Indonesian Journal of Electrical Engineering and Computer Science
Vol. 39, No. 2, August 2025, pp. 1145~1154
ISSN: 2502-4752, DOI: 10.11591/ijeecs.v39.i2.pp1145-1154

Wolfram Alpha based-inventory model for damaged items of pharmaceutics by utilizing exponential demand rate

Indrawati[1,2], Fitri Maya Puspita[2], Siti Suzlin Supadi[3], Evi Yuliza[2], Farah Nabilah Tampubolon[2]
[1]Mathematics and Natural Science Doctoral Study, Sriwijaya University, Indralaya, Indonesia
[2]Mathematics Department, Faculty of Mathematics and Natural Science, Sriwijaya University, Indralaya, Indonesia
[3]Institut of Mathematical Science, University of Malaya, Kuala Lumpur, Malaysia

Article Info

Article history:

Received Aug 7, 2024
Revised Mar 22, 2025
Accepted Jul 2, 2025

Keywords:

Complete backlogging
Deteriorating
Exponential demand rates
Inventory
Pharmaceutical items

ABSTRACT

In this study, an inventory model is developed for pharmaceutical products that deteriorate over time with an exponential demand rate. The discussion of exponential demand is rarely explored but has the advantage that the demand value toward total cost remains positive. This study assumes allowable shortages and complete backlogging, making it necessary to design an optimal policy for deteriorating goods with an exponential demand rate. The model shows that the initial stock decreases over time, potentially leading to shortages before the next order arrives. The optimal solution indicates that the inventory reaches the zero point at $t_1 = 0.0000011$ and the cycle length $T_1 = 0.012$ resulting in an average minimum total cost of $\overline{TC} = \$17{,}133.9$ per cycle by Wolfram Alpha. Sensitivity analysis measures the changes of the results in the increasing value of $\overline{TC}$ for all parameters. Exponential function variables (a and b) produces t_1 and T_1 stable values. On increasing the cost of each damage (D_C) and constant damage rate (θ) produces a t_1 stable value, but the value of T_1 increases. An increase in storage costs (h) results in a decrease in the value of t_1 and T_1. Increasing in the cost of shortages (s) resulted in an increase in the value of t_1 and a decrease in the value of T_1.

This is an open access article under the CC BY-SA license.

Corresponding Author:

Fitri Maya Puspita
Mathematics Department, Faculty of Mathematics and Natural Sciences, Sriwijaya University
Indralaya, 30662, Indonesia
Email: fitrimayapuspita@unsri.ac.id

1. INTRODUCTION

The pharmaceutical world needs a system of distribution and sales provisions to build company development and achieve company goals [1]–[6]. Therefore, pharmaceutical companies require a well-functioning distribution and sales system to meet their objectives and reduce inventory issues, like stock shortages or running out of stock [7]. Pharmaceutical goods are divided into two types, namely drugs and non-medicines. Medicines, produced by pharmaceutical companies and ordered by pharmacists, are items that can expire if stored for extended periods, leading to losses and challenges in inventory management. Procurement of inventory aims to ensure that the company's operational activities run well related to purchasing, storing, maintaining, and selling goods. The significance of having an inventory model that can adapt to fluctuating demand and address the issue of damaged goods in stock [8].

According to Fadli *et al.* [8], two key factors were is examined in developing the inventory model, the deterioration of goods and the demand levels for those goods [9]–[11]. Deterioration typically results from prolonged storage, leading to losses [12]–[16], complete backlogging may occur when customers choose to switch to other companies due to delayed orders or choose to wait until goods are available again.

Pharmaceutical goods, commonly known as medicines, are easily perishable items, which is a problem faced by the pharmaceutical inventory system in overcoming shortages and profits [17]–[19]. According to Uthayakumar and Karuppasamy [20], a small portion of the shortages that occur are unfulfilled customer demand for pharmaceutical supplies, resulting in shortage costs. Time-dependent pharmaceutical inventory models play an important role in the healthcare industry as demand levels are in a dynamic state with optimal costs [21].

Numerous studies have explored pharmaceutical inventory models, introducing various conceptual frameworks, such as Sutrisno *et al.* [22] deal with inventory systems using the linear quadratic Gaussian method, inventory management using the ABC method and as EOQ model, or system of optimal inventory of supplies of deteriorating goods of water supplies [23], Uthayakumar and Tharani [21], the key challenge is managing damaged stock while addressing dynamic demand patterns, such as quadratic and exponential demand when formulating an inventory model for damaged pharmaceutical goods, the research focuses on demand influenced by quadratic time with complete backlogging. Uthayakumar and Karuppasamy [20], developed an inventory model for the healthcare industry [24] with quadratic demand, linear carrying costs, and shortage, so in this research the demand fuction model was developed into an exponential function.

This study explores develop a model that can minimize the total cost of inventory for managing deteriorating pharmaceutical goods under exponential demand levels which are rarely discussed but have the benefit of the value of demand towards total cost will always be a positive value, with assumptions of permitted shortages and complete backlogging. The model developed is different from the previous model by Uthayakumar and Tharani [21], in terms of the demand function being an exponential function, the model is in quadratic function. The solution utilizes the Wolfram Alpha. To determine changes in variable values to produce optimal solutions and minimize the average total inventory costs for each cycle, a sensitivity analysis is carried out.

2. METHOD

The process undertaken in this study is outlined as follows:

1) Identify and establish the symbols and assumptions required to develop the inventory model for damaged pharmaceutical goods. The symbols and assumptions employed are detailed below:

a and b	: The quantity of item a, is set at 100. The quantity of item b is set at 50
D_c	: Each damaged item cost
h	: Holding cost of inventory per unit of time.
H	: Total cost of holding inventory.
$I(t)$	: Inventory amount at t.
s	: The cost of shortages per unit of time.
S	: Shortage total costs.
θ	: Rate of deterioration for available items (on hand); $0 < \theta < 1$.
T_1	: The cycle length
t_1	: Time of inventory when achieving zero point
t_1^* and T_1^*	: The optimal point
$\overline{TC}$	: The average total cost of damaged inventory per unit of time

Several assumptions are made in the development of a deteriorating inventory model, as outlined below:

- The deterioration rate is constant.
- Shortages are allowed, and complete backlogging occurs.
- The lead time is assumed to be zero.
- During shortage periods, the backlogging level is a variable depending on the waiting time for the next replenishment, with $t_1 \leq t \leq T_1$.

One of the goals of the inventory model is to obtain minimum costs to determine inventory costs. The following is the definition of components, the total cost of supplies is as follows:

i) Purchasing cost refers to the price per unit when sourced externally or the production cost per unit when produced internally. It may vary depending on order size, especially if suppliers offer discounts for larger quantities [25].
ii) Ordering costs are the expenses incurred when placing orders with suppliers, including administrative, transportation, loading/unloading, and receiving fees [26].
iii) Holding costs, or storage costs, are the expenses of maintaining inventory over time. These include warehouse rent, obsolescence, administrative fees, staff salaries, utilities, capital tied up in inventory, taxes, insurance, and damage costs [27].

iv) Deteriorating costs, or damage costs, are expenses from goods losing quality due to prolonged storage or damage, including the value of inventory lost to deterioration [28].
v) Shortage costs are incurred when inventory fails to meet demand on time. They include administrative expenses, lost profits, production or distribution delays, extra costs, and potential customer loss [29].

In the inventory model formulation, there are two influencing factors of goods that are experiencing deterioration or damage and level demand is time dependent. Deterioration of pharmaceutical goods is a thing that must be addressed in the healthcare system. So, the medicines play a role important role in patient care so a planning is a necessity. At the start of the inventory cycle, the quantity of goods will reach its maximum inventory level as [21]:

$$I\ (0) = Q$$

With:
$I\ (0)$: the current inventory total, $t = 0$
Q : maximum inventory level during the ordering cycle.

Goods inventory will progressively decrease assuming demand follows an exponential function over time. The differential equation representing this exponential demand function is typically formulated as:

$$D(t) = -Ae^{at}$$

$D(t)$: demand varies with time.
A : initial request.
e : variabel coefficient t at the demand level.
a : a constant at the request level.
t : time

2) Conduct the numerical computations using Uthayakumar and Tharani [21] data up to 3 variations to each parameter to examine model formulation.
3) Conduct a sensitivity analysis to identify which variables have the most significant influence on achieving accurate results and to evaluate changes in the model [17]. The analysis tests the impact of parameter variations on the optimal solution by adjusting each parameter by plus or minus 1 [30].
4) Provide an interpretation of the resulting model.

3. RESULTS AND DISCUSSION

3.1. Mathematical formulation of models for inventory management

The methodology is structured based on several steps, namely formulating an inventory model for pharmaceutical goods prone to deterioration, where demand levels follow an exponential pattern. Performing numerical calculations on data previously used by [21], to complete the model formulation with Wolfram Alpha software. Perform sensitivity analysis and provide an interpretation of the model obtained. On interval $[0, t_1]$, the inventory amount will be:

$$\frac{dI(t)}{dt} = -D(t) - \theta l(t), 0 \leq t \leq t_1$$
$$\frac{dI(t)}{dt} = -Ae^{at} - \theta l(t), 0 \leq t \leq t_1 \quad (1)$$

The amount of inventory when shortages occur in the interval $[t_1, T_1]$ is formulated as (1).

$$\frac{dI(t)}{dt} = -Ae^{at}, t_1 \leq t \leq T_1 \quad (2)$$

Subject to initial conditions of $I(0) = Q$ and $I(t_1) = 0$

3.2. Inventory model solution for deteriorating pharmaceutical goods with exponential demand levels

As (1), (2) suggested before, the inventory model obtained in the interval $[0, t_1]$ is as (2).

$$I\ (t) = -\frac{A}{\theta - a}\ (e^{t_r(\theta-a)-\theta t} - e^{-at}), 0 \leq t \leq t_1 \quad (3)$$

For the maximum amount of inventory obtained by providing the initial value boundary condition is $I(0) = I_0$, then calculate Q as (3).

$$I\ (t) = -\frac{A}{\theta-a}\ (e^{t_r(\theta-a)} - 1) \tag{4}$$

If the inventory level at $t = 0$, then calculate Q as (4).

$$Q = -\frac{A}{\theta-a}\ (e^{t_r(\theta-a)} - e), 0 \leq t \leq t_1 \tag{5}$$

Total requests in the interval $[0, t_1]$ are as (5).

$$\int_0^{t_1} D(t)dt = \int_0^{t_1} (Ae^{at})dt = \frac{Ae^{at_1-A}}{a} \tag{6}$$

The total number of items undergoing deterioration in the interval $[0, t_1]$, or D_T is calculated as (6).

$$D_T = Q - \int_0^{t_1} D(t_1)dt \tag{7}$$

$$D_T = \frac{-Ae^{t_1\theta-at_1-eA-At_1\theta e^{at}+Aat_1e^{at_1}}}{\theta-a} \tag{8}$$

Total storage (H) in the interval $[0, t_1]$ according to [31].

$$H = \int_0^{t_1} (a + bt)I(t)dt$$
$$H = \frac{Ae^{-at_1}}{\theta a(\theta-a)} \begin{pmatrix} (a + bt_1)(\theta - a) + a\left(ae^{\theta t_1} - \theta e^{\theta t_1}\right) + \\ \frac{b}{\theta a}\left((\theta^2 - a^2) + \left(a^2e^{\theta t_1} - \theta^2 e^{at_1}\right)\right) \end{pmatrix} \tag{9}$$

Total shortages in the interval $[t_1, T]$ as (10).

$$S = -\int_{t_1}^{T_1} I(t)dt$$
$$S = \left(-Ae^{t_1}T_1 + T_1^{\ 2} - Ae^{t_1}t_1^{\ 2} + t_1^{\ 2}\right) \tag{10}$$

3.3. Average total pharmaceutical inventory costs

The average total cost of pharmaceutical inventory over the interval $[0, T_1]$ per item per unit time $(\overline{TC})$ is as [21]:

$$\overline{TC} = \frac{1}{T_1}(A + h\,H + D_cD_T + sS)$$

$$\overline{TC} = \frac{1}{T_1}\left\{A + h.\frac{Ae^{-at_1}}{\theta a(\theta-a)}\left((a + bt_1)(\theta - a) + a\left(ae^{\theta t_1} - \theta e^{\theta t_1}\right) + \frac{b}{\theta a}\left((\theta^2 - a^2) + \left(a^2e^{\theta t_1} - \theta^2e^{at_1}\right)\right)\right) + D_c\left\{\frac{-Ae^{t_1\theta-at_1-eA-At_1\theta e^{at}+Aat_1e^{at}}}{\theta-a}\right\} + s\left(-Ae^{t_1}T_1 + T_1^{\ 2} - Ae^{t_1}t_1^{\ 2} + t_1^{\ 2}\right)\right\} \tag{11}$$

The first-order derivative of TC concerning t_1 and T_1 is as:

$$\frac{\partial\overline{TC}}{\partial t_1} = \frac{1}{T_1}\left(h\frac{\partial H}{\partial t_1} + D_c\frac{\partial D_T}{\partial t_1} + s\frac{\partial S}{\partial t_1}\right) \tag{12}$$

$$\frac{\partial\overline{TC}}{\partial T_1} = -\frac{1}{T_1^{\ 2}}A - \frac{1}{T_1^{\ 2}}hH - \frac{1}{T_1^{\ 2}}D_cD_T - \frac{1}{T_1^{\ 2}}sS + \frac{1}{T_1}s\frac{\partial S}{\partial T_1} \tag{13}$$

because $\frac{\partial D_T}{\partial T_1} = \frac{\partial H}{\partial T_1} = 0$. The solutions to the derivative in (8)-(10) are as:

$$\frac{\partial D_T}{\partial t_1} = \frac{A(\theta e^{at_1} - e^a\theta + ae^at_1)}{\theta-a} \tag{14}$$

$$\frac{\partial H}{\partial t_1} = \frac{Ae^{-at_1}}{\theta - a}\left(2b - 2a + a^2e^{\theta t_1} - \theta ae^{\theta t_1} + be^{\theta t_1} - \theta e^{at_1}\right) \tag{15}$$

$$\frac{\partial S}{\partial t_1} = \left(-3Ae^{t_1} + 2T_1\frac{T_1}{t_1} + t_1\right) \tag{16}$$

$$\frac{\partial S}{\partial t_1} = -Ae^{t_1} + 2t_1 \tag{17}$$

Substitute (14)-(16) into (12) and (17) into (13), then the following is obtained:

$$\frac{\partial \overline{TC}}{\partial t_1} = \frac{1}{T_1}\left(h\frac{dH}{dt_1} + D_c\frac{dD_T}{dt_1} + s\frac{dS}{dt_1}\right)$$

$$\frac{\partial \overline{TC}}{\partial t_1} = \frac{1}{T_1}\left(h\left(\frac{Ae^{-at_1}}{\theta - a}\left(2b - 2a + a^2e^{\theta t_1} - \theta ae^{\theta t_1} + be^{\theta t_1} - \theta e^{at_1}\right)\right) + D_c\left(\frac{A(\theta e^{at_1} - e^{a}\theta + ae^{a}t_1)}{\theta - a}\right) + s\left(-3Ae^{t_1} + 2T_1\frac{T_1}{t_1} + t_1\right)\right) \tag{18}$$

$$\frac{\partial \overline{TC}}{\partial T_1} = -\frac{1}{{T_1}^2}A - \frac{1}{{T_1}^2}hH - \frac{1}{{T_1}^2}D_cD_T - \frac{1}{{T_1}^2}sS + \frac{1}{T}s\frac{dS}{dT_1}$$

$$\frac{\partial \overline{TC}}{\partial T_1} = -\frac{1}{{T_1}^2}A - \frac{1}{{T_1}^2}h\frac{Ae^{-at_1}}{\theta a(\theta - a)}\left((a + bt_1)(\theta - a) + a\left(ae^{\theta t_1} - \theta e^{\theta t_1}\right) + \frac{b}{\theta a}\left((\theta^2 - a^2) + \left(a^2e^{\theta t_1} - \theta^2e^{at_1}\right)\right)\right) - \frac{1}{{T_1}^2}D_c\left(\frac{A(\theta e^{at_1} - e^{a}\theta + ae^{a}t_1)}{\theta - a}\right) - \frac{1}{{T_1}^2}s\left(-Ae^{t_1}T_1 + {T_1}^2 - Ae^{t_1}{t_1}^2 + {t_1}^2\right) + \frac{1}{T_1}s(-Ae^{t_1} + 2t_1) \tag{19}$$

The optimal values of t_1 and T_1 are denoted by ${t_1}^*$, and ${T_1}^*$ which are obtained by satisfying the conditions necessary to minimize the average total cost function.

$$\frac{\partial \overline{TC}}{\partial t_1} = 0 \tag{20}$$

and $\frac{\partial \overline{TC}}{\partial T_1} = 0$ (21)

3.4. Numerical calculations

In numerical calculations, data provide by Uthayakumar and Tharani [21] is used to test the model formulation. The calculations are solved until the optimal values ${t_1}^*$, ${T_1}^*$and $\overline{TC}$ are obtained using (18) and (19).

Iteration 1

– Step 1: set $T_1 = 1$ in (18) and determine t_1.

$$\frac{1}{T_1}\left(h\left(\frac{Ae^{-at_1}}{\theta - a}\left(2b - 2a + a^2e^{\theta t_1} - \theta ae^{\theta t_1} + be^{\theta t_1} - \theta e^{at_1}\right)\right) + D_c\left(\frac{A(\theta e^{at_1} - e^{a}\theta + ae^{a}t_1)}{\theta - a}\right) + s\left(-3Ae^{t_1} + 2T_1\frac{T_1}{t_1} + t_1\right)\right) = 0$$

$t_1 = 0.00331808$

– Step 2: substitute t_1 in the (19) and determine T_1 with the value of $t_1 = 0.00331808$.

$$-\frac{1}{{T_1}^2}A - \frac{1}{{T_1}^2}h\frac{Ae^{-at_1}}{\theta a(\theta - a)}\left((a + bt_1)(\theta - a) + a\left(ae^{\theta t_1} - \theta e^{\theta t_1}\right) + \frac{b}{\theta a}\left((\theta^2 - a^2) + \left(a^2e^{\theta t_1} - \theta^2e^{at_1}\right)\right)\right) - \frac{1}{{T_1}^2}D_c\left(\frac{A(\theta e^{at_1} - e^{a}\theta + ae^{a}t_1)}{\theta - a}\right) - \frac{1}{{T_1}^2}s\left(-Ae^{t_1}T_1 + {T_1}^2 - Ae^{t_1}{t_1}^2 + {t_1}^2\right) + \frac{1}{T_1}s(-Ae^{t_1} + 2t_1) = 0$$

$T_1 = 0.0728988$

– Step 3: substitute T_1 in the (18) and obtain t_1 with $T_1 = 0.0728988$.

$$\frac{1}{T_1}\left(h\left(\frac{Ae^{-at_1}}{\theta-a}\left(2b-2a+a^2e^{\theta t_1}-\theta ae^{\theta t_1}+be^{\theta t_1}-\theta e^{at_1}\right)\right)+D_c\left(\frac{A(\theta e^{at_1}-e^a\theta+ae^at_1}{\theta-a}\right)+\right.$$
$$\left.s\left(-3Ae^{t_1}+\ 2T_1\frac{T_1}{t_1}+\ t_1\right)\right)=0$$
$$t_1=0.0000177137$$

– Step 4: repeat all steps 2 and 3 until there is no change in reaching the values t_1 and T_1. The next calculation process is carried out in the same way as before, namely:

Iteration 2:

$$T_1=0.099999$$
$$t_1=0.000033331$$

Iteration 3:

$$T_1=0.059999$$
$$t_1=0.0000119997$$

Iteration 4:

$$T_1=0.01795901$$
$$t_1=0.00000107508$$

Iteration 5:

$$T_1=\ 0.0119977$$

Based on the results of the t_1 and T_1 values, it can be said that the solution has reached convergence, where the respective values of t_1 and T_1 are close to each other so that the values of ${t_1}^*$ and ${T_1}^*$(optimal values) are 0.0000011 and 0.012 subsequently, the iterative calculation ceased.

– Step 5: calculate $\overline{TC}$ by substituting the values of t_1 and T_1.

$$\overline{TC}=\frac{1}{T_1}\left\{A+h\frac{Ae^{-at_1}}{\theta a(\theta-a)}\left((a+bt_1)(\theta-a)+a\left(ae^{\theta t_1}-\theta e^{\theta t_1}\right)+\frac{b}{\theta a}\left((\theta^2-a^2)+\left(a^2e^{\theta t_1}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\theta^2e^{at_1}\right)\right)\right)+D_c\left\{\frac{-Ae^{t_1\theta-at_1}-eA-At_1\theta e^{at}+Aat_1e^{at}}{\theta-a}\right\}+s\left(-Ae^{t_1}T_1+\ {T_1}^2-Ae^{t_1}{t_1}^2+\ {t_1}^2\right)\right\}$$
$$\overline{TC}=17{,}133.9$$

The result $\overline{TC}$ obtained was $17,133.9 which shows the average minimum total cost per cycle.

3.5. Sensitivity analysis calculations

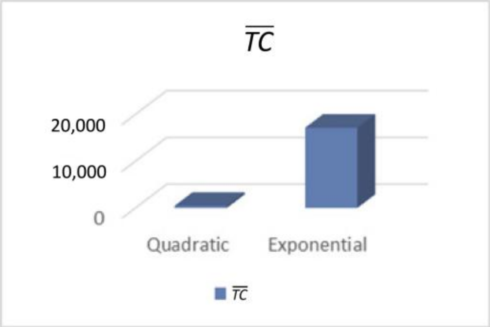
For this sensitivity analysis, the calculations for t_1, T_1, and $\overline{TC}$ are performed by varying each parameter while keeping the others constant as shown in Figure 1 by using (18) and (19) to calculate the values of the t_1 and T_1, then (11) to calculate $\overline{TC}$. Forthermore, parameter determination is carried out when for each a = 100 to 102, b = 48 to 50, Dc = 1 to 3, h = 8 to 10, s = 5 to 7, and θ = 0.0009 to 0.002. The results of the sensitivity analysis are presented in Table 1.

Based on the data presented in Table 1, the following conclusions can be drawn. On increasing the values of h, b, and D_c resulted in different values of t_1 and T_1 but at the same time the value of T_c increased. Increasing the value of $\bar{s}$ resulted in a decrease in the value of t_1 but at the same time the value of T_1 and T_c values increased. Increasing the θ value results in an increase in the t_1 and T_1values, but at the same time the value of $\overline{TC}$ decreases. An increase in the value of α results in a decrease and increase in the value of $\overline{TC},$ but at that time the same as the values of t_1 and T_1 increase. The study compares the demand rate modeled by an exponential function with the results devired from a quadratic function [21] as shown in Figure 1. According to the figure, the values of $\overline{TC}$, T_1, and t_1 are nearly identical. However, it is observed that the exponential function yields a minimum $\overline{TC}$ of $445.25 per cycle, demonstrating better performance than the quadratic function.

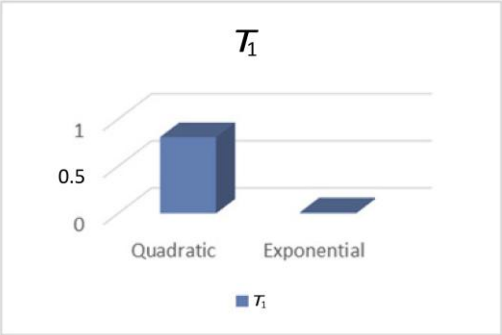
Figure 1 compares exponential and quadratic demand in three metrics. Figure 1(a) shows that the total cost $\overline{TC}$ is higher in exponential demand. Figures 1(b) and 1(c) show that the values of T_1 and t_1 is greater in the quadratic model. This indicates that the exponential pattern has more impact on the cost, while the quadratic has more impact on the time parameter. This research indicates that cost optimization with higher exponential demand is not associated with stock management in a delayed fulfillment system. The proposed method can leverage demand adjustment and the delayed fulfillment system without negatively impacting cost efficiency and stock availability.

Table 1. Sensitivity analysis result

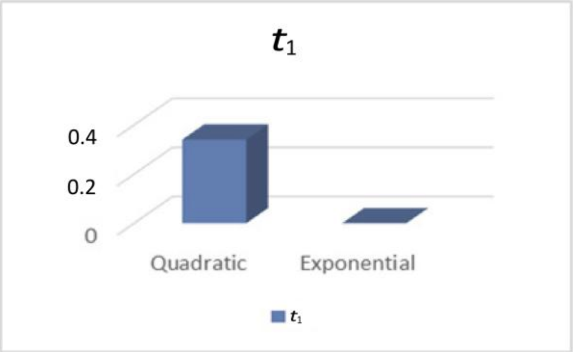
Parameter	Variation	t_1	T_1	$\overline{TC}$
θ	0.0009	1.1	0.13	108,374.04
	0.001	0.0000011	0.012	17,133.9
	0.002	2.2	0.06	7,130.05
a	100	0.0000011	0.012	17,133.9
	101	0.0000099	0.09	1,020.5
	102	0.0403905	0.021	10,792.29
b	48	11.3412	0.045	250.65
	49	1	1.0109	2,224.98
	50	0.0000011	0.012	17,133.9
D_C	1	0.01	0.046	2,835.63
	2	1.12357	1.09	9,455.64
	3	0.0000011	0.012	17,133.9
h	8	0.0561452	1.13729	29.09
	9	0.0551883	0.075	1,062.5
	10	0.0000011	0.012	17,133.9
s	5	0.00001	0.00999	21,260.08
	6	0.00001	0.00999	21,260.09
	7	0.0000011	0.012	17,133.9



(a)



(b)



(c)

Figure 1. Comparison of request rates between exponential and quadratic demand level (a) in terms of $\overline{TC}$, (b) in terms of T_1, and (c) in terms of t_1

4. CONCLUSION

Based on the outcomes of the inventory model for deteriorating goods with an exponential demand rate, the following summary can be provided. The model begins with an inventory level of Q and decreases continuously over time within the interval $[Q , t_1]$ during one cycle. Consequently, shortages may occur, and there is a waiting period until the next order is initiated (T_1), assuming zero lead time, ensuring that orders arrive promptly upon placement. The derived inventory model is expressed by (11).

In the optimal solution, t_1 and T_1 are 0.0000011 and 0.012 with an average total minimum cost was $17,133.9 per cycle. The results of sensitivity analysis to changes in value produce the value $\overline{TC}$ decrease in all parameters. On increasing the values of h, b and D_c resulted in different values of t_1 and T_1 but at the same time the value of T_c increased. Increasing the value of $\bar{s}$ resulted in a decrease in the value of t_1 but at the same time the value of T_1 and T_c values increased. Increasing the θ value results in an increase in the t_1 and T_1 values, but at the same time the value of $\overline{TC}$ decreases. An increase in the value of α results in a decrease and increase in the value of $\overline{TC}$, but at that time the same as the values of t_1 and T_1 increase. For further research, the possibility to also include the storage level due to in real implication, the storage level indeed affect maximum total cost.

ACKNOWLEDGMENTS

We would like to precise our profound appreciation to all viewers who taken part within the process of review for this issue's article. The proficient help and back of all prominent reviewers made this manuscript is qualified to be published.

FUNDING INFORMATION

We declare that we have no funding involved.

AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Indrawati	✓	✓	✓	✓	✓	✓		✓	✓	✓			✓	✓
Fitri Maya Puspita	✓	✓				✓		✓	✓	✓	✓	✓		
Siti Suzlin Supadi	✓		✓	✓			✓			✓	✓	✓		
Evi Yuliza	✓			✓		✓						✓		
Farah Nabilah Tampubolon	✓								✓				✓	

C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

We declare that we have no conflict of interest.

DATA AVAILABILITY

The data that support the findings of this study are available from Uthayakumar and Tharani [21] for use of comparing models.

REFERENCES

[1] M. M. J. Basha, N. V.S, S. T. Wani, and P. V. S. Gogi, "Study of inventory management in pharmaceuticals: a review of COVID-19 situation," *International Journal of Innovative Science and Research Technology*, vol. 5, no. 8, pp. 366–371, 2020, doi: 10.38124/ijisrt20aug257.

[2] A. William and W. Lee, "An inventory model to control multi-item medicines with consideration of space capacity in the hospital and joint replenishment," *International Journal of Industrial Engineering and Engineering Management*, vol. 2, no. 2, pp. 39–48, 2020.

[3] S. Priyan and P. Mala, "Optimal inventory system for pharmaceutical products incorporating quality degradation with expiration date: A game theory approach," *Operations Research for Health Care*, vol. 24, p. 100245, 2020, doi: 10.1016/j.orhc.2020.100245.

[4] M. Zheng, N. Du, H. Zhao, E. Huang, and K. Wu, "A study on the optimal inventory allocation for clinical trial supply chains," *Applied Mathematical Modelling*, vol. 98, pp. 161–184, 2021, doi: 10.1016/j.apm.2021.04.029.

[5] S. Chakraborty, R. D. Raut, T. M. Rofin, S. Chatterjee, and S. Chakraborty, "A comparative analysis of multi-attributive border approximation area comparison (MABAC) model for healthcare supplier selection in fuzzy environments," *Decision Analytics Journal*, vol. 8, p. 100290, 2023, doi: 10.1016/j.dajour.2023.100290.

[6] S. Romdhani, I. Nouaouri, J. Tounsi, S. Gattoufi, and H. Allaoui, "Two-echelon inventory management for sustainable pharmaceutical supply chain through waste reduction," *IFAC-PapersOnLine*, vol. 55, no. 10, pp. 1380–1385, 2022, doi: 10.1016/j.ifacol.2022.09.583.

[7] Rachmat Hidayat and Irsan Saleh, "The importance of inventory management in pharmaceutical practice," *Open Access Indonesia Journal of Social Sciences (OAIJSS)*, vol. 3, no. 1, pp. 1–9, 2020, doi: 10.37275/oaijss.v3i1.22.

[8] F. Fadli, S. Sudrajat, and E. Lesmana, "An inventory model for deteriorating items with exponential declining demand and return," *Jurnal Ilmiah Sains*, vol. 20, no. 1, p. 31, 2020, doi: 10.35799/jis.20.1.2020.27767.

[9] S. S. Kurade and R. Latpate, "Demand and deterioration of items per unit time inventory models with shortages using genetic algorithm," *Journal of Management Analytics*, vol. 8, no. 3, pp. 502–529, 2020, doi: 10.1080/23270012.2020.1829113.

[10] R. Mondal, A. A. Shaikh, A. K. Bhunia, I. M. Hezam, and R. K. Chakrabortty, "Impact of trapezoidal demand and deteriorating preventing technology in an inventory model in interval uncertainty under backlogging situation," *Mathematics*, vol. 10, no. 1, 2022, doi: 10.3390/math10010078.

[11] A. H. Md Mashud, "An EOQ deteriorating inventory model with different types of demand and fully backlogged shortages," *International Journal of Logistics Systems and Management*, vol. 36, no. 1, pp. 16–45, 2020, doi: 10.1504/IJLSM.2020.107220.

[12] H. L. Yang, "An optimal replenishment cycle and order quantity inventory model for deteriorating items with fluctuating demand," *Supply Chain Analytics*, vol. 3, no. May, p. 100021, 2023, doi: 10.1016/j.sca.2023.100021.

[13] L. A. San-José, J. Sicilia, L. E. Cárdenas-Barrón, and M. González-de-la-Rosa, "A sustainable inventory model for deteriorating items with power demand and full backlogging under a carbon emission tax," *International Journal of Production Economics*, vol. 268, no. August 2023, p. 109098, 2024, doi: 10.1016/j.ijpe.2023.109098.

[14] F. Akhtar, M. Al-Amin Khan, A. A. Shaikh, and A. F. Alrasheedi, "Interval valued inventory model for deterioration, carbon emissions and selling price dependent demand considering buy now and pay later facility," *Ain Shams Engineering Journal*, vol. 15, no. 3, p. 102563, 2024, doi: 10.1016/j.asej.2023.102563.

[15] P. Saranya and E. Chandrasekaran, "Optimal inventory system for deteriorated goods with time-varying demand rate function and advertisement cost," *Array*, vol. 19, no. April, p. 100307, 2023, doi: 10.1016/j.array.2023.100307.

[16] A. Limi, K. Rangarajan, P. Rajadurai, A. Akilbasha, and K. Parameswari, "Three warehouse inventory model for non-instantaneous deteriorating items with quadratic demand, time-varying holding costs and backlogging over finite time horizon," *Ain Shams Engineering Journal*, vol. 15, no. 7, p. 102826, 2024, doi: 10.1016/j.asej.2024.102826.

[17] M. Rukonuzzaman, M. Al-Amin Khan, A. R. Khan, A. AlArjani, M. S. Uddin, and E. A. Attia, "Effects of a quantity-based discount frame in inventory planning under time-dependent demand: a case study of mango businesses in Bangladesh," *Journal of King Saud University - Science*, vol. 35, no. 7, p. 102840, 2023, doi: 10.1016/j.jksus.2023.102840.

[18] S. Aka and G. Akyüz, "An inventory and production model with fuzzy parameters for the food sector," *Sustainable Production and Consumption*, vol. 26, pp. 627–637, 2021, doi: 10.1016/j.spc.2020.12.033.

[19] N. Mohamadi, S. T. A. Niaki, M. Taher, and A. Shavandi, "An application of deep reinforcement learning and vendor-managed inventory in perishable supply chain management," *Engineering Applications of Artificial Intelligence*, vol. 127, p. 107403, 2024, doi: 10.1016/j.engappai.2023.107403.

[20] R. Uthayakumar and S. . Karuppasamy, "A pharmaceutical inventory model for healthcare industries with quadratic demand, linear holding cost and shortages," *International Journal of Pure and Applied Mathematics*, vol. 106, no. 8, pp. 73–83, 2016, doi: 10.12732/ijpam.v106i8.11.

[21] R. Uthayakumar and S. Tharani, "An inventory model for deteriorating pharmaceutical items with time dependent demand under complete backlogging," *Communications in Applied Analysis*, vol. 22, no. 4, pp. 511–530, 2018, doi: 10.12732/caa.v22i4.2.

[22] S. Sutrisno, W. Widowati, and R. H. Tjahjana, "Control of inventory system with random demand and product damage during delivery using the linear quadratic gaussian method," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 22, no. 3, pp. 1748–1753, 2021, doi: 10.11591/ijeecs.v22.i3.pp1748-1753.

[23] O. Dwipurwani, F. M. Puspita, S. S. Supadi, and E. Yuliza, "Minitab 20 and Python based-the forecasting of demand and optimal inventory of liquid aluminum sulfate supplies," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 35, no. 3, pp. 1796–1807, 2024, doi: 10.11591/ijeecs.v35.i3.pp1796-1807.

[24] A. U. Rehman, S. H. Mian, Y. S. Usmani, M. H. Abidi, and M. K. Mohammed, "Modelling and analysis of hospital inventory policies during COVID-19 pandemic," *Processes*, vol. 11, no. 4, 2023, doi: 10.3390/pr11041062.

[25] W. H. Ritonga, Y. Samri, J. Nasution, and T. Anggraini, "Planning and monitoring operational costs for purchasing goods," *Jurnal Akuntansi dan Perbankan Syariah*, vol. 05, no. 01, pp. 93–104, 2022, doi: 10.32332/finansia.v5i1.6307.

[26] B. Chen, D. Simchi-Levi, Y. Wang, and Y. Zhou, "Dynamic pricing and inventory control with fixed ordering cost and incomplete demand information," *Management Science*, vol. 68, no. 8, pp. 5684–5703, 2022, doi: 10.1287/mnsc.2021.4171.

[27] A. Gurtu, "Optimization of inventory holding cost due to price, weight, and volume of items," *Journal Risk and Financial. Management*, vol. 14, no. 2, 2021, doi: 10.3390/jrfm14020065.

[28] B. A. Kumar and S. K. Paikray, "Cost optimization inventory model for deteriorating items with trapezoidal demand rate under completely backlogged shortages in crisp and fuzzy environment," *RAIRO - Operations Research*, vol. 56, no. 3, pp. 1969–1994, 2022, doi: 10.1051/ro/2022068.

[29] A. A. Istiningrum, L. M. Munandar, and S. Sono, "Reducing spare part inventory cost with shortage elimination through probabilistic economic order quantity," *Kinerja*, vol. 25, no. 2, pp. 179–191, 2021, doi: 10.24002/kinerja.v25i2.4371.

[30] M. Derbel, W. Hachicha, and A. M. Aljuaid, "Sensitivity analysis of the optimal inventory-pooling strategies according to multivariate demand dependence," *Symmetry (Basel).*, vol. 13, no. 2, pp. 1–24, 2021, doi: 10.3390/sym13020328.

[31] B. P. Dash, T. Singh, and H. Pattnayak, "An inventory model for deteriorating items with exponential declining demand and time-varying holding cost," *American Journal of Operations Research*, vol. 04, no. 01, pp. 1–7, 2014, doi: 10.4236/ajor.2014.41001.

BIOGRAPHIES OF AUTHORS

Indrawati obtained her S.Si degree in mathematics from Sriwijaya University, South Sumatera, Indonesia in 1996. Then she received her M.Si in actuarial science in 2004 at Bandung Institute of Technology. She has been a Mathematics Department member at The Faculty of Mathematics and Natural Sciences at Sriwijaya University South Sumatera Indonesia since 1998. She is currently a Ph.D. student in Mathematics and Natural Sciences Study Program of Sriwijaya University since 2022. Her research interests include optimization, actuarial science, insurance problems and operations research focusing on inventory problem. She can be contacted at email: indrawati@mipa.unsri.ac.id.

Fitri Maya Puspita received her S.Si degree in mathematics from Sriwijaya University, South Sumatera, Indonesia in 1997. Then she received her M.Sc. in mathematics from Curtin University of Technology (CUT) Western Australia in 2004. She received her Ph.D. in science and technology in 2015 from Universiti Sains Islam Malaysia. She has been a Mathematics Department member at Faculty of mathematics and Natural Sciences at Sriwijaya University in South Sumatera Indonesia since 1998. Her research interests include optimization and its applications such as vehicle routing problems and QoS pricing and charging in third generation internet. She can be contacted at email: fitrimayapuspita@unsri.ac.id.

Siti Suzlin Supadi received her B.Sc. in mathematics from University of Malaya in 2001. Then she received her M.Sc. from University of Malaya in 2004 and her research interest is applied mathematics. She got her Ph.D. from University of Malaya in 2012 and her research interest is applied mathematics. She has been a Institute of Mathematical Sciences at Faculty of Science University of Malaya, Kuala Lumpur. Her research interest includes operation research (inventory modelling, vendor-buyer coordination). She can be contacted at email: suzlin@um.edu.my.

Evi Yuliza obtained her S.Si degree in mathematics from Sriwijaya University, South Sumatera, Indonesia in 2000. Then she received her M.Si in Universitas Gadjah Mada in 2004. She received her Ph.D. in mathematics and natural science in 2021 from Sriwijaya University. She has been a Mathematics Department member at Faculty of mathematics and Natural Sciences Sriwijaya University South Sumatera Indonesia since 2008. Her research interests include optimization, focusing on vehicle routing problems and its varians. She can be contacted at email: eviyuliza@mipa.unsri.ac.id.

Farah Nabilah Tampubolon obtained her S.Si degree in mathematics from Sriwijaya University, South Sumatera, Indonesia in June, 2024. Her research interests include operations research, focusing on inventory model for pharmaceutical items. She can be contacted at email: farahnabilahtampubolon11@gmail.com.