n}$ for n=2, 3… 512 is calculated to be -86.4847dB.

The TVAR coefficients $a_{k,n}$ can also be used to predict the non stationary process x_n by means of Equation (23). The discrete AM Signal x_n in addition to the TVAR prediction are shown in Figure (4), and we observe that the TVAR model has effectively predicted x_n.The average squared prediction error is calculated to be 0.1753.

The time-varying power spectral density of discrete AM is computed using (36), A plot of the time-frequency distribution (TFD) of discrete AM for the TVAR model is obtained in Figure (5). At every sample instant, the TFD is projected to comprise peaks at the IF estimates at that instant. To demonstrate this, we also illustrate the analogous flat time-frequency view of the TFD in Figure (6).

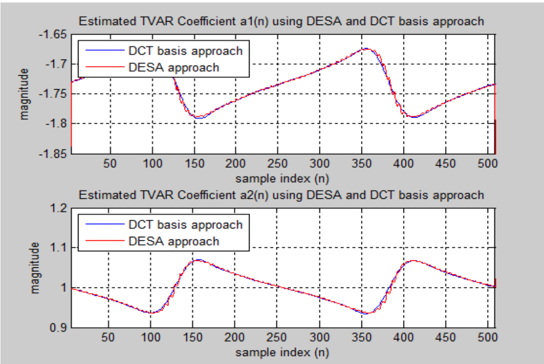

Figure 1. The estimate of the TVAR coefficients $a_{1,n}$, $a_{2,n}$ for the AM signal

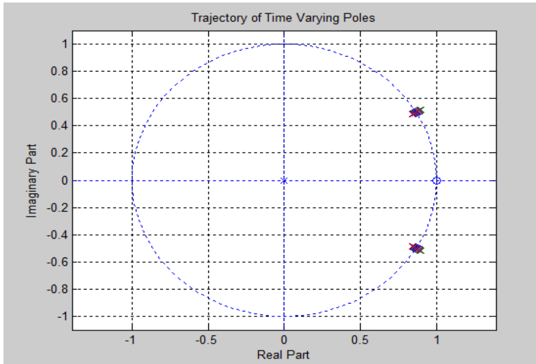

Figure 2. Trajectory of Time-Varying poles used for discrete AM Signal

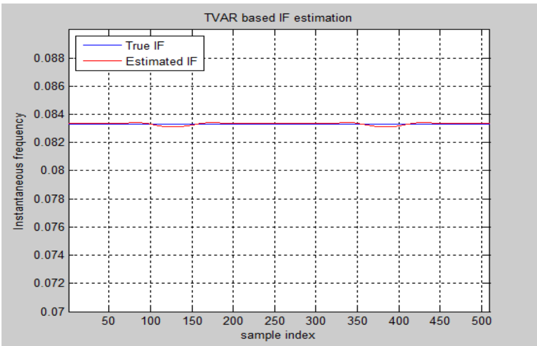

Figure 3. True and Estimated IF of Discrete AM Signal

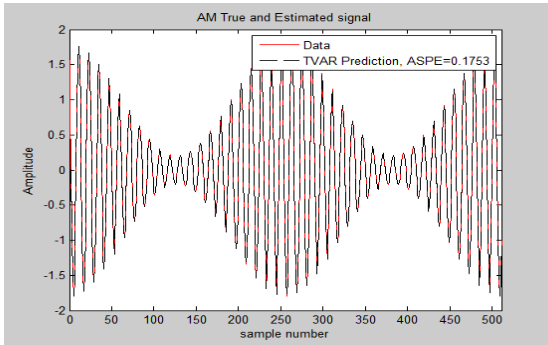

Figure 4. Original AM Signal and Predicted AM Signal

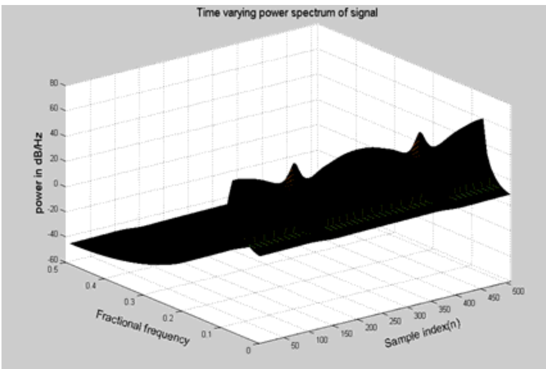

Figure 5. Time Varying Power Spectrum of the Discrete AM Signal

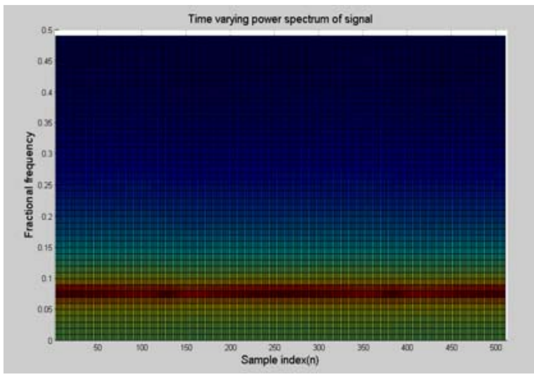

Figure 6. Time-Frequency View of the TFD of Discrete AM Signal

6.2. Discrete FM Signal

Consider the discrete FM signal:

$$y(n) = \cos\left(\theta n + \omega_m \sum_{i=1}^{n} \cos(\omega_f i)\right) \quad (40)$$

For n=1, 2…..N, where ω_m=0.2θ, $\omega_f = \frac{\pi}{128}$, $\theta = \frac{\pi}{6}$ and N=512.

The IF law of the above signal is given by:

$$f_{in} = \frac{\theta + (\omega_m * \omega_f)\cos(\omega_f n)}{2\pi} \quad (41)$$

The coefficients of the TVAR process $a_1(n)$ and $a_2(n)$ of the discrete FM signal are estimated using (18), (19) and Equation (21) and shown in Figure 7. When the TVAR coefficients are estimated using basis functions, p=2 and q=14 discrete cosine basis functions are found to give best results. Figure 7 shows the TVAR parameters estimated by the DESA-1and using basis method.TVAR parameter estimation using basis function approach requires O (p^3 $(q+1)^3$) computations, where as DESA based approach requires O (p $(q+1)^2$) computations.

Using step 2 in experimental procedure we can compute time varying poles, Trajectory of Time-varying Poles used for discrete FM Signal are plotted in Figure (8). From Figure (8) we observe that the poles are close to the unit circle as anticipated. For every sample instant n, we now come across the angles of the poles and divide by 2π to find the IF estimate of the FM component. The true IF & estimated IF of the FM component are shown in Figure (9). From Figure (9) we observe that the TVAR based technique has resulted in really nice IF estimation. The mean square error (MSE) among the true IF $f_{i,n}$and estimated IF $\hat{f}_{i,n}$ for n=2, 3… 512 is calculated to be -96. 87dB.

The TVAR coefficients $a_{k,n}$ can also be used to predict the non stationary process x_n by means of equation (23). The discrete FM Signal x_n in addition to the TVAR prediction are shown in Figure (10), and we observe that the TVAR model has effectively predicted x_n. The average squared prediction error is calculated to be 0.1065.

The time-varying power spectral density of discrete FM Signal is computed using (36), A plot of the time-frequency distribution (TFD) of discrete FM signal for the TVAR model is obtained in Figure (11). At every sample instant, the TFD is projected to comprise peaks at the IF estimates at that instant. To demonstrate this, we also illustrate the analogous flat time-frequency view of the TFD in Figure (12).

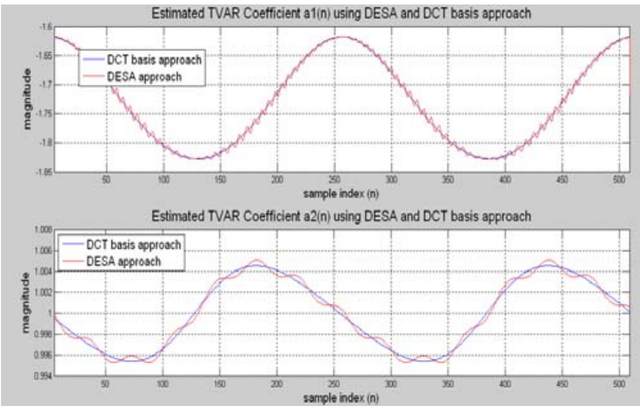

Figure 7. The Estimate of the TVAR Coefficients $a_{1,n}$, $a_{2,n}$ for the FM Signal

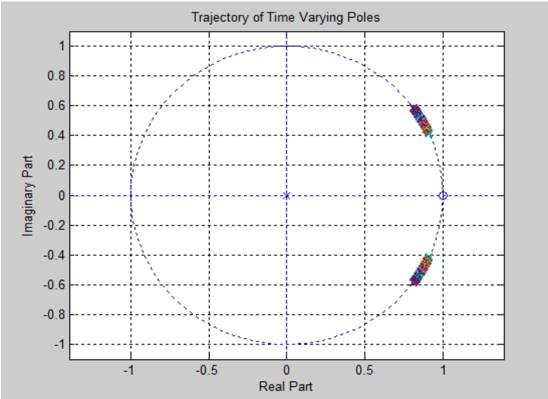

Figure 8. Trajectory of Time-varying Poles used for discrete FM Signal

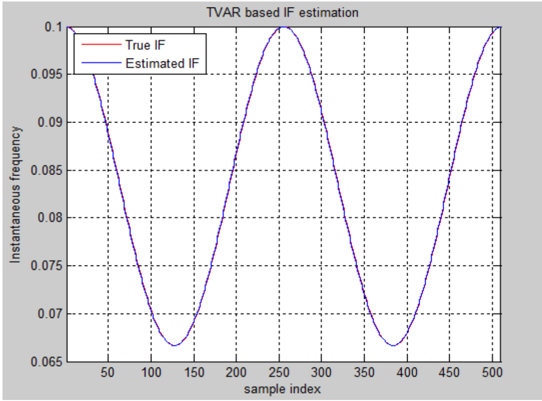

Figure 9. True and Estimated IF of discrete FM Signal

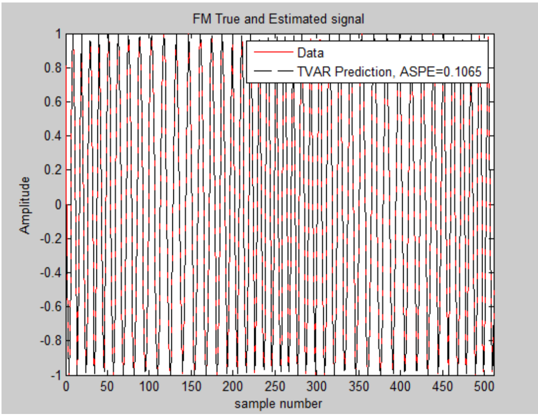

Figure 10. Original FM signal, and predicted FM signal

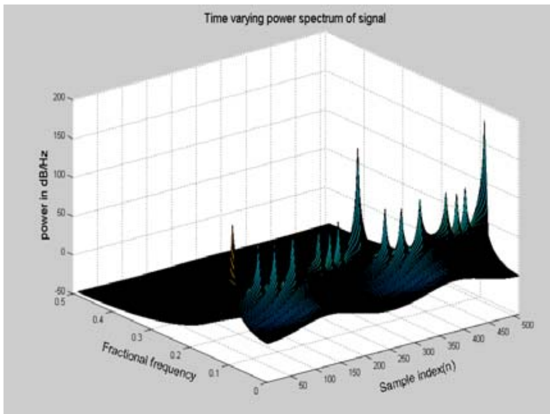

Figure 11. Time Varying Power Spectrum of the discrete FM Signal

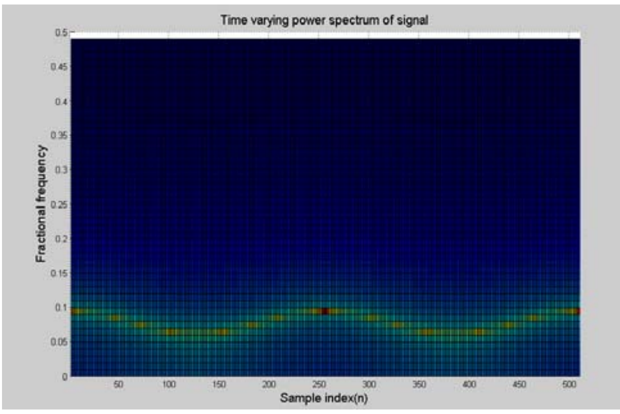

Figure 12. Time-Frequency View of the TFD of Discrete FM Signal

6.3. Discrete AM-FM Signal

Consider the discrete AM-FM signal,

$$y(n) = (1 + k\cos(\omega_a n))\cos\left(\theta n + \omega_m \sum_{i=1}^{n} \cos(\omega_f i)\right) \quad (42)$$

For n=1,….N where k=0.5, ω_m=0.5θ, $\omega_a = \omega_f = \frac{\pi}{128}$ $\theta = \frac{\pi}{6}$ and N=512.

The IF law of the above signal is given by:

$$f_{in} = \frac{\theta + (\omega_m * \omega_f)\cos(\omega_f n)}{2\pi} \quad (43)$$

The coefficients of the TVAR process $a_1(n)$ and $a_2(n)$ of the discrete AM-FM Signal are estimated using (18), (19) and Equation (21) and shown in Figure 13. When the TVAR coefficients are estimated using basis functions, p=2 and q=23 discrete cosine basis functions are found to give best results. Figure 13 shows the TVAR parameters estimated by the DESA-1and using basis method.TVAR parameter estimation using basis function approach requires O (p^3 $(q+1)^3$) computations, where as DESA based approach requires O (p $(q+1)^2$) computations.

Using step 2 in experimental procedure we can compute time varying poles, Trajectory of Time-varying Poles used for discrete AM-FM Signal are plotted in Figure (14). From Figure (14) we observe that the poles are close to the unit circle as anticipated. For every sample instant n, we now come across the angles of the poles and divide by 2π to find the IF estimate of the discrete AM-FM component. The true IF & estimated IF of the AM-FM component are shown in Figure (15). From Figure (15) we observe that the TVAR based technique has resulted in really nice IF estimation. The mean square error (MSE) among the true IF $f_{i,n}$and estimated IF $\hat{f}_{i,n}$ for n=2, 3… 512 is calculated to be -76.1437dB.

The TVAR coefficients $a_{k,n}$ can also be used to predict the non stationary process x_n by means of Equation (23). The original discrete AM-FM Signal x_n in addition to the TVAR prediction are shown in Figure (16), and we observe that the TVAR model has effectively predicted x_n.The average squared prediction error is calculated to be 0.0503.

The time-varying power spectral density of discrete AM-FM Signal is computed using (36), A plot of the time-frequency distribution (TFD) of discrete AM-FM signal for the TVAR model is obtained in Figure (17). At every sample instant, the TFD is projected to comprise peaks at the IF estimates at that instant. To demonstrate this, we also illustrate the analogous flat time-frequency view of the TFD in Figure (18).

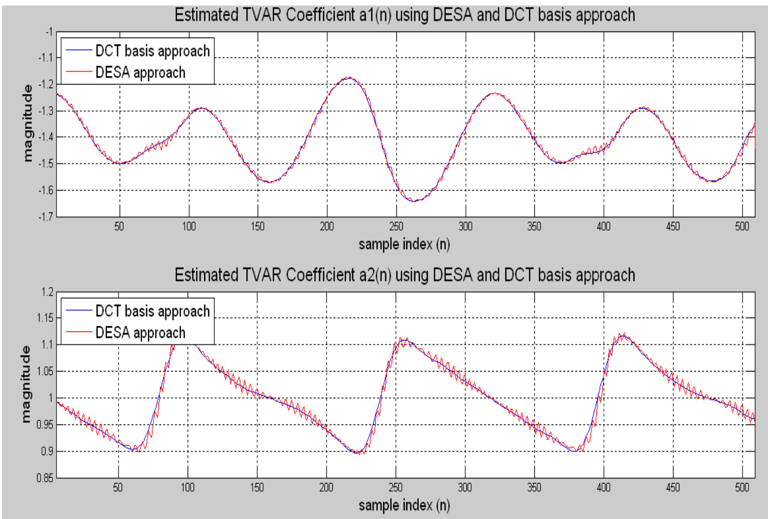

Figure 13. The Estimate of the TVAR coefficients $a_{1,n}$, $a_{2,n}$ for the AM- FM signal

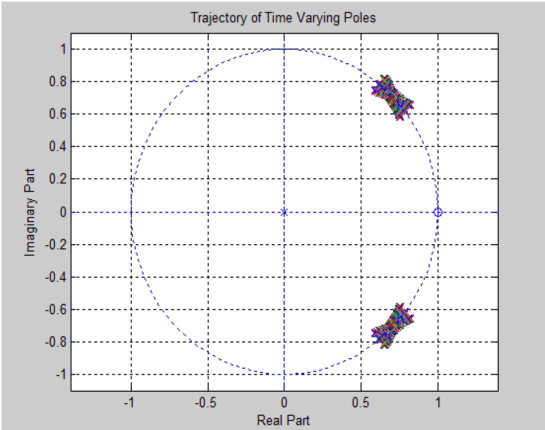

Figure 14. Trajectory of Time-varying Poles used for discrete AM-FM Signal

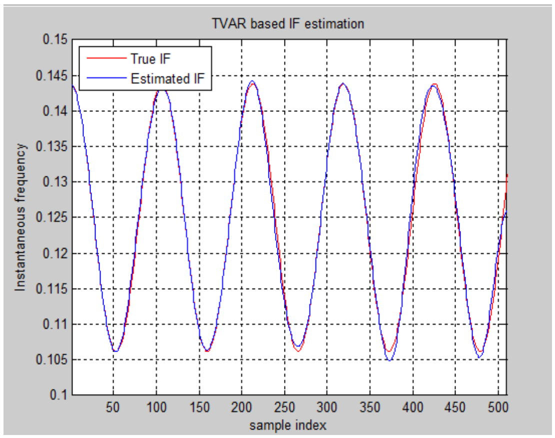

Figure 15. True and Estimated IF of discrete AM-FM Signal

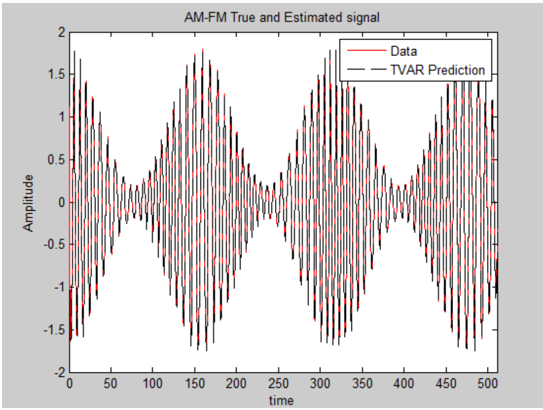

Figure 16. Original AM- FM signal, and predicted discrete AM-FM signal

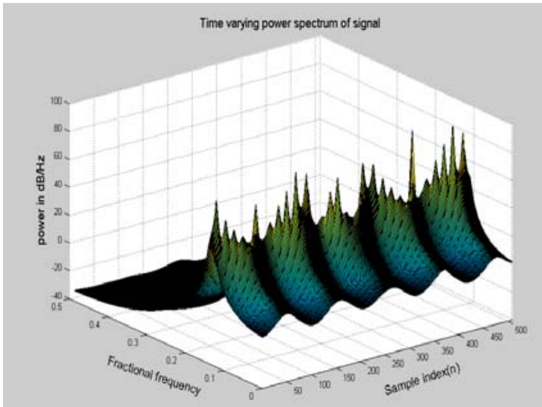

Figure 17. Time Varying Power Spectrum of the discrete AM-FM Signal

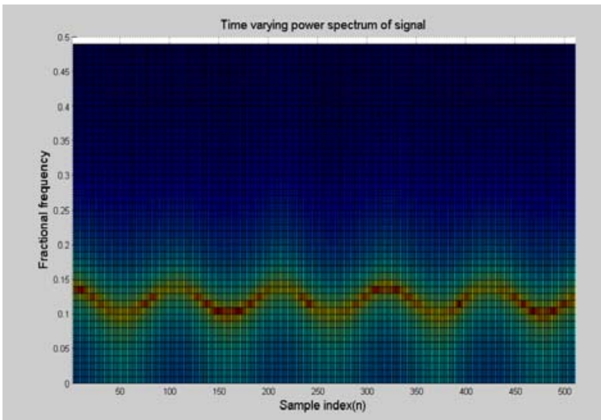

Figure 18. Time-Frequency View of the TFD of discrete AM-FM Signal

7. Conclusion

In this paper, we have shown a method for estimating the TVAR coefficients of various AM-FM signals using the Discrete Energy Separation Algorithm. The performance of the method based on DESA is shown to comparable to the existing method using basis functions for discrete AM, discrete FM,and discrete AM-FM signal models. However, the proposed method is simpler to implement in hardware and consumes considerably less computational resources compared to Basis function method. Another advantage is that the present method works well with quickly varying signals. The time varying poles can be estimated using this method and have application in model identification.

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