

Robust Controller Design for Networked Control Systems Based on State Estimation

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Abstract

For the networked control systems with uncertain time delay in both sensor-to-controller and controller-to-actuator channels, and data dropouts in sensor-to-controller channel, a new model of networked control systems based on state estimation is proposed using single-exponential smoothing method to predict the state variables. After adopting Lyapunov stability theory, the asymptotic stability for the system has been proved, and the Robust Controller has been designed by using linear matrix inequality. In the simulation experiment, comparing the state responses under different situations of diverse time delay and data dropouts, the results show that the method is effective.

Keywords: single-exponential smoothing method, state estimation, Lyapunov function, linear matrix inequality, robust controller

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1. Introduction

Networked control systems (NCSs) are distributed systems in which the communication between sensors, actuators, and controllers is supported by a shared real-time network [1-3]. Compared with the traditional point-to-point wiring, the use of the communication channels can reduce the costs of cables and power, simplify the installation and maintenance of the whole system, and improve the reliability [4-6]. But at the same time, time delays and packet dropouts will inevitably degrade the control performance of NCSs, or even cause the system to be unstable [7, 8], which complicate the analysis and comprehensiveness of the networked control systems. Therefore, time delay and data dropouts as the main problems of networked systems have received wide attention from many scholars [9].

Aiming at the networked control systems with time delay and data dropouts, in [10], when the sensor, the controller and the actuator are all clock driven, multi-rate NCS with both short time delay and packet dropouts is constructed as a switched system model. Based on the approaches of switched system and Lyapunov functions, the necessary conditions for asymptotical stability for multi-rate NCS with both short time delay and packet dropouts are given. In [11], a novel control law model is proposed to take the network-induced delay, random packet dropouts and packet-dropouts compensation into consideration simultaneously. By constructing a network-status-dependent Lyapunov function, a sufficient condition for the existence of the H_∞ output feedback controller is formulated in the form of nonconvex matrix inequality, and the cone-complementarity linearization (CCL) procedure is exploited to solve the nonconvex feasible problem. But, the network status is assumed to vary in a Markovian fashion satisfying a certain transition probability matrix. In [12], the paper is concerned with the H_∞ control issue for a class of networked control systems with packet dropouts and time-varying delays. The addressed NCS is modeled as a Markovian discrete-time switched system with two subsystems; by using the average dwell time method, a sufficient condition is obtained for the mean square exponential stability of the closed-loop NCS with a desired H_∞ disturbance attenuation level. The desired H_∞ controller is obtained by solving a set of linear matrix inequalities. In [13], the paper investigates the observer-based H_∞ fuzzy control problem for a class of discrete-time fuzzy mixed delay systems with random communication packet losses

and multiplicative noises, where the mixed delays comprise both discrete time-varying and distributed delays. The random packet losses are described by a Bernoulli distributed white sequence that obeys a conditional probability distribution, and the multiplicative disturbances are in the form of a scalar Gaussian white noise with unit variance. In the presence of mixed delays, random packet losses and multiplicative noises, sufficient conditions for the existence of an observer-based fuzzy feedback controller are derived, after that, a linear matrix inequality approach for designing such an observer-based H_∞ fuzzy controller is presented. In [14], the study is concerned with the optimal linear estimation problem for linear discrete-time stochastic systems with possible multiple random measurement delays and packet dropouts. The model is constructed to describe the phenomena of multiple random delays and packet dropouts by employing some random variables of Bernoulli distribution. By state augmentation, the system with random delays and packet dropouts is transferred to a system with random parameters. The estimators are recursively computed in terms of the solutions of a Riccati difference equation and a Lyapunov difference equation. In [15], Aiming at the networked systems with sensor delays, missing measurements and packet dropouts, a design method about adaptive Kalman filter is proposed. Two different adaptive filters are considered to estimate unknown parameter vector associated with the system matrices, then, the estimation of state and parameters of the system based on the minimization of square of the output prediction error is adopted in bootstrap manner. An estimator-based robust controller design has been proposed for asymptotic stability of the system whose parameters can vary within a known bound.

It needs to point out, in the above references, the significant limitations have been found in the methods of controller design for the networked control systems with time delay with restraint that less than a sampling period. Meanwhile, it is difficult to achieve in practical project to create the conditions with time delay or data dropouts that reaches certain known stochastic distribution. However, the system design with time delay and packet dropouts only under the situation of sensor presence is lack of comprehensive factors. In this paper, based on the method of state estimation, a robust controller is designed for networked control systems which have uncertain time delay in both sensor-to-controller and controller-to-actuator channels, and data dropouts in sensor-to-controller channel. In this design, there are no restrictions with the length or distribution of time delay. Considering the complex situation of modeling uncertainty and external disturbances, the driving method of controller which is combined with event-driven and time-driven is adopted to design the robust controller based on state estimation.

2. Model of Control System Based on State Estimation

The model of networked control systems based on state estimation is shown in Figure 1. Sensor and controller are connected through communication network, so is the connection between controller and actuator. Here, τ_{sc} is the networked uncertain delay in sensor-to-controller channel, and τ_{ca} is the networked uncertain delay in controller-to-actuator channel. Compared with τ_{sc} or τ_{ca} , the networked delay in plant-to-sensor or actuator-to-plant channel is so short that it can be ignored. Therefore, the networked uncertain delay in the networked control systems is $\tau_k = \tau_{sc} + \tau_{ca}$. For convenience, the assumptions are as follow:

- (1) Sensor is time-driven, and the sampling time is T .
- (2) Controller adopts the compound driving mode. The controller is time-driven when it generates the data based on state estimation at every sampling period; the controller is event-driven when the data transmit from sensor to controller after the time delay of τ_{sc} .
- (3) The sequence disorder doesn't exist in the process of data transmission from sensor or controller.

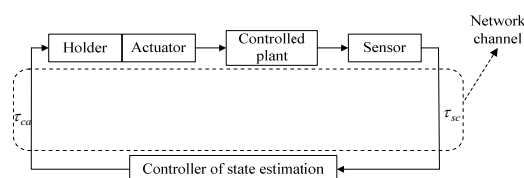


Figure 1. The Model of Networked Control Systems Based on State Estimation

The sequence diagrams of state estimation control system which exist time delay is shown in Figure 2. When the time delay τ_k is smaller than a constant of a period of sampling time, sensor samples data at kT time. Meanwhile, the output of controller is $u[x_c(k)]$, based on estimate state variables. The output of controller is $u[x(k)]$ based on sampling value $x(k)$ which is transferred to the controller after time delay τ_{sc} . $u[x_c(k)]$, the output of controller at kT time is transferred to actuator after time delay τ_{ca} . The output $u[x(k)]$ of controller at $kT + \tau_{sc}$ time is transferred to actuator after time delay τ_{ca} . Therefore, During a sampling period $[kT, (k+1)T)$, the control inputs of the control system can be divided into three parts respectively as following: (1) there is a networked time delay τ_{ca} in controller-to-actuator channel, so during $[kT, kT + \tau_{ca})$, the control input of the system is $u[x(k-1)]$. (2) During $[kT + \tau_{ca}, kT + \tau_k)$, the control input of the system is $u[x_c(k)]$. (3) During $[kT + \tau_k, (k+1)T)$, the control input of the system is $u[x(k)]$.

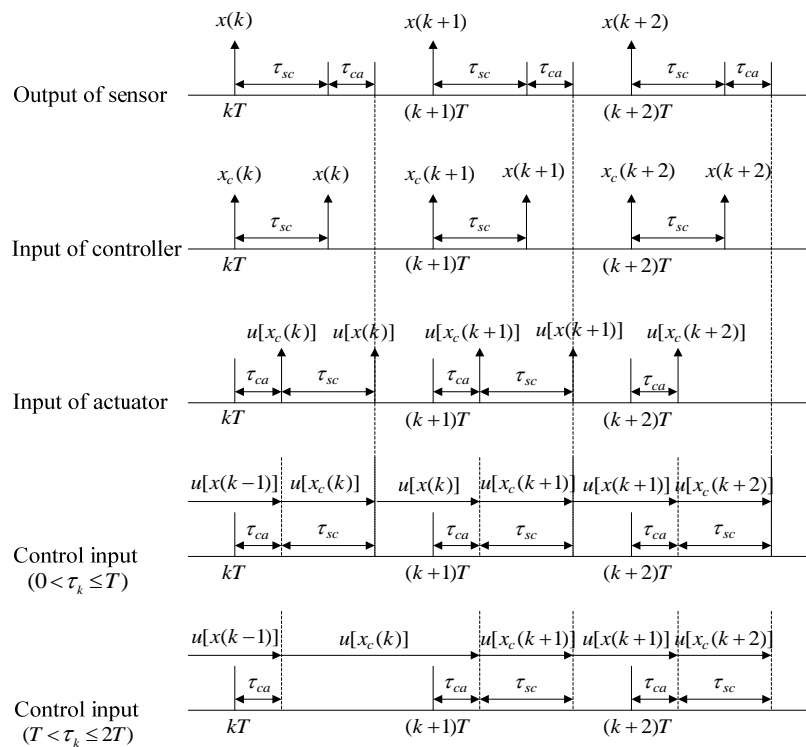


Figure 2. The Sequence Diagrams of State Estimation Control System which Exist Time Delay

During the sampling time of $[kT, (k+1)T)$, if the time delay τ_k is longer than one sampling period but shorter than or equal to two sampling periods, namely $T < \tau_k \leq 2T$, The situation is similar to the analysis of the time delay that τ_k is smaller than one sampling period. In this case, the control inputs of the control system can be divided into two parts respectively as following: (1) During $[kT, kT + \tau_{ca})$, the control input of the system is $u[x(k-1)]$. (2) During $[kT + \tau_{ca}, (k+1)T)$, the control input of the system is $u[x_c(k)]$.

When the time delay meets $nT < \tau_k$ (n is an integer larger than 1), by that analogy, the control inputs of the networked control systems can be obtained. When there are data packet

dropouts in sensor-to-controller channel, the same method is used to estimate the state variable which isn't transferred to the controller.

The prediction model based on state estimation is as follow:

$$\mathbf{x}_c(k+1) = \alpha \mathbf{x}_c(k) + (1-\alpha)x_\theta(k) \quad (1)$$

Where $\mathbf{x}_c(k+1)$ is the predicted value of state variable of control system at time $(k+1)T$, $\mathbf{x}_c(k)$ is the predicted value of state variable of control system at time kT , $x_\theta(k)$ is the actual input value of controller at time $[(k+1)T - \varepsilon_0]$ (where, $\varepsilon_0 \rightarrow 0$). That is, when time delay is smaller than a sampling time, $x(k)$ is considered as the controller input; when time delay is larger than a sampling time or data dropouts, $x_c(k)$ is considered as the controller input.

Firstly, considering about the model of general continuous system:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{z}(t) = \mathbf{C}\mathbf{x}(t) \end{cases} \quad (2)$$

Here $\mathbf{x}(t) \in \mathbf{R}^p$ is the state, $\mathbf{u}(t) \in \mathbf{R}^q$ is the control input, $\mathbf{z}(t) \in \mathbf{R}^m$ is the controlled signal output. $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are matrixes which have appropriate dimension, p, q, m are positive integers.

Considering about the time delay and data dropouts in the networked control systems, a continuous time state feedback law $\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t)$ is proposed. According to Sequence diagram 2, the Equation (2) is discretized into Equation (3).

$$\begin{cases} \mathbf{x}(k+1) = \tilde{\mathbf{A}}\mathbf{x}(k) + \hat{\mathbf{B}}_1\mathbf{K}\mathbf{x}(k-1) + \hat{\mathbf{B}}_2\mathbf{K}x_c(k) + \hat{\mathbf{B}}_0\mathbf{K}\mathbf{x}(k) \\ \mathbf{z}(k) = \mathbf{C}\mathbf{x}(k) \end{cases} \quad (3)$$

Where $\tilde{\mathbf{A}} = e^{\mathbf{A}T}$, $\hat{\mathbf{B}}_1 = \int_0^{\tau_{ca}} e^{\mathbf{A}s} \mathbf{B} ds$, $\hat{\mathbf{B}}_2 = \int_{\tau_{ca}}^{T-\tau_k} e^{\mathbf{A}s} \mathbf{B} ds$, $\hat{\mathbf{B}}_0 = \int_{\tau_k}^T e^{\mathbf{A}s} \mathbf{B} ds$. According to matrix theory, $\hat{\mathbf{B}}_1 = \tilde{\mathbf{B}}_1 + \mathbf{E}\mathbf{F}(\tau_k)\mathbf{H}_1$, $\hat{\mathbf{B}}_2 = \tilde{\mathbf{B}}_2 + \mathbf{E}\mathbf{F}(\tau_k)\mathbf{H}_2$, $\hat{\mathbf{B}}_0 = \tilde{\mathbf{B}}_0 + \mathbf{E}\mathbf{F}(\tau_k)\mathbf{H}_0$, $\mathbf{F}(\tau_k)$ is a time-dependent quantity based on uncertain time delay τ_k , and it meets $\mathbf{F}^T(\tau_k)\mathbf{F}(\tau_k) \leq \mathbf{I}$, $\mathbf{E}\mathbf{F}(\tau_k)\mathbf{H}_1 + \mathbf{E}\mathbf{F}(\tau_k)\mathbf{H}_2 + \mathbf{E}\mathbf{F}(\tau_k)\mathbf{H}_0 = \mathbf{0}$.

Consider modeling uncertainty and external disturbance in the networked control systems, then,

$$\begin{cases} \mathbf{x}(k+1) = \bar{\mathbf{A}}\mathbf{x}(k) + \bar{\mathbf{B}}_1\mathbf{K}\mathbf{x}(k-1) + \bar{\mathbf{B}}_2\mathbf{K}x_c(k) + \mathbf{F}_1\omega(k) \\ \mathbf{z}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{F}_2\omega(k) \end{cases} \quad (4)$$

Where $\bar{\mathbf{A}} = \tilde{\mathbf{A}} + \tilde{\mathbf{B}}_0\mathbf{K} + \Delta\mathbf{B}_0$, $\bar{\mathbf{B}}_1 = \tilde{\mathbf{B}}_1 + \Delta\mathbf{B}_1$, $\bar{\mathbf{B}}_2 = \tilde{\mathbf{B}}_2 + \Delta\mathbf{B}_2$. $\Delta\mathbf{B}_0 = \mathbf{E}\mathbf{F}(\tau_k)\mathbf{H}_0\mathbf{K} + \Delta\bar{\mathbf{B}}_0$, $\Delta\mathbf{B}_1 = \mathbf{E}\mathbf{F}(\tau_k)\mathbf{H}_1 + \Delta\bar{\mathbf{B}}_1$, $\Delta\mathbf{B}_2 = \mathbf{E}\mathbf{F}(\tau_k)\mathbf{H}_2 + \Delta\bar{\mathbf{B}}_2$ are the total modeling uncertainty and meets $\Delta\mathbf{B}_0 = \mathbf{G}\mathbf{F}\mathbf{E}_0$, $\Delta\mathbf{B}_1 = \mathbf{G}\mathbf{F}\mathbf{E}_1$, $\Delta\mathbf{B}_2 = \mathbf{G}\mathbf{F}\mathbf{E}_2$. $\mathbf{G} \in \mathbf{R}^{p \times l}$, $\mathbf{E}_0, \mathbf{E}_1, \mathbf{E}_2 \in \mathbf{R}^{l \times p}$ are known matrixes, $\mathbf{F} \in \mathbf{R}^{l \times l}$ is unknown matrix and meets $\mathbf{F}^T\mathbf{F} \leq \mathbf{I}$. $\Delta\bar{\mathbf{B}}_0$, $\Delta\bar{\mathbf{B}}_1$, $\Delta\bar{\mathbf{B}}_2$ are unknown model errors, $\omega(k) \in \mathbf{R}^r$ are the disturbance inputs, which belong to $\omega(k) \in L_2[0, \infty)$, where r is a positive integer.

Definition: Given γ as a positive constant, if the control system (4) with the uncertainty which meets $\mathbf{F}^T\mathbf{F} \leq \mathbf{I}$ has characters as follow: 1) System is asymptotically stable; 2) Under

zero initial condition, $\|z(k)\|_2 \leq \gamma \|w(k)\|_2$ for any $w(k) \in L_2[0, \infty)$. Then we call the system (4) has the performance γ of H_∞ . Where $\|z(k)\|_2 \leq \gamma \|w(k)\|_2$ responds the interference suppression capabilities for control system, so, the smaller of γ , the better of the system's performance.

3. Design of Robust Controller Based on State Estimation

Lemma [16]: Given matrices Y, H, E and R of appropriate dimensions and with Y and R symmetrical and $R > 0$, then:

$$Y + HFE + E^T F^T H^T < 0$$

For all F that satisfying $F^T F \leq R$, if and only if there exists some $\varepsilon > 0$ such that:

$$Y + \varepsilon^2 HH^T + \varepsilon^{-2} E^T R E < 0$$

Theorem: For a given $\gamma > 0$, the control system (4) has the performance γ of H_∞ if there are positive-definite matrices Q, S_1, S_2, M and constant $\varepsilon > 0$ such that.

$$\begin{bmatrix} -Q + \varepsilon GG^T + \tilde{B}_1 S_1 \tilde{B}_1^T & \tilde{A}Q + \tilde{B}_0 M & \tilde{B}_2 & 0 & F_1 & 0 & 0 & 0 \\ Q\tilde{A}^T + M^T \tilde{B}_0^T & -Q & 0 & 0 & 0 & C^T & M^T & QE_0^T \\ \tilde{B}_2^T & 0 & S_2 & 0 & 0 & 0 & 0 & E_2^T \\ 0 & 0 & 0 & -S_2 & 0 & 0 & 0 & 0 \\ F_1^T & 0 & 0 & 0 & -\gamma^2 I & F_2^T & 0 & 0 \\ 0 & CQ & 0 & 0 & F_2 & -I & 0 & 0 \\ 0 & KQ & 0 & 0 & 0 & 0 & -S_1 & 0 \\ 0 & E_0 Q & E_2 & 0 & 0 & 0 & 0 & -\varepsilon I + E_1 S_1 E_1^T \end{bmatrix} < 0$$

Proof: Firstly, a Lyapunov function is defined as:

$$V(k) = x^T(k)Px(k) + x^T(k-1)K^T R_1 Kx(k-1) + x_c^T(k-1)K^T R_2 Kx_c(k-1)$$

Where P, R_1, R_2 are real symmetry matrixes. Taking the difference of the Lyapunov function,

$$\begin{aligned} \Delta V(k) &= Vx(k+1) - Vx(k) \\ &= x^T(k+1)Px(k+1) - x^T(k)(P - K^T R_1 K)x(k) - x^T(k-1)K^T R_1 Kx(k-1) \\ &\quad + x_c^T(k)K^T R_2 Kx_c(k) - x_c^T(k-1)K^T R_2 Kx_c(k-1) \end{aligned} \quad (6)$$

When $w(k) = 0$,

$$\Delta V(k) = \begin{bmatrix} x(k) \\ Kx(k-1) \\ Kx_c(k) \\ Kx_c(k-1) \end{bmatrix}^T U_1 \begin{bmatrix} x(k) \\ Kx(k-1) \\ Kx_c(k) \\ Kx_c(k-1) \end{bmatrix} < 0 \quad (7)$$

Where U_1 is defined:

$$U_1 = \begin{bmatrix} \bar{A}^T P \bar{A} - P + K^T R_1 K & \bar{A} P \bar{B}_1 & \bar{A} P \bar{B}_2 & 0 \\ \bar{B}_1^T P \bar{A} & \bar{B}_1^T P \bar{B}_1 - R_1 & \bar{B}_1^T P \bar{B}_2 & 0 \\ \bar{B}_2^T P \bar{A} & \bar{B}_2^T P \bar{B}_1 & \bar{B}_2^T P \bar{B}_2 + R_2 & 0 \\ 0 & 0 & 0 & -R_2 \end{bmatrix}$$

When $w(k) \neq 0$,

$$\Delta V(k) = \begin{bmatrix} x(k) \\ Kx(k-1) \\ Kx_c(k) \\ Kx_c(k-1) \\ \omega(k) \end{bmatrix}^T U_2 \begin{bmatrix} x(k) \\ Kx(k-1) \\ Kx_c(k) \\ Kx_c(k-1) \\ \omega(k) \end{bmatrix} < 0 \quad (8)$$

Where U_2 is defined:

$$U_2 = \begin{bmatrix} \bar{A}^T P \bar{A} - P + K^T R_1 K & \bar{A}^T P \bar{B}_1 & \bar{A}^T P \bar{B}_2 & 0 & \bar{A}^T P F_1 \\ \bar{B}_1^T P \bar{A} & \bar{B}_1^T P \bar{B}_1 - R_1 & \bar{B}_1^T P \bar{B}_2 & 0 & \bar{B}_1^T P F_1 \\ \bar{B}_2^T P \bar{A} & \bar{B}_2^T P \bar{B}_1 & \bar{B}_2^T P \bar{B}_2 + R_2 & 0 & \bar{B}_2^T P F_1 \\ 0 & 0 & 0 & -R_2 & 0 \\ F_1^T P \bar{A} & F_1^T P \bar{B}_1 & F_1^T P \bar{B}_2 & 0 & F_1^T P F_1 \end{bmatrix}$$

When matrix inequality (8) is satisfied, closed-control system (4) is asymptotic stability. It's clear that if inequality is satisfied, the inequality (7) is certain true.

In the next place, assume the zero initial condition and let us introduce:

$$J = \sum_{k=0}^{\infty} [z^T(k)z(k) - \gamma^2 \omega^T(k)\omega(k)] \quad (9)$$

Where $\gamma > 0$ is constant, then:

$$J \leq \sum_{k=0}^{\infty} [z^T(k)z(k) - \gamma^2 \omega^T(k)\omega(k) + \Delta V(k)] \quad (10)$$

Plugging inequality(8) and equality(9) into the inequality (10).

$$J \leq \begin{bmatrix} x(k) \\ Kx(k-1) \\ Kx_c(k) \\ Kx_c(k-1) \\ \omega(k) \end{bmatrix}^T U_3 \begin{bmatrix} x(k) \\ Kx(k-1) \\ Kx_c(k) \\ Kx_c(k-1) \\ \omega(k) \end{bmatrix} \quad (11)$$

Where U_3 is defined:

$$U_3 = \begin{bmatrix} \bar{A}^T P \bar{A} - P + K^T R_1 K + C^T C & \bar{A}^T P \bar{B}_1 & \bar{A}^T P \bar{B}_2 & 0 & \bar{A}^T P F_1 + C^T F_2 \\ \bar{B}_1^T P \bar{A} & \bar{B}_1^T P \bar{B}_1 - R_1 & \bar{B}_1^T P \bar{B}_2 & 0 & \bar{B}_1^T P F_1 \\ \bar{B}_2^T P \bar{A} & \bar{B}_2^T P \bar{B}_1 & \bar{B}_2^T P \bar{B}_2 + R_2 & 0 & \bar{B}_2^T P F_1 \\ 0 & 0 & 0 & -R_2 & 0 \\ F_1^T P \bar{A} + F_2^T C & F_1^T P \bar{B}_1 & F_1^T P \bar{B}_2 & 0 & F_1^T P F_1 - \gamma^2 I + F_2^T F_2 \end{bmatrix} < 0 \quad (12)$$

If $U_3 < 0$ is satisfied, under zero initial condition, there is $\|z(k)\|_2 \leq \gamma \|w(k)\|_2$ satisfied. So, when inequality(12) is satisfied, the designed system controller makes control system (4) has the performance γ of H_∞ .

Applying Schur complement, inequality (12) is transformed into:

$$\begin{bmatrix} -P^{-1} & \bar{A} & \bar{B}_1 & \bar{B}_2 & 0 & F_1 & 0 & 0 \\ \bar{A}^T & -P & 0 & 0 & 0 & 0 & C^T & K^T \\ \bar{B}_1^T & 0 & -R_1 & 0 & 0 & 0 & 0 & 0 \\ \bar{B}_2^T & 0 & 0 & R_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -R_2 & 0 & 0 & 0 \\ F_1^T & 0 & 0 & 0 & 0 & -\gamma^2 I & F_2^T & 0 \\ 0 & C & 0 & 0 & 0 & F_2 & -I & 0 \\ 0 & K & 0 & 0 & 0 & 0 & 0 & -R_1^{-1} \end{bmatrix} < 0 \tag{13}$$

Plug $\bar{A} = \tilde{A} + B_0 K + \Delta B_0$, $\bar{B}_1 = \tilde{B}_1 + \Delta B_1$, $\bar{B}_2 = \tilde{B}_2 + \Delta B_2$ into inequality(13), Adopt **Lemma**, inequality (13) is transformed into

$$\begin{bmatrix} -P^{-1} + \varepsilon G G^T & \tilde{A} + \tilde{B}_0 K & \tilde{B}_1 & \tilde{B}_2 & 0 & F_1 & 0 & 0 & 0 \\ \tilde{A}^T + K^T \tilde{B}_0^T & -P & 0 & 0 & 0 & 0 & C^T & K^T & E_0^T \\ \tilde{B}_1^T & 0 & -R_1 & 0 & 0 & 0 & 0 & 0 & E_1^T \\ \tilde{B}_2^T & 0 & 0 & R_2 & 0 & 0 & 0 & 0 & E_2^T \\ 0 & 0 & 0 & 0 & -R_2 & 0 & 0 & 0 & 0 \\ F_1^T & 0 & 0 & 0 & 0 & -\gamma^2 I & F_2^T & 0 & 0 \\ 0 & C & 0 & 0 & 0 & F_2 & -I & 0 & 0 \\ 0 & K & 0 & 0 & 0 & 0 & 0 & -R_1^{-1} & 0 \\ 0 & 0 & E_0 & E_1 & E_2 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{14}$$

By using the elementary matrix transformation and Schur complement, inequality (14) is equivalent to:

$$\begin{bmatrix} -P^{-1} + \varepsilon G G^T + \tilde{B}_1 R_1^{-1} \tilde{B}_1^T & \tilde{A} + \tilde{B}_0 K & \tilde{B}_2 & 0 & F_1 & 0 & 0 & 0 \\ \tilde{A}^T + K^T \tilde{B}_0^T & -P & 0 & 0 & 0 & C^T & K^T & E_0^T \\ \tilde{B}_2^T & 0 & R_2 & 0 & 0 & 0 & 0 & E_2^T \\ 0 & 0 & 0 & -R_2 & 0 & 0 & 0 & 0 \\ F_1^T & 0 & 0 & 0 & -\gamma^2 I & F_2^T & 0 & 0 \\ 0 & C & 0 & 0 & F_2 & -I & 0 & 0 \\ 0 & K & 0 & 0 & 0 & 0 & -R_1^{-1} & 0 \\ 0 & 0 & E_0 & E_2 & 0 & 0 & 0 & -\varepsilon I + E_1 R_1^{-1} E_1^T \end{bmatrix} < 0 \tag{15}$$

Two sides of Equation (15) are multiplied proper matrix as follow:

$$\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -P^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

The result can be written:

$$\begin{bmatrix} -P^{-1} + \varepsilon GG^T + \tilde{B}_1 R_1^{-1} \tilde{B}_1^T & \tilde{A}P^{-1} + \tilde{B}_0 KP^{-1} & \tilde{B}_2 & 0 & F_1 & 0 & 0 & 0 \\ P^{-1} \tilde{A}^T + P^{-1} K^T \tilde{B}_0^T & -P^{-1} & 0 & 0 & 0 & C^T & P^{-1} K^T & P^{-1} E_0^T \\ \tilde{B}_2^T & 0 & R_2 & 0 & 0 & 0 & 0 & E_2^T \\ 0 & 0 & 0 & -R_2 & 0 & 0 & 0 & 0 \\ F_1^T & 0 & 0 & 0 & -\gamma^2 I & F_2^T & 0 & 0 \\ 0 & CP^{-1} & 0 & 0 & F_2 & -I & 0 & 0 \\ 0 & KP^{-1} & 0 & 0 & 0 & 0 & -R_1^{-1} & 0 \\ 0 & E_0 P^{-1} & E_2 & 0 & 0 & 0 & 0 & -\varepsilon I + E_1 R_1^{-1} E_1^T \end{bmatrix} < 0 \quad (16)$$

Using some changes of variables, $P^{-1} = Q$, $R_1^{-1} = S_1$, $R_2 = S_2$, $KP^{-1} = M$, inequality(16) is changed to inequality(5).

4. Numerical Example

Consider a system which meets the system (2):

$$A = \begin{bmatrix} 0.693147 & 0.3794685 \\ 0 & 0.4054651 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{The sampling time is set as}$$

$T=1s$, the control plant is discretized, the parameters of the discrete system(4) with uncertain time delay, data dropouts, modeling uncertainty and external disturbance as follow:

$$\tilde{A} = \begin{bmatrix} 2.0000 & 0.6595 \\ 0 & 1.5000 \end{bmatrix}, \quad \tilde{B}_1 = \int_0^{0.1} e^{As} B ds = \begin{bmatrix} 0.1035 & 0.0020 \\ 0 & 0.1021 \end{bmatrix}, \\ \tilde{B}_2 = \int_{0.1}^{0.4} e^{As} B ds = \begin{bmatrix} 0.3574 & 0.0332 \\ 0 & 0.3322 \end{bmatrix}, \quad \tilde{B}_0 = \int_{0.4}^1 e^{As} B ds = \begin{bmatrix} 0.9817 & 0.2412 \\ 0 & 0.7989 \end{bmatrix},$$

Suppose the other parameters of the system (4) as follow:

$$\Delta B_0 = \begin{bmatrix} 0.08 & 0 \\ 0 & 0.18 \end{bmatrix}, \quad \Delta B_1 = 0, \quad \Delta B_2 = 0, \quad F_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad \text{Accordingly, } G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_0 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad F = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.9 \end{bmatrix}, \quad \text{Here,}$$

taking $T = 1s$, $\gamma = 0.8$, $\alpha = 0.3$, using software MATLAB, the discrete time state feedback gain

$$\text{is } K = \begin{bmatrix} -0.9489 & -0.2390 \\ -0.2380 & -0.7536 \end{bmatrix}.$$

For the case that there exist data dropouts and time delay which is larger than a sampling time in the networked systems, the conventional method of controller design is that the actuator take the value of last cycle as the current value. Using the same value of K, this paper made three different analysis and the simulations to the methods of conventional controller design and controller design based on state estimation.

In the first case, the initial state is $x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, the time delay is smaller than a sampling

time ($\tau_k \leq T$), there is impulse interference signal such as $\omega(k) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ at the fifth and sixth

sampling time, the data dropouts in sensor-to-controller channel at the tenth sampling time. Using the conventional method of controller design of the control system, the responses of state

variables of x_1 and x_2 are shown in Figure 3. Applying the method of controller design based on state estimation, the responses of state variable of x_1 and x_2 shown in Figure 4.

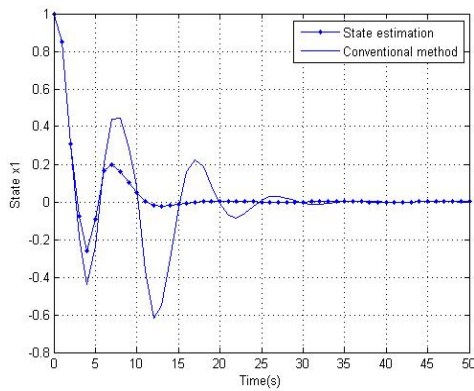


Figure 3. The Response Curve of State Variable x_1 when the Time Delay Meets $0 < \tau_k \leq T$

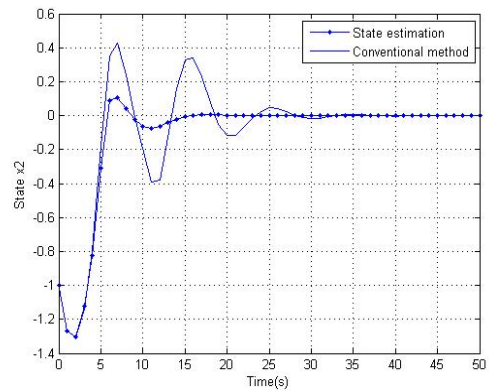


Figure 4. The Response Curve of State Variable x_2 when the Time Delay Meets $0 < \tau_k \leq T$

In the second case, the initial state is $x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, and the impulse interference signal such as $\omega(k) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ has been applied at the fifth and sixth sampling time. The time delay is larger than a sampling time ($T < \tau_k \leq 2T$) at the fifth and sixth sampling time, while it is smaller than a sampling time at others sampling times. The data dropouts in sensor-to-controller channel have been found at the tenth sampling time. Using the conventional method of controller design of the control system, the responses of state variables of x_1 and x_2 are shown in Figure 5. Applying the method of controller design based on state estimation, the response of state variable of x_1 and x_2 is shown in Figure 6.

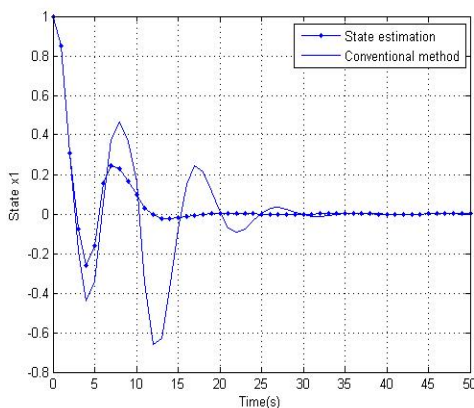


Figure 5. The Response Curve of State Variable x_1 when the Time Delay Meets $T < \tau_k \leq 2T$

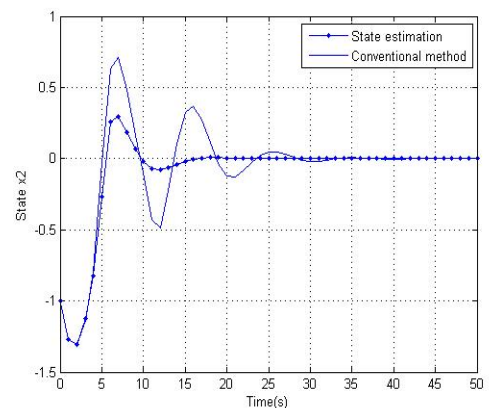


Figure 6. The Response Curve of State Variable x_2 when the Time Delay Meets $T < \tau_k \leq 2T$

In the third case, the initial state is $x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, the impulse interference signal such as $\omega(k) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ was applied at the fifth and sixth sampling time, the time delay is larger than two sampling times but smaller than three sampling times ($2T < \tau_k \leq 3T$) at the fifth, sixth and seventh sampling times, the time delay is smaller than a sampling time at others sampling times, the data dropouts in sensor-to-controller channel has been found at the tenth sampling time. Using the conventional method of controller design of the control system, the responses of state variable of x_1 and x_2 are shown in Figure 7. Applying the method of controller design based on state estimation, the responses of state variable of x_1 and x_2 is shown in Figure 8.

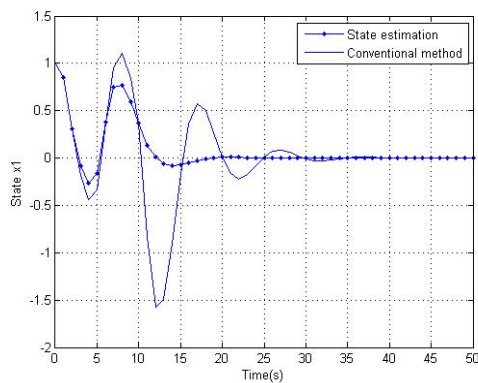


Figure 7. The Response Curve of State Variable x_1 when the Time Delay Meets $2T < \tau_k \leq 3T$

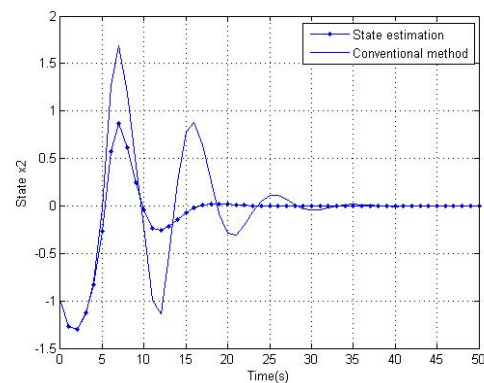


Figure 8. The Response Curve of State Variable x_2 when the Time Delay Meets $2T < \tau_k \leq 3T$

Comparing the three cases, for the control system with external disturbance or time delay which is larger than a sampling time from the fifth sampling time, after applying the conventional method, the responses of the state variable of x_1 and x_2 have fiercely jump, but the responses of the state variable of the system based on state estimation are smooth. Compare to the case which has the data dropouts at the tenth sampling time, the responses of the state variable also have the significant improvement. Obviously, applying the method of controller design state estimation, the regulation time and overshoot are obviously shorter. Control precision of the networked systems is enhanced; the influences of the control system caused by modeling uncertainty and external disturbance are validly restrained.

5. Conclusion

Aiming at the networked control systems with uncertain time delay in both sensor-to-controller and controller-to-actuator channels, and data dropouts in sensor-to-controller channel in this paper, a new method of robust control design based on state estimation is proposed. To get over the effectiveness of data dropouts and the delay of data transfer caused by time delay, A state prediction arithmetic is added in the end of controller input. The arithmetic refers to apply single-exponential smoothing method as the prediction model of control system, and prospect the state at every sampling time. To increase the rapidity of the drive of controller, controller applies the way of composite-driven of time-driven and event-driven. Adopting Lyapunov function and linear matrix inequality, the design of robust controller based on state estimation is completed. The method of robust controller design based on state estimation doesn't have restrictions on distribution or length of time delay for the system. Simulation results demonstrate that the designed robust controller based on state estimation not only resolves the problems of

transmission errors caused by time delay and data dropouts, but also restrains the influences of external disturbances and model inaccuracy caused by modeling uncertainty for system performance. In conclusion, the control precision of system is improved.

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