

Improved Reaching Law Sliding Mode Control Algorithm Design for DC Motor Based on Kalman Filter

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Abstract

Aiming at the inaccurately modeling and some uncertain existing in servo system seriously affected the control quality and the instability problem, sliding mode control algorithm with improved reaching law is proposed in this paper. The improved reaching law is used to weaken the chattering problem existing in the sliding mode control. Also the kalman filter is used to inhibit the interference, which make the servo system have strong anti-interference ability and the ability of weakening the chattering problem existing in the sliding mode control. The simulation results show that the algorithm can effectively inhibit the external disturbance and noise existing in the system, and make the system have strong anti-interference ability. At the same time, the chattering also is obviously inhibited, and the method makes the system stability and control quality been further improved.

Keywords: DC motor, SMC, Kalman filter, improved reaching law, chattering, simulation

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1. Introduction

Train control system, abbreviated as TCS and its task is to control the velocity [1] and the distance [2] between trains in the railway and to protect the safe and high efficiency of train running [3, 4]. The train position is very important in the TCS simulation environment. An ideal TCS testing and simulation platform should simulate the real railway operating environment and study the TCS theory. In testing and simulation platform, different people have the different concern about the train, such as the policymaker and researcher, etc. Therefore, the 3D visualization of train can be separated two aspects.

Sliding mode variable structure control is a special nonlinear control method, which is actually the discontinuity of control. The control strategy is different from other control method, because the structure of system is mainly unfixed, but the system can be changed in the dynamic process according to the current state of the system which is changed by some destination. And the method makes system move according to the state trajectory of sliding mode. Because the sliding mode can be designed and has nothing to do with the object parameter and disturbance, which makes the variable structure control have quick response, and be not sensitive to the disturbance and parameter change [1]. But due to the influence of factors such as time delay switch, spatial lag switch and the system inertia [2], it is difficult for the system to strictly sliding along the sliding mode for the balance, but crossing back and forth on both sides of the sliding mode surface to produce chattering. Due to vibration, it is easy to motivate the high frequency modeling dynamic in the system, and damage the system performance. In order to weaken the chattering of the system, Gao Weibing proposed the reaching law to weaken chattering problem produced by variable structure control. The paper [3] designed a variable structure controller with a filter, which can effectively eliminate the chattering of control signal. With the development of artificial intelligence, scholars have put forward the method based on artificial intelligence to solve the chattering problems existing in the sliding mode control, Such as the fuzzy method, neural network method; Genetic optimization algorithm, etc. [1].

This paper has proposed a sliding mode control algorithm with improved reaching law and kalman filter technology for the DC servo system, which can make the system have fast tracking performance and improve the ability of weakening the chattering. The simulation results

show that the proposed control scheme can effectively improve the dynamic characteristic of the system, and has strong anti-interference ability and weaken the chattering ability.

2. System Description

2.1. Mathematic Model

With the development of the modern industry, DC motor is used as the Executive Termination for the servo system widely. The mathematical model of the magnetic brushless DC motor is established based on the working principle of magnetic brushless DC motor [4]. The working principle and the equivalent circuit diagram of DC motor was depicted in the following figure (entitled Figure 1).

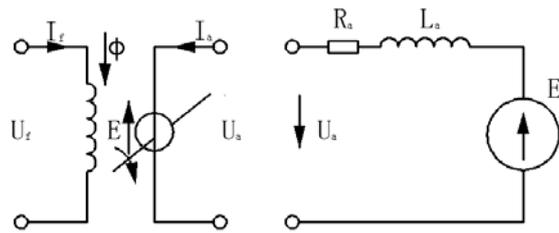


Figure 1. Working principle and the equivalent circuit diagram of DC motor

From the working principle of magnetic brushless DC motor, we can obtain the voltage balance equation of DC motor armature circuit:

$$U_a = E_a + R_a I_a = C_E \Phi n + I_a R_a = K_E n + R_a I_a \quad (1)$$

The DC motor's dynamic equation of is given as follows:

$$u_a = K_E n + R_a i_a + L_a \frac{di_a}{dt} \quad (2)$$

In the Equation (2), the parameters R_a is the loop resistance, I_a is the loop current, E_a is the induction electromotive force, U_a is the voltage of the circuit, C_E is the electromotive force constant, n is the motor's speed, K_E is the electromotive force which is produced by unit speed.

Balanced equation of electrodynamics is as follows:

$$T_e = Bn + T_L + J \frac{dn}{dt} = C_E \Phi i_a = K_T i_a \quad (3)$$

Where T_e is the instantaneous electromagnetic torque, T_L is the load torque, B is damping coefficient, J is the moment of inertia, K_T is torque constant.

Assuming the initial conditions is zero, the motor load is constant, the equations are transformed by the Laplace. The transfer function of DC motor can be obtained as follows:

$$G(s) = \frac{n(s)}{U(s)} = \frac{K_T}{L_a J s^2 + (R_a J + L_a B) s + R_a B + K_T K_E} \quad (4)$$

2. Design for Discrete Sliding Mode Controller

2.1. Reaching Law

Domestic experts proposed the reaching law approach to reduce or inhibit the chattering of SMC in the premise of ensuring the condition of sliding existence $S\dot{S} < 0$ has been meet. And given four different methods of reaching law, such as constant reaching law, exponential approach law, power reaching law and general reaching law, where the exponential approach is applied widely [2], but the discrete form of exponential reaching law's switching zone is zonal. So which can not be close to the origin ultimately but a chattering near the origin during the process moving [2]. In order to solve the chattering phenomenon of exponential reaching law near the origin [5, 6] proposed a variable rate reaching law for continuous system, it's discrete form as follows:

$$S(k+1) - S(k) = -\varepsilon T \|X\|_1 \operatorname{sgn}(S(k)) \quad (5)$$

Where: $\|X\|_1$ -- 1 norm of x.

Reaching speeds of variable rate reaching law is $\varepsilon \|X\|_1$, and is proportional to $\|X\|_1$. The switching zone pass through the origin with two rays, which can make $S = 0$ in the middle of the two rays, can be stabilized at the origin, However, when the system just entered to switching zone $\|X\|_1$ will get a large value, and have a big chattering in SMC. In order overcome the problem of the variable rate reaching law and exponential approach law, [7] has proposed a new reaching law as follows:

$$S(k+1) = (1 - Tq)S(k) - \varepsilon T \tan \operatorname{sig}(\|X\|_1) \operatorname{sgn}(S(k)) \quad (6)$$

Where:

$$\tan \operatorname{sig}(\|X\|_1) = 2 \operatorname{sig}(\|X\|_1) - 1 = \frac{1 - e^{-\|X\|_1}}{1 + e^{-\|X\|_1}}$$

2.2. Design of Sliding Model Controller

In this paper, in order to enhance the ability of anti-interference, and reduce the chattering of VSC, we made use improved reaching law and kalman filter to design the controller, the structure of the controller was shown in Figure 4 [8].

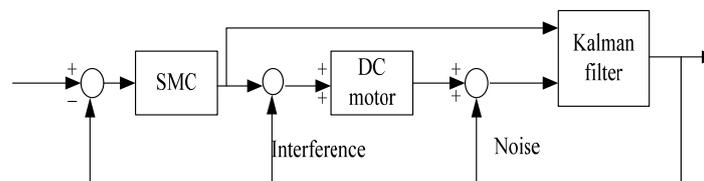


Figure 4. The diagram of controller

Assuming $G(s)$ is the transfer function of servo system, and its state space equation as follow:

$$x(k+1) = Ax(k) + Bu(k) \quad (7)$$

Where: $x(k) = [x_1(k) \quad x_2(k)]^T$

$x_1(k)$ -- The actual velocity.

$x_2(k)$ -- The changing rate of velocity.

Supposed $r(k)$, $dr(k)$ as the velocity order and it's change rate. $R_k = [r(k), dr(k)]$, predicted by linear extrapolation R_{k+1} is:

$$r(k+1) = 2r(k) - r(k-1) \quad (8)$$

$$dr(k+1) = 2dr(k) - dr(k-1) \quad (9)$$

Define switching function is:

$$S(k) = C_e E = C_e (R_k - x(k)) \quad (10)$$

$$S(k+1) = C_e E = C_e (R_{k+1} - x(k+1)) = C_e [R_{k+1} - Ax(k) - Bu(k)] \quad (11)$$

The novel reaching law's discrete form is:

$$S(k+1) = (1 - Tq)S(k) - \varepsilon T \tan \text{sig}(\|X\|_1) \text{sgn}(S(k)) \quad (12)$$

From the above equation. The sliding mode control law of servo system is:

$$u(k) = (C_e B)^{-1} [C_e R_{k+1} - C_e Ax(k) - (1 - Tq)S(k) + \varepsilon T \tan \text{sig}(\|X\|_1) \text{sgn}(S(k))] \quad (13)$$

Where: $C_e = [c, 1]$

3. Kalman Filtering Operator

The continuous model of control system is converted into a discrete model, the discrete state equation and measurement equation are given as:

$$\begin{aligned} x(k) &= Ax(k-1) + B(u(k) + w(k)) \\ y_v(k) &= Cx(k) + v(k) \end{aligned} \quad (14)$$

Where $x(k)$ and $y_v(k)$ respectively are the state vector and observation vector. A is the state matrix, B is the control matrix, C is the output observation matrix, $w(k)$ is the process noise signal, $v(k)$ is the observation noise signal.

The flow chart of kalman filtering algorithm is shown in Figure 5 [9]:

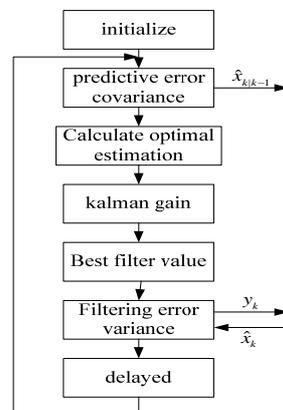


Figure 5. The flow chart of kalman filtering

10]: The recursive algorithm of the corresponding kalman filter of flow chart is given as [9,

The best estimation is:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_{k-1} \hat{\mathbf{x}}_{k-1}; \quad (15)$$

The error variance of expect estimates is:

$$\mathbf{P}_{k|k-1} = \mathbf{A}_{k-1} \mathbf{P}_{k-1} \mathbf{A}_{k-1}^T + \mathbf{B}_{k-1} \mathbf{Q} \mathbf{B}_{k-1}^T; \quad (16)$$

The kalman gain is:

$$\mathbf{K}_k = \mathbf{p}(k) \mathbf{C}^T [\mathbf{C} \mathbf{p}(k) \mathbf{C}^T + \mathbf{R}]^{-1}; \quad (17)$$

The update estimation is:

$$\hat{\mathbf{x}}_k = \mathbf{A}_{k-1} \hat{\mathbf{x}}_{k-1} + \mathbf{K}_k (\mathbf{y}_v - \mathbf{C} \mathbf{A}_{k-1} \hat{\mathbf{x}}_{k-1}); \quad (18)$$

The updated estimate covariance is:

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{C}) \mathbf{P}_{k|k-1}; \quad (19)$$

The output of filter is:

$$\mathbf{y}_e = \mathbf{C} \hat{\mathbf{x}}_k; \quad (20)$$

Where Q, R are the covariance matrix of random noise $w(k), v(k)$ respectively.

4. Numerical Simulation

In order to verify the effectiveness of the controller, use MATLAB to make simulation for the DC. the servo system's status equation [8]:

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{A} \mathbf{x}(k-1) + \mathbf{B}(u(k) + w(k)) \\ \mathbf{y}(k) &= \mathbf{C} \mathbf{x}(k) + v(k) \end{aligned} \quad (21)$$

Where:

$$\mathbf{A} = \begin{bmatrix} 1.0 & 0.0010 \\ 0 & 0.9753 \end{bmatrix}; \quad \mathbf{B} = [0.000 \quad 0.1314]; \quad \mathbf{C} = [1 \quad 0]; \quad \mathbf{D} = 0;$$

$w(k)$ is the process white noise signal; $v(k)$ is the observation white noise signal. In order to prove the effect of system, the system with kalman filter and the system without kalman filter are respectively simulated. The simulation parameters are given as: $c = 40, \varepsilon = 130, q = 280, x = [-0.5, -0.5], \Delta = 0.05$.

The simulation results are given in the Figure 6-Figure 9:

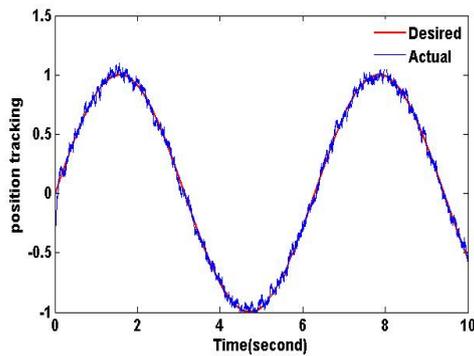


Figure 6. Position trajectory without filter

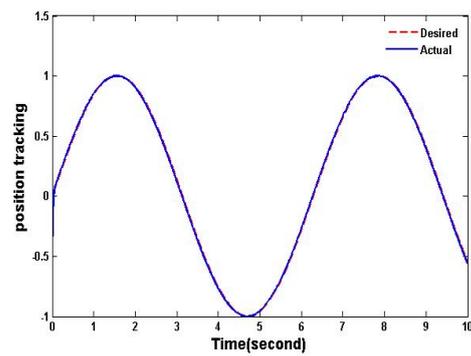


Figure 7. Position trajectory with filter

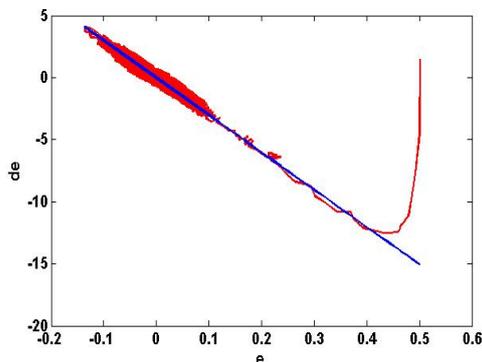


Figure 8. Phase trajectory without filter

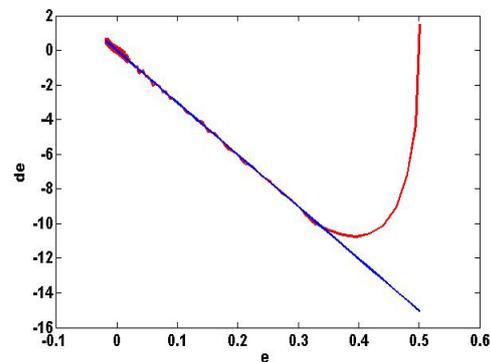


Figure 9. Phase trajectory with filter

The Figure 6-Figure 7 is the position trajectory of the Figure 8-Figure9 is the phase trajectory. From the simulation results (Figure 6-Figure 9) we can obtain the conclusions that the external disturbance and noise of system after adding kalman filter can be effectively restrained, the chattering of the sliding mode variable structure control is also inhibited, and can better realize the function of the controller.

5. Conclusion

Aim at the inaccurately modeling and some uncertain existing in servo system seriously affected the control quality and the instability problem, the sliding mode variable structure is applied to the servo system. Considering the chattering problem of servo sliding mode variable structure control and the existing interference, which affect the quality and stability of system control, a reaching law approach is used to weaken the chattering problem existing in the sliding mode control. At the same time, kalman filter is used to inhibit the interference, thus to improve the quality and stability of servo system. System simulations show that the scheme can effectively suppressed external disturbance and noise, which makes the system have strong anti-interference ability. And the chattering of the sliding mode variable structure control also had been obviously inhibited, the system stability and control quality has also been further improved. stability and control quality been further improved.

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