

Review of the Urban Traffic Modeling

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Abstract

Nowadays, the urban traffic modeling, which is helpful in planning and controlling the traffic system, has becoming a research hotspot of traffic engineering. After decades of research and development, there now exists hundreds of models choosing different modeling methods to simulate the traffic flow. It is important for us to understand these models by classifying them and analyzing their features. The features of traffic models, including the scalability, accuracy and computability, are becoming important indicators to measure their performance. In this paper, we introduce and compare some grounded models. In particular, we analyze the advantages and disadvantages of existing models, and classify them into three categories according their granularity: macroscopic, mesoscopic and microscopic models.

Keywords: macroscopic models, mesoscopic models, microscopic models, survey, car-following theory, cellular automaton models, gas-kinetic theory

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1. Introduction

With the increasing number of vehicles, the urban traffic system faces many problems one of which is traffic congestion becoming more serious day after day. The urban traffic modeling is helpful for mitigating traffic congestion, because it allows us to better understand, plan, design and optimize the traffic system. There exist hundreds of urban traffic models, which choose different kinds of methods, such as probability and statistics, differential equations and numerical methods. It is necessary to classify these traffic models for comparing their advantages and disadvantages.

According to the model granularity, which is the level of detail considered in the model, we classify traffic models into three categories, macroscopic, mesoscopic (hybrid) and microscopic (sub-microscopic) models.

Macroscopic models view all vehicles as a whole, and study the characteristics of the entire traffic flow. In particular, they measure the variation of traffic flow parameters, which include flow rate, velocity and density, and analyze the relationship among these parameters. Although these models can describe the variation of some traffic phenomena (e.g., the stop-and-go wave), they cannot explain the formation of these phenomena due to the ignorance of the individual vehicle's behavior. We divide macroscopic models into two categories, time-independent static models and time-related dynamic models, according to the correlation of time. The time-related dynamic models consider the effect of space and time correlation, and thus, they could be more realistic than time-independent static ones in some cases. Typical time-related dynamic models (e.g., LWR mode) apply the fluid dynamics, which is a theory of fluid mechanics that deals with the natural science of fluids (liquids and gases) in motion, to characterize the variation of the traffic flow.

Mesoscopic models assume a set of nearby vehicles as a unit, a so-called "platoon", and describe the inflow and outflow of each platoon. Specifically, these models study the common behavior of vehicles in a same platoon. We group mesoscopic models into two categories: gas-kinetic models and hybrid models. The first gas-kinetic model proposed by Prigogine and Herman applies the gas-dynamics, which is a law that explains the behavior of a hypothetical ideal gas, to describe the platoon. Hybrid models usually combine different models (e.g., a microscopic model mixed with a mesoscopic model) to combine their advantages and remedy their disadvantages.

Microscopic models focus on the behavior of individual vehicles, and study how one vehicle dynamically interacts with another. These models attempt to describe the overall characteristics of the system by integrating the characteristic of each individual vehicle. Microscopic models have three categories: car-following models, cellular automaton models (particle hopping models) and sub-microscopic models. Car-following models analyze the vehicle following behavior in one lane. Cellular automaton models view individual vehicles as self-driven particles, which is a collection of particles respond to a random perturbation by the motion of the other nearby particles. Compared with other two kinds of models, sub-microscopic models describe more details, such as driver's psychological reactions, response to the traffic and car lights, etc.

This paper classifies traffic models mainly in model granularity. And we respectively introduce some important models of each type and summarize the characteristic of these models in section macroscopic models, mesoscopic models and microscopic models. At the end of each section we list the comparison of the advantage, disadvantage, applicable environment and modeling methods of the most important models. Besides, there's a conclusion about characteristics of existing models at the end of this article.

2. Macroscopic Models

Macroscopic models consider traffic flow as an entirety and they do not care about the behavior of individual vehicles. These models contain static macroscopic models and dynamic macroscopic models. The standard static models include the recursive model, the start-arrive model and the start-destination model. The dynamic models contain the first-order continuum model (e.g., the LWR model), and second-order continuum models such like the Payne model and the Papageorgiou model. The classification of macroscopic models is shown in Figure 1.

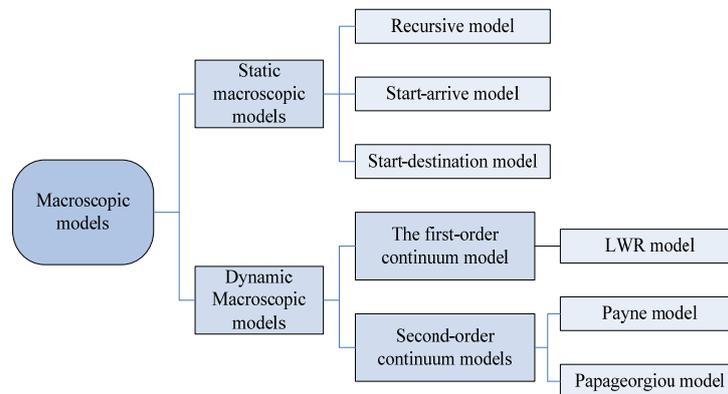


Figure 1. The Classification Figure of Macroscopic Models

2.1. Static Macroscopic Models

Static macroscopic models research on the time-independence relationship among traffic parameters such as traffic flow velocity $v(x)$, flow rate $q(x)$ and density $\rho(x)$ at the traffic flow location x . There are three important Static macroscopic models: the recursive model, the start-arrival model and the start-destination model [1].

The recursive model: The recursive model aims to figure out the traffic flow rate through the calculation of the traffic flow rate at each section recursively. The model uniformly divides the traffic flow into N sections and let each section i owns at most one entrance r_i and one exit s_i . Let q_{i-1} and q_i respectively be the traffic inflow rate and the outflow rate of section i . By Equation (1), we can compute q_i according to parameters q_{i-1} , r_i and s_i . As the model's name implies, we can calculate the traffic flow rate of any section recursively from the flow rate of previous ones.

$$q_i = q_{i-1} + r_i - s_i; i = 1, 2, \dots, N \quad (1)$$

The schematic diagram of the recursive model is shown in Figure 2. Section 1 is the beginning section of the traffic flow, and q_0 is the initial flow rate. Each section is seen uniformly which contains at most one entrance and one exit. The output q_i of section i is the input of its forward section $i + 1$.

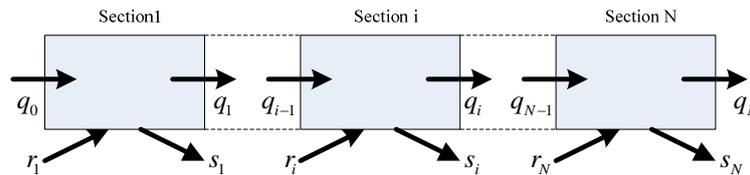


Figure 2. Schematic Diagram of the Recursive Model

The Start-arrive model: The start-arrive model attempts to count the traffic flow rate through the arrival flow rate of each section. The schematic diagram of this model is the same as the recursive model. The difference is that the start-arrive model define a proportional variable a_{ij} as the traffic flow rate of section j which entered from entrance r_i . Then we can calculate each section's flow rate according to form (2):

$$q_j = \sum_{i=1}^j r_i a_{ij}; j = 1, 2, \dots, N, (0 \leq a_{i,N} \leq a_{i,N-1} \leq \dots \leq a_{i,i+1} \leq a_{i,i} \leq 1) \quad (2)$$

To calculate the traffic flow rate at each section, we import the $N \times N$ order start-arrive matrix of a_{ij} :

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,N} \\ 0 & a_{2,2} & \dots & a_{2,N} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{N,N} \end{pmatrix} \quad (3)$$

Then we set the traffic flow vector $q = [q_1, q_2, \dots, q_N]$, and the entrance traffic flow vector $r = [r_1, r_2, \dots, r_N]$.

And we simplify the matrix form as:

$$q = rA \quad (4)$$

That is to say the traffic flow rate at each section can be figured out from matrix A and the entrance flow vector r .

The start-destination model: The start-destination model is used on a specific condition that the road is closing at the end. This model defines a proportional variable b_{ij} the rate of the exit traffic flow at s_i which entered from entrance r_j . Then we can calculate each section's exit flow rate in from (5):

$$s_j = \sum_{i=1}^j r_i b_{ij}; \quad j = 1, 2, \dots, N \quad (5)$$

And all vehicles exit at the end of section N , so we have:

$$\sum_{j=i}^N b_{ij} = 1; \quad j = 1, 2, \dots, N \quad (6)$$

Then we also import the $N \times N$ order start-destination Matrix of b_{ij} in the form as follow:

$$S = rB \quad (7)$$

We can also express a_{ij} in Formula (8):

$$a_{ij} = \sum_{k=j+1}^N b_{ik}; \quad i = 1, 2, \dots, N \quad (8)$$

Static macroscopic models are constant coefficient models. The usage of these models is limited practically because they can not make predictions for accidental events. Nevertheless, studying on static macroscopic models is still meaningful due to the unstable measurement in dynamic model. In that case, only the mean value of a short period is valuable which usually fluctuate strongly.

2.2. Dynamic Macroscopic Models

Dynamic macroscopic models mainly describe the spatio-temporal association rules of the traffic flow features, including traffic flow rate, velocity and density. The theoretical basis of dynamic macroscopic models is the fluid dynamics model, which is also known as the continuum model of traffic flow. Such models consider traffic flow as a compressible fluid formed by a large number of vehicles and do not mention the individual behavior of these vehicles. We can divide the dynamic microscopic models into two categories [2]. One category is the first-order continuum models contain relations between the traffic flow velocity-density or flow rate-density. The other category is the second-order continuum models contain additional relaxation time to adapt the velocity of vehicles with the surrounding ones. The major difference between these 2 categories is that whether the model contains inertia term. It make these two categories no difference if the time constant of the inertia term is set to zero, which means vehicles can instantaneous change their velocity.

2.2.1. The First-order Continuum Model

The representative model of the first-order continuum model introduced in this paper is the LWR model proposed by Lighthill and Whitham.

LWR model: Lighthill and Whitham established their traffic model with the one-dimensional kinetic theory of traffic flow in 1955. They choose the principle of mass conservation of fluid dynamics in traffic flow and then form their first-order macroscopic traffic flow model. They let ρ as the traffic flow density, q as the traffic flow rate, t as time variable, x as the spatial displacement of traffic flow. Thus, $s(x, t)$ is the traffic flow generation rate. There are three cases: the first is $s(x, t) = 0$, which indicates the conservation of the flow rate; the second is $s(x, t) > 0$, which means the inlet traffic flow; the last is $s(x, t) < 0$, which means the outlet traffic flow. The continuity equation [3] of the traffic flow is shown in (9):

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = s(x, t) \quad (9)$$

LWR model and many models based on LWR assume a relationship between the traffic flow velocity and density under equilibrium state in (10). v_e is the dynamic equilibrium velocity of the traffic flow:

$$v(x, t) = v_e(\rho(x, t)) \quad (10)$$

So the equation can be transformed into:

$$\frac{\partial \rho}{\partial t} + (v_e + \rho \frac{\partial v_e}{\partial \rho}) \frac{\partial \rho}{\partial x} = s(x, t) \quad (11)$$

LWR model can correctly describe the formation of the shock waves and the dispersing of traffic congestion, but it can not describe none-equilibrium traffic flow phenomena like the ghost traffic. In order to describe these phenomena, second-order continuous models based on LWR model was proposed later.

2.2.2. Second-order Continuum Models

Second-order continuous models include models like the Payne model and Papageorgiou model which add a relaxation time to LWR model, etc.

Payne model: To describe the none-equilibrium traffic phenomenon like ghost traffic, scholars add vary momentum equation to LWR model and formed fluid dynamic model such as Payne model.

According to the idea of the car-following theory, Payne proposes the corresponding dynamic equations [4] in Formula (12). The model defines μ as the pressure index; $-\frac{\mu}{\rho\tau} \frac{\partial \rho}{\partial x}$ as the pressure term which indicates the driver's reaction process to stimulations; τ as the relaxation time; $\frac{1}{\tau}(v_e - v)$ as the relaxation term which indicates the relax process that driver adapts to the equilibrium speed.

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = -\frac{\mu}{\rho\tau} \frac{\partial \rho}{\partial x} + \frac{1}{\tau}(v_e - v) \quad (12)$$

The Payne model is able to simulate the propagation of nonlinear wave in real road. And it is the basis model of the extensively used simulation software FREFLO.

Payne model's main contribution lies in the relationship formula of the dynamic traffic flow velocity-density in Equation (13). τ is the relaxation time and Δx is the flow's spatial displacement during relaxation time τ .

$$v(x, t + \tau) = v[\rho(x + \Delta x, t)] \quad (13)$$

The model can simulate preliminary the backward-spread of traffic congestion. There are problems on the adaptive process and numerical calculation. Yet the main problem is the relationship assumption between the traffic flow velocity and density.

Payne also summarizes the second-order fluid dynamic model's general form. Most of the fluid dynamic models can be written in velocity equation from their continuity equation in Formula (14). $V \frac{\partial V}{\partial x}$ is the transport term, P is the traffic pressure, τ is the relaxation time,

V_e is the dynamic equilibrium traffic velocity determined by local vehicles' density, $-\frac{1}{\rho} \frac{\partial P}{\partial x}$ is

the pressure term, and $\frac{1}{\tau}(V_e - V)$ is the relaxation term.

$$\frac{\partial V}{\partial t} + \underbrace{V \frac{\partial V}{\partial x}}_{\text{Transport Term}} = - \underbrace{\frac{1}{\rho} \frac{\partial P}{\partial x}}_{\text{Pressure Term}} + \underbrace{\frac{1}{\tau} (V_e - V)}_{\text{Relaxation Term}} \quad (14)$$

The main difference between these models ^[5] is the traffic pressure P , relaxation time τ and the dynamic equilibrium traffic velocity V_e .

The relaxation time in LWR model is set to 0. Payne model and Papageorgious model assume $P(\rho) = [V_0 - V_e(\rho)] / (2\tau)$ [6], and the average free/expect speed $V_0 = V_e(0)$. Phillips model assumes $P = \rho\theta$, and θ is the variance of the velocity [7]. Kühne model [8], Kerner

and Konhäuser model [9] define $P = \rho\theta_0 - \eta \frac{\partial V}{\partial x}$, while θ_0 is a positive constant, η is the coefficient of viscosity. The term $-\eta \frac{\partial V}{\partial x}$ in Kerner and Konhäuser model means the viscosity

term similar with the term $\frac{\eta}{\rho} \frac{\partial^2 V}{\partial x^2}$ proposed by Whitham before, which is important to filter the shock front. Michalopoulos model define the relaxation time τ as a variable inversely proportional to the traffic flow density [10]. Wu model introduces a momentum equation of the one-dimensional pipe flow into traffic flow model in the condition of the hybrid and low-speed traffic in China [11]. Bellouquid researches on the hyperbolic asymptotic limit of the discrete kinetic theory model of vehicular traffic [12]. Daganzo researches on the analysis of the stability of macroscopic traffic flow [13]. Ngoduy thinks widely scattered traffic flow rate-density relationship is caused by the random variations in driving behavior. And he solves this problem by adopting a multi-class first-order model with a stochastic setting in his model parameters [14].

The most prominent feature of microscopic models is that these models do not take vehicle's individual behavior into account. We summarize the features of these models in Table 1.

Table 1. Features of Macroscopic Models

Model	Features
Static macroscopic models	Being a constant model, static model is limited used in real cases.
Dynamic macroscopic models	<ul style="list-style-type: none"> ● Being different from common fluid, the velocity inversely proportional to the density in traffic flow is an inexplicable phenomenon in the dynamic conservation equation. ● Dynamic models are only suitable for the crowded, equilibrium and stable traffic flow.
	<ul style="list-style-type: none"> ● The model assumes the relationship between velocity-density in equilibrium state. ● It can simulate the form of traffic shock and the dissipation of congestion. But it can't simulate the traffic flow in non-equilibrium state. ● There contains no relaxation time. ● These models assume a dynamic relationship between traffic velocity-density. ● It can simulate phenomena like the stop-and-go and the propagation of nonlinear waves. ● There contain relaxation time.
	<ul style="list-style-type: none"> First-order continuum model Second-order continuum models

2.3. Mesoscopic Models

Mesoscopic models include gas-kinetic models and hybrid models. These models usually combine different models among macroscopic and microscopic models. Mesoscopic models describe single vehicle's dynamic response to the variation of the traffic flow density (flow rate or velocity). These models treat traffic flow as 'platoons' formed by a set of nearby

vehicles to describe the behavior of the inflow and outflow of each platoon. The description of the movement of the vehicles in these models is similar with them in macroscopic models, which means vehicles in the same platoon have a same speed. The classification of mesoscopic models is shown in Figure 3.

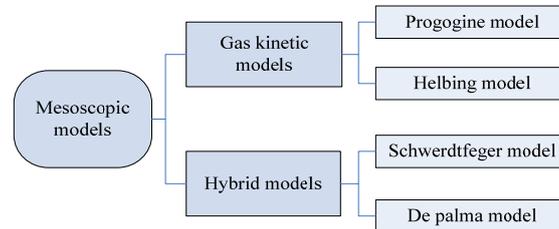


Figure 3. Classification Figure of Mesoscopic Models

2.4. Gas Kinetic Models

Prigogine model: Prigogine-Herman model proposed the first mesoscopic model of traffic flow^[15] in 1971. Such model deduces that the LWR model is a limitation case according to its kinetic theory. The model uses a partial differential equation to express the spatio-temporal evolution of the velocity and density of vehicles. Then they import an approximation relation to close the Boltzmann equation in order to obtain the model equation. In a historical view, the gas kinetic model contributes on the basis of theoretical derivation of the macroscopic equation.

The conservation equation contains the relaxation term $\left(\frac{\partial f}{\partial t}\right)_{rel}$ and the interaction term $\left(\frac{\partial f}{\partial t}\right)_{int}$:

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x} \left(f \frac{dx}{dt} \right) + \frac{\partial}{\partial v} \left(f \frac{dv}{dt} \right) = \left(\frac{\partial f}{\partial t} \right)_{rel} + \left(\frac{\partial f}{\partial t} \right)_{int} \quad (15)$$

If the quantity of vehicles remains the same, which means there has no other entrances and exits. The conservation equation is:

$$\frac{\partial f}{\partial t} = - \frac{\partial}{\partial x} \left(f \frac{dx}{dt} \right) - \frac{\partial}{\partial v} \left(f \frac{dv}{dt} \right) \quad (16)$$

That means the location x and the velocity v determine vehicle's state in this model.

Fontana model: Fontana [16] extended the Prigogine model in 1975. He assumes that all vehicles have their individual expected velocity. The location x , velocity v and expected velocity v_0 determine the state of the vehicle. The governing equation is as follow:

$$\frac{\partial f}{\partial t} + \frac{\partial(fv)}{\partial x} + \frac{\partial}{\partial v} \left(f \frac{dv}{dt} \right) + \frac{\partial}{\partial v_0} \left(f \frac{dv_0}{dt} \right) = \left(\frac{\partial f}{\partial t} \right)_{tr} \quad (17)$$

Helbing model: The gas kinetic based models had no further improvement due to the mathematical difficulty of it's gas kinetic frame until Helbing proposed his model [17] in 1995. Helbing bings in the interaction between the acceleration of vehicle N and $N + 1$ to form the kinetic-based continuum model, a so-called Helbing model. He defines P as the traffic pressure, $v_e(\rho)$ as the dynamic equilibrium velocity in his model:

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\tau} [v_e(\rho) - v] \quad (18)$$

The well-known traffic software MASTER adopts Helbing model for the advantages of its fast computation and strong robustness, etc. This model is able to simulate stop-and-go wave and nonlinear dynamic phenomenon such as congestion of synchronization, etc.

Hoogendoorn and Bovy proposed a generalized gas kinetic traffic flow model [18] in 1999, which became an unitive framework of mesoscopic models.

2.5. Hybrid Models

Hybrid models are usually the combination models mixed with macroscopic, mesoscopic and microscopic models. For example, Burghout presents a hybrid mesoscopic-microscopic model that applies microscopic simulation to areas of specific interest while simulation a large surrounding network in less detail with a mesoscopic model [19]. McCrea presents a hybrid approach combined the complementary features and capabilities of both continuum mathematical models and knowledge-based models to describe effectively traffic flow in road networks [20]. Depalma mixes microscopic method with macroscopic method [21]. In his model, movements of vehicles are modeled in macroscopic way with the policy of vehicles modeled in microscopic way. Depalma model and Schwerdtfeger model [22] are adopted by software METROPOLIS and DYNEMO.

2.6. Microscopic Models

We usually call microscopic models the entity-based models. These models focus on the individual vehicles' modeling to describe their movements and interactions. These models attempt to describe the overall characteristics of the system by integrating the characteristic of each individual vehicle. That is, each vehicle gathers information of surrounding ones and then generates its own driving strategy to forms the actual traffic flow. Vehicle's individual behaviors such like car-following, lane-changing and overtaking can actual reflected in these model. Being different from macroscopic models, microscopic models do not consider about the specific situation of the features of traffic flow, such as traffic flow rate, density and velocity.

Macroscopic models contain car-following models, sub-microscopic models and particle hopping models (also known as cellular automaton models). Microscopic models' classification is shown in Figure 4.

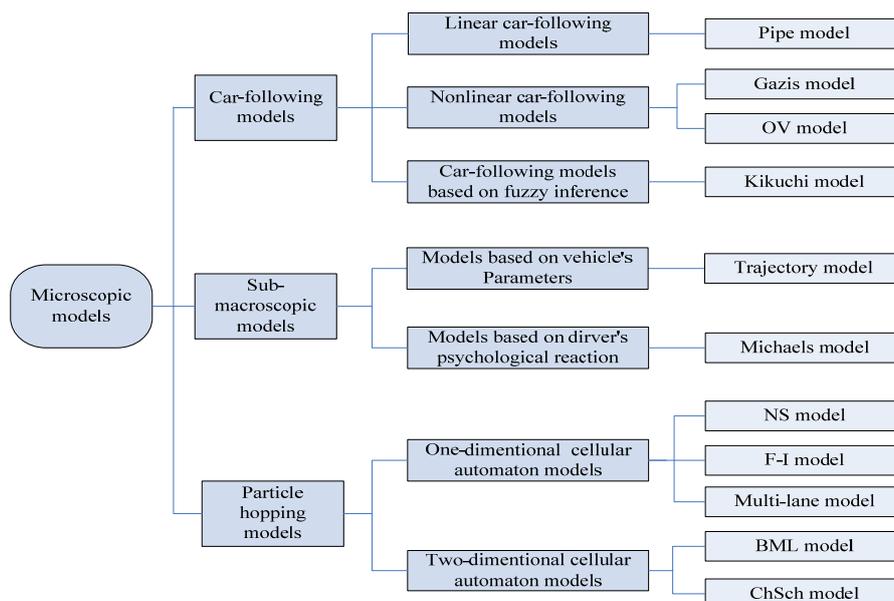


Figure 4. Classification Figure of Microscopic Models

2.7. Car-following Models

Car-following models aim to study the propagating of the traffic flow on a single lane. Pipes proposed the earliest car-following model and its theory in 1953 [23]. It uses mathematical model to analyze its dynamic theory in order to simulate vehicle's following behavior on a single lane. Such models research on the states of traffic flow mainly in the synchronized flow of traffic, which are defined in three-phase traffic theory. The synchronized flow has characteristics including conditionality, retardance and transitivity, which makes the state of traffic propagating backward intermittently and continuously like pulse do. These models are "stimulus-response" models that research on vehicle's following behaviors in synchronized flow by analyzing driver's responses to different stimulations. The form is: car-following response = sensitivity \times stimulus.

Car-following models contain linear car-following models, nonlinear car-following models and car-following models based on fuzzy inference system.

2.7.1. Linear car-following Models

Pipes model, the earliest proposed car-following model, is a representative linear car-following model introduced in our paper.

Pipes model : The model defines $s(t)$ as the distance between vehicle N and $N+1$ that two vehicles won't crash when vehicle n breaks; T as the reaction time, during which the velocity of vehicle $N+1$ does not change. The schematic diagram of Pipes model is shown in Figure 5:

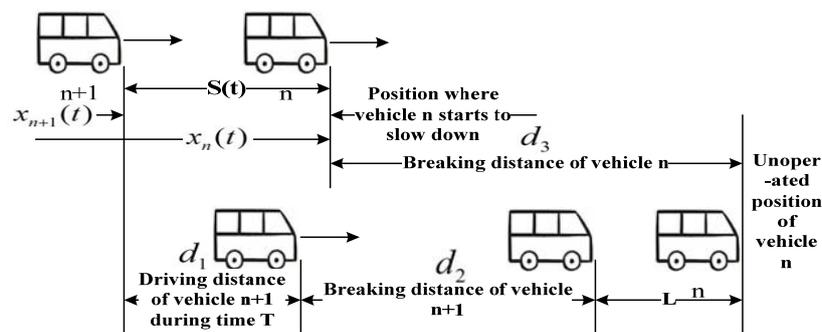


Figure 5. Schematic Diagram of Pipes Model

The model assumes the breaking distance of vehicle n d_3 equals to the breaking distance of vehicle $n+1$ d_2 . The gap between two vehicles is:

$$s(t) = x_n(t) - x_{n+1}(t) = T\dot{x}_{n+1}(t+T) + L$$

The type t differential, we can get $\ddot{x}_{n+1}(t+T)$ and $\dot{x}_n(t) - \dot{x}_{n+1}(t)$, separately the acceleration (reaction) of vehicle $n+1$ at time $t+T$ and the velocity difference between vehicle n and $n+1$ at time t :

$$\ddot{x}_{n+1}(t+T) = \frac{1}{T} [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \quad (19)$$

We notice that the reaction of vehicle $n+1$ is proportional to the velocity difference between n and $n+1$ at time t . Thus we name the model linear car-following model.

The equation $d_2 = d_3$ in the model is assumed, which does not exist in physical truth. Hence, researchers adopt the reaction coefficient λ to replace the sensitivity $\frac{1}{T}$ and then form the general type of linear car-following models:

$$\ddot{x}_{n+1}(t+T) = \lambda [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \quad (20)$$

That is: car-following reaction = sensitivity(or reaction coefficient) \times stimulus.

There are various reaction coefficients λ assumed in different models. Some models assume λ as a constant (e. g., $\lambda = a$). Some models assume λ in distribution function such as:

$$\lambda = \begin{cases} a, & d_1 + d_2 > c \\ b, & d_1 + d_2 < c \end{cases} \quad (21)$$

a, b, c is constant.

2.7.2. Non-linear Car-following Models

Nonlinear car-following models contain models improved based on linear car-following models such as Gazis model, OV model, etc.

Gazis model: The assumption that the acceleration (reaction) of vehicle $n+1$ in Pipes model relates only with two vehicles' relative velocity which is not agreed with Gazis. Thus, he proposed a nonlinear car-following model in which the reaction coefficient λ was inversely proportional to the gap between two vehicles in 1959 [24]. Gazis defines that a as the proportionality coefficient which is proportional to the critical velocity of traffic flow V_m and inversely proportional to the gap of two vehicles $V_f : a = V_m = \frac{1}{2} V_f$. Gazis model is shown as follow:

$$\ddot{x}_{n+1}(t+T) = \frac{a}{x_n(t) - x_{n+1}(t)} [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \quad (22)$$

Gazis proposes the general form of car-following models in his subsequent research [25]. He defines $\frac{x_{n+1}^m(t+T)}{[x_n(t) - x_{n+1}(t)]^l}$ as the sensitivity, m, l are constants.

$$\ddot{x}_{n+1}(t+T) = a \frac{x_{n+1}^m(t+T)}{[x_n(t) - x_{n+1}(t)]^l} [\dot{x}_n(t) - \dot{x}_{n+1}(t)] \quad (24)$$

It is the nonlinear model formula when $m = 0$ and $l = 1$; Yet it is the general form of linear model when $m = 0$ and $l = 0$.

Gipps model (Safety-distance model): Gipps model assumes a safety distance that vehicles always keep in their following behavior to avoid crashing. Thus the model is known as the car-following model based on safety distance^[26]. The original form of the model is expressed in differential equations of basic Newtonian motion but not in the form of stimulus-response:

$$x_n(t) - x_{n+1}(t) = \alpha \dot{x}_n^2(t) + \beta_l \dot{x}_{n+1}^2(t+T) + \beta \dot{x}_{n+1}(t+T) + b_0 \quad (25)$$

Gipps imports some factors that other model omitted before, including driver's extra safety reaction time $T/2$, the maximum possible breaking probability of vehicles, etc. The model can simulate the propagating interference of vehicles. However, the assumption that drivers must keep a safety distance is not a rule that drivers must abide by in real cases.

OV model (optimization speed model): The optimization speed model was presented by Bando in 1995. The model assumes that drivers should adjust their velocity dynamically according to the gap between him and his forward vehicle [27]. The model defines $V(\Delta x_n)$ as the optimization speed determined by the gap between two vehicles Δx_n ; κ as the sensitivity coefficient. The model is shown as follow:

$$\ddot{x}_n(t) = \kappa [V(\Delta x_n) - \dot{x}_n(t)] \quad (26)$$

The outcome appears that the resultant acceleration and deceleration does not conform to actual situation according to the simulating with field data executed by Helbing.

Generalized force model: To solve the problem of the unrealistic acceleration appeared in the OV model, Helbing presents the generalized force model [28]. The model imports the effect of the velocity-difference between two vehicles while the forward one is slower than the following one. He defines H as the heaviside-funtion, λ and κ as different sensitivity coefficients. The model is shown as follow:

$$\ddot{x}_n(t) = \kappa [V(\Delta x_n) - \dot{x}_n(t)] - \lambda H [\dot{x}_n(t) - \dot{x}_{n+1}(t)]^2 \quad (27)$$

Besides, there are other non-linear car-following models: Jiang proposes his full-velocity-difference model (FVD) which considers the effect of the velocity-difference of two vehicles in any situation [29]. Peng improves FVD model by adding multi-vehicles' relationship and forms multi-car-following model [30]. Shamoto presents a new car-following model based on relative velocity and velocity of forward vehicle. The acceleration is assumed infinitely in order to simulate metastable critical density of homogeneous flow [31]. Tang develops a new car-following model with the consideration of the driver's forecast effect and the analytical and numerical results shows that the stability of traffic flow is enhanced with the increase of the forecast effect coefficient and the forecast time [32]. Farah researches on the driving method combination with special instrument in order to reduce error in subjective judgment. And it is proved that co-operative system has appositive impact on drivers' car-following behavior [33]. Feng proposes a non-linear traffic flow time sequence prediction model aiming at the periodic and stochastic characteristics of the traffic flow [71].

2.7.3. Car-following Models Based on Fuzzy Inference

Car-following model based on fuzzy inference can accurately represent people's driving behavior in his next logical step. The model divides inputs into several fuzzy subsets which describe how adequately a variable closes to a condition. For example, a subset describes and quantifies a condition 'too close'. They assume the distance for example below 0.5m as 'too close' with a membership of 1. Otherwise, the distance above 0.5m is given a membership of 0. Once defined, they can obtain the fuzzy output subsets according to the logical operation on input subsets (e.g., IF 'close' AND 'closing' Then 'brake'). All possible outcomes can be figured out by evaluating the output sets with actual cases.

Kikuchi and Chakroborty presented an original form of the car-following model based on fuzzy inference in 1992 [34]. The model fuzzifies inputs like gap, velocity difference and forward vehicle's acceleration into several subsets. They figure out memberships to estimate the belonging subsets of inputs according to experience and probability statistics. They firstly fuzzify the memberships of the input subsets, and then fuzzy infer them in fuzzy control rules to obtain the fuzzy output (acceleration of following vehicle). The fuzzy rules is shown as follow:

$$\begin{aligned} \text{If } \Delta x = \text{'adequate'} \\ a_n = (\Delta v + a_{n-1}xT) / \gamma \end{aligned} \quad (28)$$

Chakroborty and Kikuchi improved their model's efficiency and veracity by using look-up table [35]:

$$\begin{aligned} & \text{If } \Delta x \in A_i \ \& \ \Delta v \in B_i \ \& \ a_{n-1} \in C_i \\ & a_n \in D_i \end{aligned} \quad (29)$$

Chakroborty and Kikuchi completed their fuzzy model by calibrating the membership function of the fuzzy inference system with field data in their later work [36].

In recent years, researches on fuzzy model contains as below: Qipeng Xiong employs least squares to fit discrete data to obtain membership functions [37]. Khodayari and Alireza presents a car-following model developed in adaptive neuro fuzzy inference system to simulate and predict the future behavior of a driver-vehicle-unit. The model can be recruited in drier assistant device and other ITS applications [38]. Then they propose a model developed based on a new idea for estimation the instantaneous reaction of a driver-vehicle-unit, which is used as an input of fuzzy model. And the result shows that fuzzy model based on instantaneous reaction delay outperformed the other car-following models [39].

Car-following models are the earliest models widely used and researched. Researches on car-following models are trend to combination models with sub-microscopic models. Some features of car-following models are listed in Table 2:

Table 2. Features of Car-following Models

models	Features
Linear car-following models	<ul style="list-style-type: none"> ● The velocity difference of two vehicles determines the reaction of the following one. ● Such models are suitable for the one-lane and non-free traffic flow in high density. ● These models are the deterministic models. ● The sampling of parameters influenced by many factor is difficult to obtain, which makes the models non-universality.
Nonlinear car-following models	<ul style="list-style-type: none"> ● The reaction of the following vehicle is not only determined by velocity difference of two vehicles, but also determined by parameters such like the gap and safety distance between vehicles. ● Other features of these models are similar with the linear car-following models.
Car-following models based on fuzzy inference	<ul style="list-style-type: none"> ● These models obtain fuzzy outputs (reaction) from estimating the membership functions. ● Such models are combined with sub-microscopic models closely. ● The measurement of the membership functions is difficult. ● Multi-lane is supported in latest fuzzy-logical models. ● These models are stochastic models which make the simulation of the traffic phenomena much better.

2.8. Sub-microscopic Models

Sub-microscopic models are more detailed than microscopic models. These models contain 2 categories: One category focuses on the human psychology such as the driver's behavior indicator and the threshold of driver's reaction to traffic lights and surrounding, etc. The other category concentrates on the performance of vehicles, such as the shifting of vehicles, trajectories, performance of different vehicles and vehicle's parameters like acceleration/braking curve, etc.

Michaels defines that the following vehicle's behavior thresholds are determined by the visual estimated relative velocity in his sub-microscopic model [40]. Then a model based on distance threshold appears [41]. After that, a series of experiments based on physiological perception lead out the physiological-perception-based models [42, 43] (e.g., the driver's perception based model [44]).

The Curves of thresholds in the model divide the car-following process into 4 states according to different relationships between the relative velocity and distance of vehicles. The section graph is shown in Figure 6.

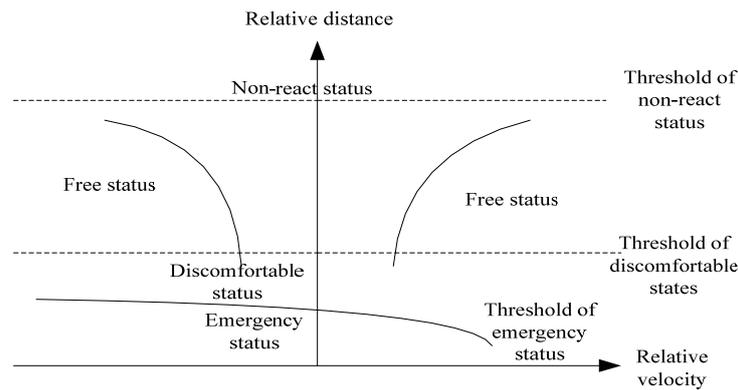


Figure 6. Section Graph of Driver's Mental Model

The problem of sub-microscopic models is that the final conclusion has yet been reached on the effectiveness of this matter. Although these models can simulate driver's behavior and describe most of the driver's characteristics, the measurement and calibration of the threshold is the key problem need to be solved.

2.9. Cellular Automaton Models (particle hopping models)

The cellular automaton (CA) is a formation of cells, grids, neighborhoods and update rules. It is a discrete, separate and space-diffusible system formed by a large number of simple, consistent and correlation partial entities. The ideal cellular automaton models are not only discretely in time and space, but also limited discretely in their belonging states. Cells that formed the CA models are localized interaction in space and causality in time. Being different from normal dynamic models, CA models are grid dynamic models contain a series model construction rules without definite equation forms.

Being equal to the logical model Turing machine in theory, CA models are especially good at parallel processing, and in principle they are able to computing any tasks [45].

2.9.1. One-dimensional Cellular Automaton Models

One-dimensional cellular automaton models mainly study on the interaction of vehicles on one-dimension roads. The major one-dimensional cellular automaton models include: CA-184 the simplest form CA model, the NS model, the F-I model and other models improved based on the NS model such as the slow-start model, the cruise control model and multi-lane models, etc.

CA-184 model (The deterministic CA model): The deterministic CA model is the basic form of CA models. The model's rule is as follow:

Let d_n be the gap between vehicle n and $n+1$ at time t , v_n be the velocity of vehicle n at time t , v_{\max} be the max velocity of all vehicles.

1. Acceleration: $v_n \rightarrow \min(v_n + 1, v_{\max}), v_n < d_n$;
2. Deceleration: $v_n \rightarrow d_n, v_n > d_n$;
3. Propagation: $x_n \rightarrow x_n + v_n$;

The model is equivalent to the CA-184 model named by Wolfram [46] while $v_{\max} = 1$. The schematic diagram of CA-184 model is shown in Figure 7, t is the time step; the number above each vehicle in the picture is the current velocity of this vehicle:

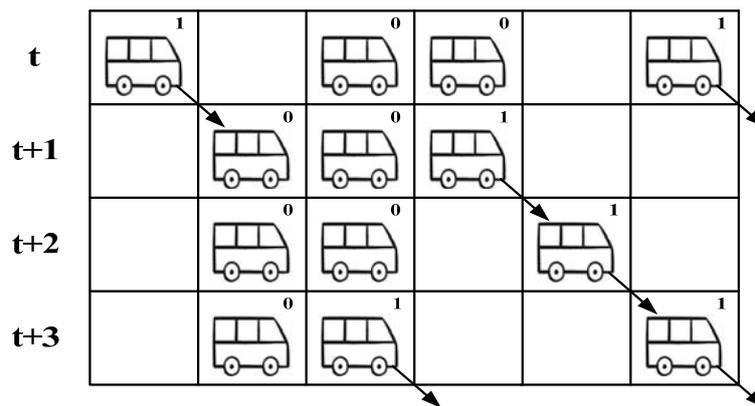


Figure 7. Schematic Diagram of CA-184 Model

Model's rule is as follow:

1. $v_n \rightarrow 1, d_n \geq 1$ and $v_n = 1$, that means vehicles maintain their highest speed 1.
2. $v_n \rightarrow 1, d_n > 2$ and $v_n = 0$, that means vehicles only start if and only if the distance from the vehicle ahead is equal to or greater than 2.

It is not only the basis of many CA-based models, but also the basis of two dimensional cellular automaton models.

N-S model : The NS model is an important single-lane cellular automaton model which adds random deceleration rules to the deterministic CA model [47]. NS model defines rulls as follow:

Let x_n and v_n respectively be the location and velocity of vehicle n . The model defines v_n as an integer from $0 \rightarrow v_{max}$ and d_n as the gap between vehicle n and its forward vehicle $n+1$. The model stores the complete configuration at time t , and compute the configuration at time $t+1$ in parallel. The Schematic diagram of N-S model is shown in Figure 8:

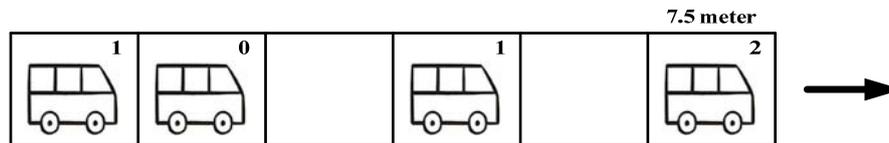


Figure 8. Schematic Diagram of N-S Model - The length of a grid is 7.5 m and figure on top right corner refer to velocity on that state

The rules of this model are as follow:

1. Acceleration: $v_n \rightarrow \min(v_n + 1, v_{max}), v_n < d_n$;
2. Deceleration: $v_n \rightarrow d_n, v_n > d_n$;
3. Randomization: $v_n \rightarrow \max(v_n - 1, 0), 0 < P < 1 \text{ and } v_n > 0$, P is the deceleration probability;
4. Propagation: $x_n \rightarrow x_n + v_n$;

The model chooses a randomization rule to simulate phenomena such as: speed fluctuation on free flow; overreaction to braking and the delay of acceleration.

Cruise control limit NS model (N-S-CC): Such model [48] assumes that vehicles reach their maximum speed without affecting by any other vehicles should maintain their highest speed. That means there's no speed fluctuation in that condition, the randomization tends to 0.

The model define that P and $P_{v_{\max}}$ are both the deceleration probabilities. The deceleration and randomization rule are as follows:

$$v_n \rightarrow \max(v_n - 1, 0), P = \begin{cases} P_{v_{\max}}, & \text{If } v = v_{\max} \\ P, & \text{If } v < v_{\max} \end{cases}, P_{v_{\max}} \rightarrow 0, 0 < P < 1 \quad (30)$$

Slow-start model: The slow-start model is a NS-based model extended only on the rules of vehicle's start. In example, a start probability ρ is set in the condition that vehicles only start when there are at least 2 blank grids ahead. That means vehicles have a probability $1 - \rho$ to start when there is only one space ahead. Slow-start model is designed to simulate traffic phenomena caused by different start policies of different drivers.

TT slow-start N-S model: The TT slow-start N-S model attempts to simulate the traffic phenomena that some drivers are unwilling to start when there's no 'enough' space ahead [49].

P_d is the start probability, and the slow-start rule is as follow:

$$v_n \rightarrow \min(v_n + 1, v_{\max}), P = \begin{cases} 1 - p_d, & \text{If } d_n = 1 \\ p_d, & \text{If } d_n > 1 \end{cases}, v_n = 0, 0 < p_d < 1 \quad (30)$$

BJH slow-start model: This model defines that only vehicles affected to stop should abide by the slow-start rule [50]. In example, when vehicle n is forced to stop by vehicle $n + 1$, its start probability is P at time-step t with his probability is 1 at time-step $t + 1$.

VDR slow-start model: The VDR model assumes that there is a relationship between the vehicle's velocity and a random deceleration parameter [51]:

$$p = p(v_n)$$

That is to represent the static delay of vehicle's start by function of random deceleration parameter and velocity.

F-I model: This model allows vehicles to accelerate to the maximum speed instantaneously [52].

The acceleration/deceleration rule is as follow:

$$v_n \rightarrow \min(d_n - 1, v_{\max})$$

The feature of F-I model makes it not able to simulate stop-and-go phenomenon. However, the significance of the model is the theoretical research value of its exact solution.

Velocity-effect model (VE): The velocity-effect model is to solve problems that the resultant simulation data is different from resultant field data caused by the velocity-effect of vehicle ahead [53]. The acceleration/deceleration rule of The N-S model is modified to simulate that effect:

$$\text{Acceleration/Deceleration: } v_n \rightarrow \min(v_{\max}, v_{n+1} + 1, d_n + v'_n),$$

$$\text{And } v'_n = \min(v_{\max} - 1, v_n, \max[0, d_n - 1])$$

Other rules of the model are similar with them in the N-S model.

Multi-lane N-S model: The lane changing rules in multi-lane model is mainly based on the N-S model. The model considers a symmetry case and a asymmetry case according to the difference between lanes (fast/slow lanes) and vehicles (small/large vehicles) [54].

The model firstly executes the lane changing, and then simulates vehicle's propagate under one-lane rules. Chowdhury presents his lane changing rules as follows [55]. $gap(n)$ is the distance between vehicle n and its forward one on the same lane: $gap_o(n)$ is the distance

between vehicle n and its forward one on the other lane; $gap_{o,back}(n)$ is the distance between vehicle n and its following one on the other lane; l is a distance constant; $rand()$ is a random number between $(0,1)$, p_c is the lane changing probability:

$$\begin{aligned} gap(n) &< l, \\ gap_o(n) &> l_o, \\ gap_{o,back}(n) &> l_{o,back}, \\ rand() &< p_c \end{aligned} \quad (32)$$

Asymmetric multi-lane model generates different Lane changing rules according to the difference between lanes and vehicles themselves. The schematic diagram of the multi-lane model is shown in Figure 9:

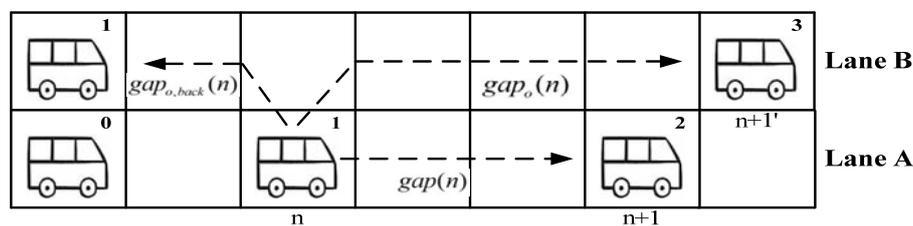


Figure 9. Schematic Diagram of Multi-lane Model

Correlational researches on one-dimensional CA models in recent years are as follow: Kerner uses simple cellular automaton model and three-phase traffic model to analyze the movement and contraction effect of synchronized flow [56]. Bin Jia presented a model that randomization effect is enhanced in with the decrease of time gap and the long time stopped vehicle has large randomization probability [57]. Jin adjusts the anticipated velocity and the acceleration threshold through parameters, and conclude that the acceleration threshold is the major factor affection the F to S phase transition [58]. Chu presents an improved cellular automaton model for symmetric two-lane traffic, which incorporates anticipation effects, sensitive driving technique and information interaction synthetically [59].

2.9.2. Two-dimensional Cellular Automaton Models

The appearance of two-dimensional cellular automaton models has authentic practical significance because they can exactly describe the realistic road traffic. Two-dimensional cellular automaton models mainly includes their fundamental form the BML model and the ChSch model which models road in cells.

BML model: The BML model [60] defines a square grids in $N \times N$ size with each grid being able to contain one vehicle in east-west or south-north direction. The model random distributes vehicles and customize the number of vehicles. The model defines rules as below: vehicles in east-west direction move 1 step in odd time. Vehicles in south-north direction move 1 step in even time. Vehicles do not move when there is a vehicle in grid ahead. That means the model considers every grid a junction with traffic light which phase changes in a time step. The traffic densities are the number of vehicles divides the number of grids respectively in that direction. There are 2 density-related phase transition points according to the three-phase traffic theory. One is the transition from the free phase to the synchronize phase, the other is the transition from the synchronize phase to the wild moving jam phase.

The BML model is wildly researched in two directions. One is to improve the BML model by extending in actual traffic factors in order to make it closer to the actual situation (e.g., ChSch model). The other focuses on the causes and the regularity of the phase transformation based on three-phase traffic theory.

ChSch model: The ChSch model has more practical significance than the BML model because it abstracts the road in grids. Such model divides road in grids and model the road similar with the NS model does [61]. The model defines each road intersection is a junction with traffic lights changed in period T . Vehicle's velocity related only with the status of forward grids and traffic lights. The schematic diagram of the ChSch model is shown in Figure 10:

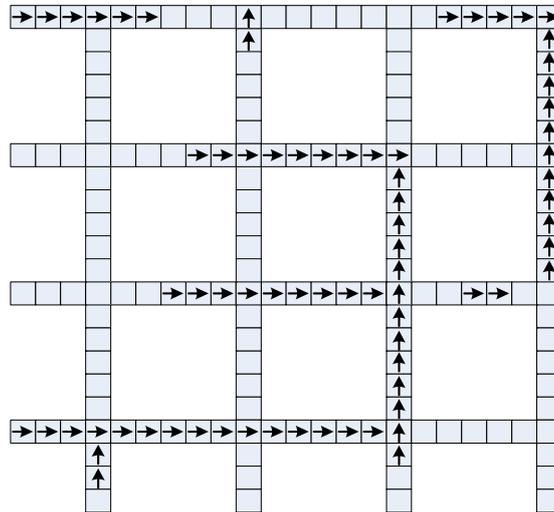


Figure 10. Schematic Diagram of ChSch Model

The model defines that d_n is the gap between vehicle n and $n+1$, s_n is the distance between vehicle n and the intersection ahead, T_{signal} is the state of signal, τ is the time that till lights turned to red. The deceleration process indicates that vehicle's velocity is depended on the time whether he could pass the intersection during green light. The rules of ChSch model are as follow:

1. Acceleration: $v_n \rightarrow \min(v_n + 1, v_{max})$;
2. Deceleration:

$$v_n \rightarrow \begin{cases} \min(v_n, d_n - 1, s_n), T_{signal} = Red \\ \min(v_n, d_n - 1), d_n < s_n, T_{signal} = Green \\ \min(v_n, d_n - 1), d_n > s_n, T_{signal} = Green, \text{ and } \min(v_n, d_n - 1) \times \tau > s_n \\ \min(v_n, s_n - 1), d_n > s_n, T_{signal} = Green, \text{ and } \min(v_n, d_n - 1) \times \tau < s_n \end{cases} ;$$

3. Randomization: $v_n \rightarrow \max(v_n - 1, 0)$, $0 < P < 1$ and $v_n > 0$;

4. Propagation:
$$\begin{cases} x_n = x_n + v_n \\ y_n = y_n + v_n \end{cases} ;$$

Scholars conduct a lot of researches on the basis of the BML and ChSch model in recent years: Sun studies on the behavior that traffic flow varying with traffic lights by stages under free flow and wild moving jam [62]. Ding studies on the driving behavior that ignores traffic lights in BML model. The result shows that the violator increases the average velocity of free flowing phase while decreases the critical car density [63]. Ding also studies on phase transformation under boundary condition in a stochastic version of the BML model with random update rule [64] as well as the effect to traffic flow caused by configuration and quantity of bridges in BML model [65]. Zhao studies on the reason and solution of increasing phase

transformation from free flow to wild moving jam [66]. Fukui researches on the changing phases caused by intermittent block of the road [67]. Ding develops a mean-field theory which successfully predicts the average velocity in moving phase. And the dependence of the average velocity, the density and the flow rate on the injection probability in the moving phase have also been obtained through the mean-field theory [68]. Sui studies on the effect that slow-start probability and parameter of traffic lights influence to the phase of traffic [69]. Yang builds a mixed traffic flow model considering the transit based on BML model [70].

3. Conclusion

We can clearly see the applicability, advantages and disadvantages of macroscopic, microscopic and mesoscopic models by classifying them and analyzing their features. Macroscopic models focus on the relationship among the flow rate, velocity and density of the whole traffic. It is much more suitable for steady and homogeneous traffic flow compared with free or intermittent ones. Macroscopic models are better in computational efficiency because they have nothing to do with the quantity of vehicles. So they are appropriated for large-scale modeling in computer applications. The Gas-kinetic model, an important mesoscopic model, has fulfilled the theoretical gap between macroscopic fluid-dynamic theory and microscopic car-following theory with its gas kinetic theory. Its modeling method has already been adopted in much simulation software. A difficulty to overcome is the error of the measurement and the complexity of calculation on the undetermined parameters and the relationship equations in Gas-kinetic theory. Microscopic models are widely studied due to the characteristics of the random model can essentially simulate the complex, dynamic, strong randomness traffic system. Cellular automaton models are still a research hotspot for their features. They discretize time-space and state, and replicate macro phenomena through simple rules. Two-dimensional cellular automaton models are more suitable for road traffic modeling that makes it in a great research value. The phase transition of three-phase traffic theory and sub-microscopic models based on driver's psychological reaction are also widely researched. Adopting microscopic models is more accurate than choosing macroscopic and mesoscopic models in computer simulation. But the problem of adopting microscopic models is the bottom-up structure and the strict time-synchronization requirements, which makes the large-scale simulation hard to achieve.

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