

## Noise Analysis of wet Multi-disc Brake Used Complex Eigenvalue

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### Abstract

*This paper proposed an analysis model and a complex-eigenvalue-based algorithm of wet multiple disc brake (WMDB) noise. Firstly, the damping and stiffness between the brake disc and the friction plate were introduced based on the consideration of the slight rotation of the braking system and damping characteristics of brake oil. Then, the friction coupling mathematical model and the finite element model (FEM) of the braking system were established. The occurrence tendency of brake squeal was judged according to the real part (positive or negative value) of characteristic roots of the complex eigenvalue. The relative error between the numerical analysis results and the finite element analysis results was 4%, indicating that braking speed and braking pressure had little effect on brake noise. During the braking system design, under the premise that the braking performance is not affected, the friction material with the lower stiffness should be used as possible. Moreover, properly decreasing the friction coefficient between the friction linings and dual steel disc will reduce the brake noise. Therefore, the proper combination of the parameters was deduced to obtain the stable frequency response of the braking system.*

**Keywords:** automotive engineering, wet multiple disc brake, complex eigenvalue, solid-liquid coupling, brake noise

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### 1. Introduction

With the anti-pollution performance, large braking torque, small wear, long service life, wet multi-disc brakes are widely used in engineering machinery. Nevertheless, braking noise deteriorated automobile braking stability, affected occupant comfort and reduced the component life [1].

Nowadays, brake noise had been extensively studied. Shih-iti Kung [2] had changed the resonant frequency by lowering the stiffness of the brake discs, thus weakening the coupled interaction and eliminating the dynamic instability. T. S. Shi [3] had found that brake discs produced the low-frequency normal squeal and the high-frequency tangential squeal and decreased the brake noise by changing the geometric structure, choosing suitable friction materials and other optimization methods. Júnior MT [4] had proved that with the increasing braking pressure, the wear was increased and the system instability was also increased. In addition, the temperature rise is helpful to improve the system stability. AbuBakar and Ou-yang [5-7] had found that the braking system involved with a large number of nonlinear factors including non-linear contact stiffness at the friction interface, physical nonlinear material, the friction coefficient and the complex relationship among loads, speed and temperature.

Terry [8] found that wet brakes noise and vibration are caused by the "stick-slip" effect. The negative slope relationship between the friction coefficient and the rotation speed may induce self-excited vibration. Dong-Ye Sun [9] had discovered that when the ratio of the static friction coefficient to the kinetic friction coefficient was about 1 and the second braking phase was extended, the vibration could be reduced to eliminate the noise. Wenqing Zhao [10-12] had established the mathematical model for wet brake, deduced the theoretical equation of brake noise cause with the modal analysis method and exploited the generation mechanisms, reasons and factors of brake noise. XueJie Fu et al [13] had proposed the complex modal analysis

theory, analyzed the main influence factors of brake noise, and proposed the measures of reducing brake noise.

The modal characteristics of wet disc brakes in brake oil fluids have not been reported at home and abroad. Compared to conventional dry brakes, wet brakes operate it well in the closed oil space. The natural frequency of solid materials in the liquid is different from that in the vacuum. To make the analysis results close to the real situation, during the establishment of the model, the damping and stiffness between steel plate and friction brake linings were introduced to consider slight rotation of the braking system. The analysis model of wet brake noise vibration was established and the wet vibration noise algorithm based on complex eigenvalues was proposed. it compared numerical analysis with finite element analysis, verified the reliability of analysis results, studied the effects of friction coefficient, braking speed, braking pressure and material stiffness on the braking system stability.

**2. Wet Brake Noise Generate Mechanisms and Complex Eigenvalue Analysis Theory**

Primary brake noise is a high-frequency or low-frequency squeal. As for disc brakes, brake noise mainly comes from circumferential and axial vibration caused by the friction of brake discs and the vibration of the brake caliper also strengthens the system vibration. When the brake begins to work, under the definite pressure, the moving parts contact each other and are engaged in the relative movement. The opposite movement between moving parts produces friction, thus stimulating the movement part to vibrate and then generate noise [14], as shown in Figure 1. In the paper, it analyzed the complex eigenvalue to predict whether the noise would occur or not.

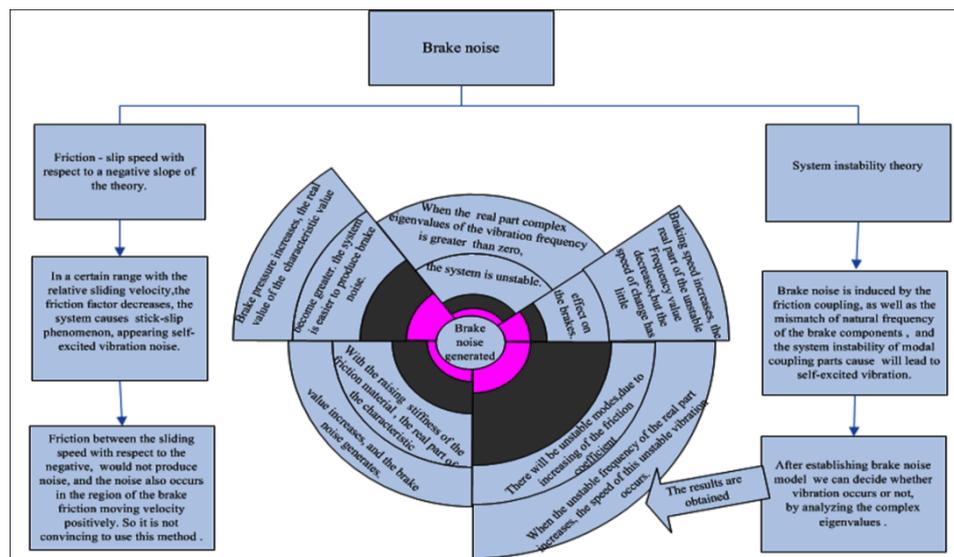


Figure 1. The Link between Complex Eigenvalues and Brake Noise

During braking, modal coupling of braking components leads to vibration, thus making the brake system unstable. Along with the modal coupling of friction linings, vibration is enhanced. Under working conditions, the modal vibration of brake discs is induced by the friction centrifugal force and the modal coupling between brake discs and the friction linings appears, then low-frequency noise is generated. The friction process of contacted surfaces is a dynamics process which belongs to the transient mode. Therefore, the transient vibration mode is called the instantaneous vibration mode. Since the contact characteristics is affected by many factors, such as contact stiffness, surface roughness, pressure, temperature, speed, and viscous damping of brake fluid. The instantaneous vibration mode is often coupled with system mode, disc mode and friction linings mode. The continuous energy input and accumulation produces the resonance, as shown in Figure 2.

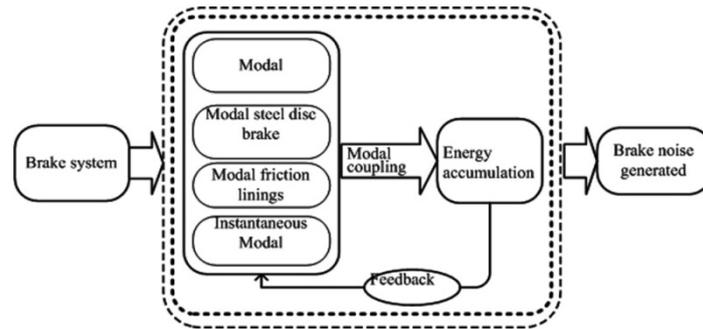


Figure 2. Brake Vibration System

Introduce the stiffness matrix  $[K_f]$  of brake friction among parts of the model; there is the equation of motion for brake vibration system.

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K - K_f]\{u\} = 0 \tag{1}$$

Where: M, C, K and  $K_f$  respectively, for the system mass matrix, damping matrix, stiffness matrix and friction stiffness matrix; u represents the vector of the system. The stiffness matrix is asymmetric due to friction stiffness matrix  $[K_f]$  of the formula (1). The characteristic matrix stiffness is failing to symmetric because of asymmetric stiffness matrix, and the eigenvalue will be plural under certain conditions induced by asymmetric matrix. When the real part of eigenvalue is positive, the system tends to be unstable.

$$\lambda_{i1}, \lambda_{i2} = \alpha_i \pm j\omega_i \quad (i = 1, \dots, n) \tag{2}$$

Where:  $\alpha_i$  represents the modal damping,  $\omega_i$  means natural frequency of i order modal. Each complex eigenvalue corresponds to a feature vector, reflecting the real part of the brake system stability, if  $\alpha_i$  is positive, then the i order modal damping ratio is negative, the amplitude increases with time, causing the brake system instability, vibration and noise. Noise frequency depends on modal natural frequency of the brake instability.

### 3. Analyze the Model of Wet Vibration Noise

#### 3.1. The Establishment of the Wet Vibration Noise Analysis Model

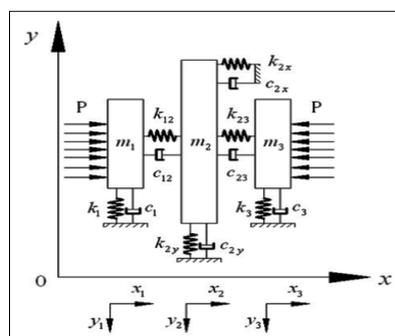


Figure 3. The Equivalent Model of Wet Brake System

The following equations are obtained by Newton's second law:

For m1:

$$\begin{cases} m_1 \ddot{x}_1 = P - k_{12}(x_1 - x_2) - c_{12}(\dot{x}_1 - \dot{x}_2) \\ m_1 \ddot{y}_1 = \{k_{12}(x_1 - x_2) + c_{12}(\dot{x}_1 - \dot{x}_2)\} \mu_1 \\ -k_1 y_1 - c_1 \dot{y}_1 \end{cases} \quad (3)$$

For m2:

$$\begin{cases} m_2 \ddot{x}_2 = k_{12}(x_1 - x_2) + c_{12}(\dot{x}_1 - \dot{x}_2) - \\ k_{23}(x_3 - x_2) - c_{23}(\dot{x}_3 - \dot{x}_2) - k_{2,x} x_2 - c_{2,x} \dot{x}_2 \\ m_2 \ddot{y}_2 = \{k_{12}(x_1 - x_2) + c_{12}(\dot{x}_1 - \dot{x}_2)\} \mu_1 - \\ \{k_{2,y} y_2 - c_{2,y} \dot{y}_2 + \{k_{23}(x_3 - x_2) + c_{23}(\dot{x}_3 - \dot{x}_2)\} \mu_2 \end{cases} \quad (4)$$

For m3:

$$\begin{cases} m_1 \ddot{x}_3 = -P + k_{23}(x_2 - x_3) + c_{23}(\dot{x}_2 - \dot{x}_3) \\ m_1 \ddot{y}_3 = \{k_{23}(x_2 - x_3) + c_{23}(\dot{x}_2 - \dot{x}_3)\} \mu_2 \\ -k_3 y_3 - c_3 \dot{y}_3 \end{cases} \quad (5)$$

Make the following assumptions: where  $m_1, m_3$  represent the mass of dual steel plates;  $m_2$  is the mass of the friction linings;  $k_1, k_3$  is the stiffness between the steel plate and the brake housing;  $c_1, c_3$  are the damping between the steel plate and the brake housing;  $k_{12}, k_{23}$  are the stiffness of dual steel plate and the friction lining;  $c_{12}, c_{23}$  are the stiffness between the friction lining and drive shaft in the x-direction and y-direction;  $c_{2,x}, c_{2,y}$  represent the stiffness between the drive shaft and friction linings in the x-direction and y-direction;  $p$  is the braking pressure applied on the steel plate.

$$\begin{cases} \mu_1 = \mu_a - \xi_\mu \omega_1 \\ \mu_2 = \mu_a - \xi_\mu \omega_2 \end{cases} \quad (4)$$

$$\begin{cases} \omega_1 = v_0 + (\dot{y}_2 - \dot{y}_1) \\ \omega_2 = v_0 + (\dot{y}_2 - \dot{y}_3) \end{cases} \quad (5)$$

Where  $u_1, u_2$  are the friction coefficient between the friction lining and steel disc,  $\omega_1, \omega_2$  are the angular velocity which the friction lining compared with dual steel plate,  $u_a$  stands for the static friction coefficient,  $\varepsilon_u$  is the rate of change of friction coefficient,  $v_a$  means the initial velocity of the friction pair.

### 3.2. Numerical Solution of the Model for Wet Vibration Noise

The formula (1), (2) and (3) are written in matrix:

$$[M] \{\ddot{X}\} + [C] \{\dot{X}\} + [K] \{X\} = [F] \quad (6)$$

There are:

$$\{X\} = \begin{Bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \end{Bmatrix}, \{\dot{X}\} = \begin{Bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{x}_2 \\ \dot{y}_2 \\ \dot{x}_3 \\ \dot{y}_3 \end{Bmatrix}, \{\ddot{X}\} = \begin{Bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{x}_2 \\ \ddot{y}_2 \\ \ddot{x}_3 \\ \ddot{y}_3 \end{Bmatrix},$$

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_6 \end{bmatrix},$$

$$[C] = \begin{bmatrix} c_{12} & 0 & -c_{12} & 0 & 0 & 0 \\ -\mu_1 c_{12} & c_1 & \mu_1 c_{12} & 0 & 0 & 0 \\ -c_{12} & 0 & c_{12} - c_{23} + c_{2x} & 0 & c_{23} & 0 \\ -\mu_1 c_{12} & 0 & \mu_1 c_{12} + \mu_2 c_{23} & c_{2y} & -\mu_2 c_{23} & 0 \\ 0 & 0 & -c_{23} & 0 & c_{23} & 0 \\ 0 & 0 & -\mu_2 c_{23} & 0 & \mu_2 c_{23} & c_3 \end{bmatrix}$$

$$[K] = \begin{bmatrix} k_{12} & 0 & -k_{12} & 0 & 0 & 0 \\ -\mu_1 k_{12} & c_1 & \mu_1 k_{12} & 0 & 0 & 0 \\ -k_{12} & 0 & k_{12} - k_{23} + k_{2x} & 0 & k_{23} & 0 \\ -\mu_1 k_{12} & 0 & \mu_1 k_{12} + \mu_2 k_{23} & k_{2y} & -\mu_2 k_{23} & 0 \\ 0 & 0 & -k_{23} & 0 & c_{23} & 0 \\ 0 & 0 & -\mu_2 k_{23} & 0 & \mu_2 k_{23} & k_3 \end{bmatrix}$$

$$[F] = [p \ 0 \ 0 \ 0 \ -p \ 0]^T$$

$$z_1 = x_1, z_2 = \dot{x}_1, z_3 = y_1, z_4 = \dot{y}_1,$$

$$z_5 = x_2, z_6 = \dot{x}_2, z_7 = y_2, z_8 = \dot{y}_2,$$

$$z_9 = x_3, z_{10} = \dot{x}_3, z_{11} = y_3, z_{12} = \dot{y}_3,$$

Assume that the state vector is:

$$[Z] = [z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8, z_9, z_{10}, z_{11}, z_{12}]^T$$

Get system state equation:

$$\dot{Z} = AZ + BV \quad (7)$$

A is the Jacobi matrix of the system:

$$A = \begin{bmatrix} Z_{(1,1)} & Z_{(1,2)} & \dots & Z_{(1,12)} \\ Z_{(2,1)} & Z_{(2,2)} & \dots & Z_{(2,12)} \\ \vdots & \vdots & & \vdots \\ Z_{(12,1)} & Z_{(12,2)} & \dots & Z_{(12,12)} \end{bmatrix}$$



Table 1. Materials Properties of Wet Brake Parts

Parts	Young's modulus/Mpa	Density/kg-m-3	Poisson's ratio	Material name
Dual steel plates	175000	7228	0.3	65Mn steel
Friction linings	175000	7228	0.3	65Mn steel
Friction materials	1500	1450	0.25	Paper based friction material without asbestos

### 4.3. The Boundary Conditions

During braking, structural components of wet multiple disks produce coupling modes among various components. Especially the friction pair produces coupling modes because of excitation force, which makes the brake to produce vibration noise. Since friction contact is a time-variant dynamic process, the contact relationship between the friction linings and steel plates should be taken into consideration. It analyzed nonlinear pre-stress of friction pairs and obtained the contact states of the friction. Then it analyzed the prestressed mode on the basis of modal analysis system. The static mechanical analysis boundary conditions are provided as follows. Fixed constraints are imposed on radial friction linings while the axial and tangential movements are free and the speed is 3 rad/s. Fixed constraints are imposed on dual steel plates in radial and tangential direction while the axial movement is free. The 4-Mpa pressure is applied on both sides. The contact interface between dual steel plate and friction linings is defined as the friction pairs and the friction coefficient is 0.3. In order to facilitate the convergence of analysis and solution, it selected a non-symmetric contact and the enhanced Lagrangian algorithm. The prestressed wet modal analysis boundary conditions are provided as follows. Based on the static analysis, the natural frequency of the entire model may be affected by the stress state. Static analysis results are extracted from boundary conditions directly. Because of the introduction of asymmetric friction contact, through the command "modopt, unsym, 30,,, on", the first 30-order modes were extracted.

### 4.4. Comparison between Wet Modal Results and Dry Modal Results

The results of static analysis were selected as initial conditions. After setting the rotation speed of the friction linings, it performed the modal analysis of the effect of oil medium on the braking system. The comparison analysis results of dry modal braking system in air contrast and wet braking system in oil medium are shown in Figure 6.

As shown in Figure 6, the modal frequency of braking system in the fluid medium is lower than that in the air. The decline can be interpreted in the following two aspects. First, before braking, the space between dual steel discs and friction linings are filled with fluid; when braking starts, the additional mass of oil leads to the mass changes of friction lining and dual steel plate. Second, during braking, the fluid medium coupling between the fluid and solid affected the modal characteristics of the braking system.

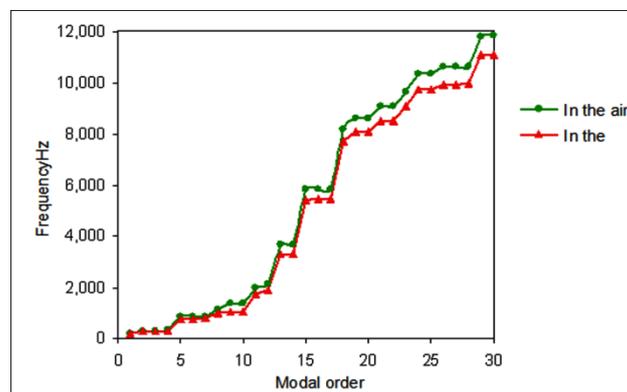


Figure 6. The Contrast Results of Wet and Dry Model

**4.5 The results of finite element analysis**

Testing results indicate that wet brake system instability frequency is generally 100-300 Hz<sup>[9]</sup>. Thus, produced noise belongs to low-frequency vibration noise. Considering the influence of brake oil on the braking system stability, it extracted the first 30-order modals, as shown in Figure 7. An imaginary part and two mutually opposite real parts occurred at unstable frequencies. The black dots represented the value of the real part at the unstable frequencies in the upper portion of Figure 7. As illustrated in Figure 7, the unstable modes belong to seven orders, whose frequencies are respectively 286.75 Hz, 756.32 Hz, 1056.8 Hz, 5435.4 Hz, 9951 Hz, 11126 Hz, and 11229 Hz.

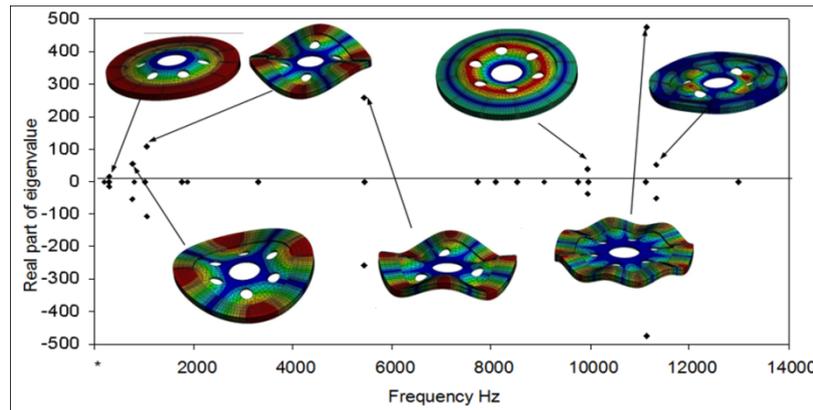


Figure 7. The Complex Eigenvalues of Braking System

**4.6. Numerical Analysis Results**

Table 2 shows the wet brake stiffness and damping parameters of the structure, there is a possibility that the equilibrium points are appear when  $\dot{Z} = 0$  :

$$\left( \frac{p}{k_{12}} + \frac{2p}{k_{2x}}, 0, \frac{2p}{k_{2x}}, 0, \frac{p}{k_{12}} + \frac{2p}{k_{2x}}, 0, \frac{\mu_0 p}{k_1}, 0, 0, 0, \frac{\mu_0 p}{k_3}, 0 \right), \mu_0 = \mu_a - \xi v_0$$

Bringing the wet brake parameter into Jacobi matrix, matrix eigenvalues are calculated, as shown in Table 2:

Parameter	Value	Parameter	Value
P	33209N	$k_{12} = k_{23}$	13000 N/s
$m_1 = m_3$	0.3Kg	$c_{12} = c_{23}$	6N·s/m
$m_2$	0.5Kg	$k_{2x} = k_{2y}$	9600N/s
$k_1 = k_3$	9600N/s	$c_{2x} = c_{2y}$	4N·s/m
$c_1 = c_3$	6N·s/m	$v_0$	2.77m/s

Bring the parameter into the Jacobi matrix, obtained the characteristic values which are shown in Table 3:

Table 3. The Eigenvalues of the Jacobi Matrix

Number	Eigenvalue	Number	Eigenvalue
1	-7.50 + 180.12i	7	3.98+276.05i
2	-7.50 - 180.12i	8	3.98-276.05i
3	-3.02+792.04i	9	-9.06+153.78i
4	-3.02-792.04i	10	-9.06-153.78i
5	-7.86+267.45i	11	-2.64+352.58i
6	-7.86-267.45i	12	-2.64-352.58i

From the above results, it can be seen that the eigenvalues are divided into two parts, a real part and an imaginary part. Imaginary eigenvalues appeared in pairs. When the characteristic value is the real part and smaller than zero, the system is in a stable state. When the characteristic value of the real part is greater than zero, the system is in a flux state. Since there are two characteristic values of the real part and greater than zero, which can predict the system is unstable, the unstable frequency is 276.05Hz.

#### 4.7. The Contrast between Numerical Analysis and Finite Element Analysis

Table 4. The Contrast Results

numerical	Finite element		Result <sup>[9]</sup>
	In the air	In the oil	
276.05Hz	300.49 Hz	286.75 Hz	100-300 Hz

From the result it can be seen that, the unstable frequency result got from the finite element analysis is 286.75 Hz which is very close to the experimental result that is 276.05 Hz, the relative error is 4% ,the results indicated the reliable of the proposed method.

#### 5. Effect of Wet Brake Parameters on System Stability

Wet brake noise is influenced by many parameters, take friction coefficient, braking speed, braking pressure, friction lining and double plate stiffness for example. If the quality, elastic and damping properties of the system are the same. Simulation parameters are shown in Table 5.

Table 5. Simulation Parameters

Effects of parameters	Test parameter values			
Friction coefficient	0.15	0.3	0.45	0.6
Brake speed ( rad/s )	1	3	5	8
Brake pressure ( Mpa )	2	4	6	8
Friction lining stiffness(EC)	0.8	1	1.2	1.4
Dual steel plate stiffness(E0)	0.8	1	1.2	1.4
EC—Friction material Young's modulus, EC=1500MPa				
EO—Dual steel plate Young's modulus, EO=175000MPa				

##### 5.1. Effect of Friction Coefficient on Brake Noise

Instability of Braking system, due to friction, resulting in braking noise, thus changing the trend of the friction coefficient on the influence of system instability. Study the stability impact of the system, friction coefficient was increased from 0.15 to 0.6, as shown in Figure 8.

From the Figure 8 it can be seen that, it will continue to the emergence of new unstable modes as increase of the friction coefficient, causing instability original value of the real part of complex eigenvalue frequency increasing, making the system more unstable. Therefore, appropriate to reduce the friction coefficient between friction linings and the steel plates without affecting the braking performance can reduce the braking noise.

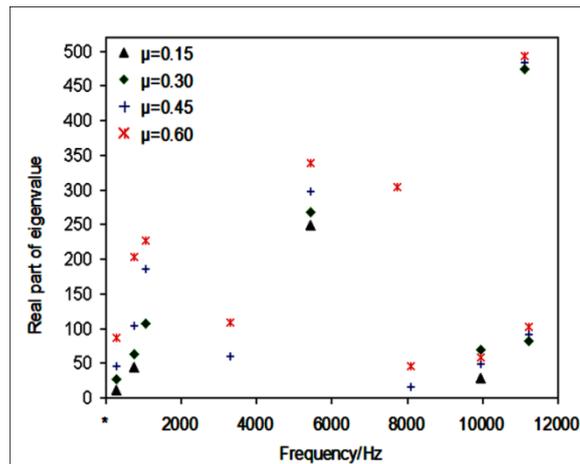


Figure 8. Effect of Friction Coefficient on the Brake System Stability

### 5.2. Effect of Braking Speed on Brake Noise

Brake noise is affected by the speed of the brake. Analyze the effects on brake noise of speed. Figure 9 represents the change of braking speed from 1rad/s to 8 rad/s have an effect on the brake noise.

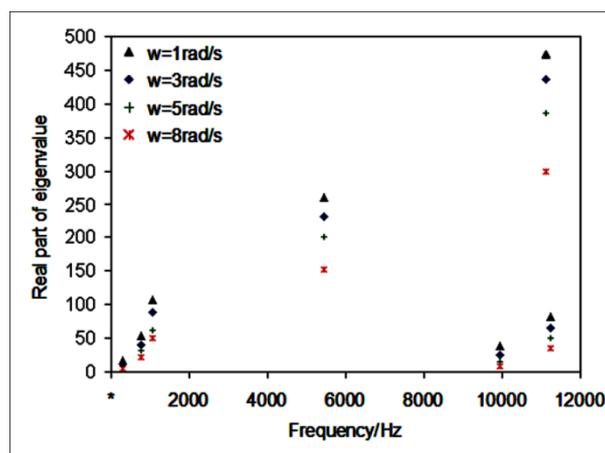


Figure 9. Effect of Braking Speed on the Brake System Stability

The speed increases, with the frequency instability of the real part of the complex eigenvalue decreases, while the system tends to be more stable, but there is no significant reduction of the real part of each rotation speed corresponding to the value. That is, the speed changes of little effect on the brake noise.

### 5.3. Effect of Braking Pressure on the Brake Noise

Brake piston is subjected to brake fluid pressure during braking, which makes the dual steel plate and brake lining friction decelerate and eventually stop. Figure 10 shows brake pressure changes from 2MPa to 8MPa, the brake system generates the braking noise.

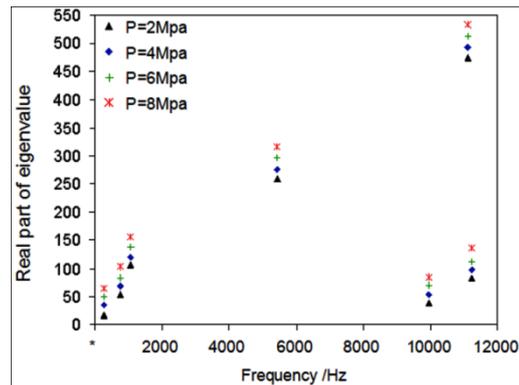


Figure 10. Effect of Brake Pressure on the Brake System Stability

With increasing pressure on the brake, the real numerical eigenvalue is increasing constantly. Namely the braking system is more prone to noise, which is due to the increase of brake pressure which makes the friction lining between the plate and the steel plate increase. However, with the increasing of brake pressure, the real part will be not significantly increased, and the value is very close to each other, the change of brake pressure has little impact on brake noise.

#### 5.4. The Influence of Friction Lining Stiffness on the Brake Noise

Friction lining is mainly composed of friction material and friction chips. The change of the friction material stiffness will be a certain influence on brake noise. Changing its young's modulus value and analyzing the friction material. Young's modulus changes from 0.8EC to 1.4EC, including EC = 1500MPa, Figure 11 shows the variation of Young's modulus to the brake noise.

With the stiffness of friction material increased, the number of the real part of complex eigenvalue also increased, the braking system is more prone to brake noise, therefore, during the braking system design, under the premise that the braking performance is not affected, the friction material with the lower stiffness should be used as possible.

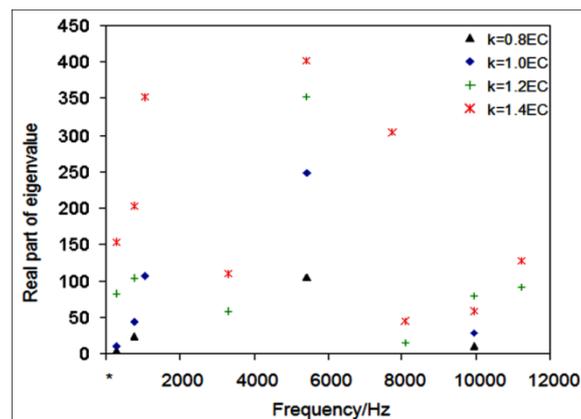


Figure 11. Effect of Fiction Lining Stiffness on the Brake System Stability

#### 5.5. The Influence of Dual Steel Plate Stiffness on Brake Noise

Dual steel plate stiffness changes will be a certain influence on brake noise. Changing its young's modulus value, analyze the Young's modulus of dual steel plate changes from 0.8EO to 1.4EO, including EO = 175000 MPA. Figure 12 shows the impact of brake noise on changing its stiffness.

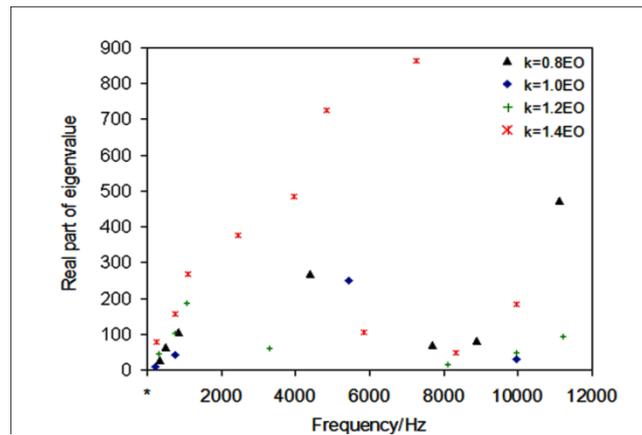


Figure 12. Effect of Dual Steel Disc Stiffness on the Brake System Stability

With dual steel plate stiffness increases, the real part of the complex eigenvalue of the system also gradually becomes larger, and produces brake squeal where did not happen previously. Therefore, when design a braking system, under the condition of not affecting the braking performance, the friction material with the lower stiffness should be used as possible.

Friction coefficient, braking speed, braking pressure, friction lining and plate stiffness are the important factors affecting wet brake noise. The smaller friction coefficient and the lower stiffness of friction lining and steel plate lead to the more faint brake noise. While the braking speed and braking pressure has little impact on the brake noise.

## 5. Conclusion

With the solid-liquid coupling method, it studied the modal characteristics of the brake system in fluid medium and established the wet brake vibration analysis model. Through the analysis of parameter characteristics, obtained the influencing laws of all the major system parameters on dynamic noise in the finite element optimization design and drew the following conclusions based on the finite element design for guidance.

(1) The paper proposed the analysis model of wet brake noise vibration, in which it comprehensively considered the stiffness, the damping characteristics and the normal rotation angle between steel plates and linings, the effect of damping characteristics of the fluid medium mode.

(2) Based on complex eigenvalues, the wet vibration noise algorithm was proposed and corresponding calculation equations were deduced. The relative error between numerical analysis results and the finite element analysis results is 4%, which is close to testing data and reliable.

(3) Under the premise that the braking performance is not affected, it is appropriate to reduce the friction coefficient between steel plates and friction linings and to adopt the friction materials with the lower stiffness.

(4) The initial rotation speed of dynamic friction linings and the brake pressure change have little effect on the brake noise. Brake noise is a complex physical behavior. Many parameters affect brake noise generation, including temperature, humidity and other parameters. The current analysis does not include these parameters. It may be more accurate to forecast the brake noise generated by considering the impact of these parameters on brake noise.

## References

- [1] Di-hua Guan, Xin-dong Su. An Overview on Brake Vibrations and Noise. *Engineering Mechanics*. 2004; 21(4): 150-155.
- [2] Shih-Wei Kung, K Brent Dunlap, Robert S Ballinger. Complex Eigenvalue Analysis for Reducing Low Frequency Brake Squeal. SAE Paper 2000-01-0444.

- 
- [3] TS Shi, O Dessouki, T Warzecha, et al. Advances in Complex Eigenvalue Analysis for Brake Noise, SAE Paper. 2001-01-1603.
  - [4] Ma'rio Triche's Ju' nior, Samir NY Gerges, Roberto Jordan. Analysis of brake squeal noise using the finite element method: A parametric study. *Applied Acoustics*. 2008; 69:147-162.
  - [5] AbuBakar AR, Ouyang HJ, James S, et al. *Finite element analysis of wear and its effect on squeal*. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering. 2008; 222(7):1153- 1165.
  - [6] Ouyang HJ, AbuBakar AR, Li L. A combined analysis of heat conduction, contact pressure and transient vibration of a disk brake. *International Journal of Vehicle Design*. 2009; 51(1-2): 190-206.
  - [7] Abu Bakar AR, Ouyang HJ. Complex eigenvalue analysis and dynamic transient analysis in predicting disc brake squeal. *International Journal of Vehicle Noise and Vibrations*. 2006; 2(2): 143-155.
  - [8] Terry V Friesen. Chatter in wet brakes, SAE Paper 921660.
  - [9] Dong-ye Sun. A Study on Vibration and Noise in Wet Multiple Disc Brakes. *Transactions of the Chinese Society of Agricultural Engineering*. 1996; 12(1):104-108.
  - [10] Wen-qing Zhao. Modelling and Restraining Study of Brake Noise on Wet Multiple Brakes. *China Journal of Highway and Transport*. 2002; 15(4): 118-120.
  - [11] Wen-qing Zhao, Yu-ling Gai. Modeling of Braking Noise from Wet Multiple-disc Brakes. *Transactions of the Chinese Society of Agricultural Machinery*. 2003; 34(2): 11-13.
  - [12] Wen-qing Zhao. Modeling and Restraint of Braking Noise of Wet Multi-disc Brakes. *Acta Armamentarii*. 2004; 25(6): 662-665.
  - [13] Xue-jie Fu, Bo-qiang Shi, Yong Jiang, et al. Analysis on Wet Multi-disc Brake Noise Based on ABAQUS. *Coal Mine Machinery*. 2012; 33(02): 99-101.
  - [14] Guang-rong Zhang, Ming-song Xie, et al. Vehicle Brake Moan Noise Induced by Brake Pad Taper Wear. *Journal of Mechanical Engineering*. 2013; 49(09): 09-081.
  - [15] Ruo-fu Xiao, Cai-xin Wei, Feng-qin Huan, et al. Study on Dynamic Analysis of the Francis Turbine Runner. *Large Electric Machine and Hydraulic Turbine*. 2001; 7: 41-43.