

Step Responses of Tuned Conventional Controller for Three Tank System

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Abstract

In this paper modeling of a temperature measuring tank system has been done and then a tuned PID Controller have been used for controlling the step responses of the system. The proposed system extends to a three tank system & each tank has same amount of liquid. The results of computer simulation for the system with Proportional, Integral and Derivative (PID) Controllers are shown here using MATLAB (R2007b) software.

Keywords: temperature, tanksystem, control, nonlinear control, PID controller

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1. Introduction

The temperature measurement of liquid in a tank can be controlled by classical and advance control algorithm PID. Here we are considering a three tank non-interacting system. We observed that tank1 affects the dynamic behavior of tank2, similarly for tank2 affects the dynamic behavior [2] of tank3 and vice versa, because the flow rate depends on the difference between liquid levels h_1 and h_2 . Thus a change in the inlet flow rate affects the liquid level in the tank, which intern affects the temperature of the liquid. Basically it is a thermal process. Various type of temperature sensor RTD, T/C, thermistor [1], [9-10]. In that particular project we used a mercury thermometer as sensor. Mathematical model of three tank method give a third order [6] equation. Each tank gives a transfer function of first order system. A lot of work has been carried out on the temperature control in terms of its stabilization. Many attempts have been made to control the response of temperature measuring system. This method is utilized to investigate [3] global properties of the designed controller.

2. Mechanical Construction

The system comprises of a mercury-in-glass thermometer placed in a liquid tank to measure the temperature of the liquid which is heated by steam through a coil system. The temperature of the liquid (T_F) varies [5] with time. T is the temperature of the mercury in the well of the thermometer. The following assumptions are made to determine the transfer function relating the variation of the thermometer (T) for change in the temperature of the liquid (T_F).

(1) The expansion or contraction of the glass walled well containing mercury is negligible (that means the resistance offered by glass wall for heat transfer is negligible)

(2) The liquid film surrounding the bulb is the only resistance to the heat transfer.

(3) The mercury assumes isothermal condition throughout. The system is shown in Figure 1.

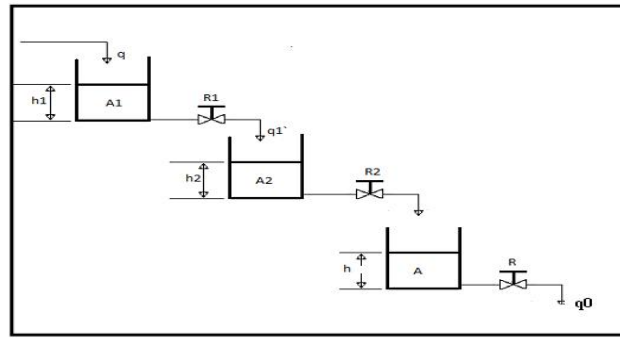


Figure 1. Three tank system

3. Proposed Mathematical Model

Applying unsteady state heat balance for the bulb, we get
 Input heat rate- Output heat rate=Rate of heat accumulation.

$$\begin{aligned}
 UA(T_F - T) - 0 &= MC_P \frac{dT}{dt} \\
 UA(T_F - T) &= MC_P \frac{dT}{dt}
 \end{aligned} \tag{1}$$

Where, A=surface area of the bulb for heat transfer in m^2
 M=Mass of mercury in the bulb, kg
 C_P =Heat capacity of the mercury in $kJ/kg\cdot K$
 U=Film heat transfer co-efficient, $kw/m^2\cdot K$

At steady state, the equation (1) can be rewritten as:

$$UA(T_{FS} - T_F) = 0 \tag{2}$$

Subtracting Equation (2) from Equation (1).

$$UA[(T_F - T_{FS}) - (T - T_S)] = MC_P \frac{d(T - T_S)}{dt}$$

Defining the deviation variables, $T_F - T_{FS} = T_1$ and $T - T_S = T_1$ and substituting in the above equation, we get:

$$\begin{aligned}
 UA(T_{F1} - T_1) &= MC_P \frac{dT_1}{dt} \\
 (T_{F1} - T_1) &= \frac{MC_P}{UA}
 \end{aligned} \tag{3}$$

Defining time constant t_p for the Thermometer,

$$t_p = M \frac{C_P}{UA}$$

Equation (3) can be rewritten as:

$$T_{F1} - T_1 = t_p \frac{dT_1}{dt} \tag{4}$$

Taking Laplace transform, we get:

$$\begin{aligned}
 T_{F1}(S) - T_1(S) &= t_p s T_1(S) \\
 \frac{T_1(S)}{T_{F1}(S)} &= \frac{1}{1 + t_p s} \text{ transfer function of Tank 1}
 \end{aligned}$$

Similarly, for tank2 & tank3 we can get a first order system. So we can able to say that the entire system is a third order system. Here we can construct the overall transfer function of the three tank system as:

$$G(S) = G_1(S) \times G_2(S) \times G_3(S) \\ = \left(\frac{K_1}{1+t_{p1}S} \right) \times \left(\frac{K_2}{1+t_{p2}S} \right) \times \left(\frac{K_3}{1+t_{p3}S} \right)$$

4. Transfer Function Modeling

As per our problem, let us assume:

t_{p1} = time constant for tank1 = 0.5 minute

t_{p2} = time constant for tank2 = 1.2 minute

t_{p3} = time constant for tank3 = 1.5 minute

$K_1 = R_1 = 0.25$ min

$K_2 = R_2 = 0.30$ min

$K_3 = R_3 = 0.35$ min

$$G(S) = \frac{0.02625}{0.9S^3 + 3.15S^2 + 3.2S + 1}$$

This transfer function is called plant transfer function. The entire experimental set up is given in Figure 2.

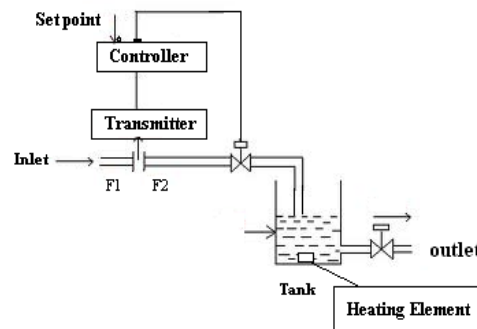


Figure 2. Proposed experimental set up

5. Closed Loop Tuning Process

As per our problem, applying a closed loop Tuning technique we can write:

$$1 + G(S)H(S) = 0 \\ \text{Or, } 1 + \frac{0.02625K}{0.9S^3 + 3.15S^2 + 3.2S + 1} = 0 \\ \text{Or, } 0.9S^3 + 3.15S^2 + 3.2S + 0.02625K = 0$$

Now according to Routh Process:

$$\begin{array}{r} S^3 \quad 0.9 \qquad \qquad \qquad 3.2 \\ S^2 \quad 3.15 \qquad \qquad \qquad (1+0.02625K) \\ S^1 \quad \frac{0.9(1+0.02625K) - 3.15 \times 3.2}{3.15} \quad 0 \end{array}$$

So by solving it,

$$K = 388.57 = K_{\text{critical}}$$

Now its Auxiliary equation:

$$3.15s^2 + (1 + 0.02625k) = 0$$

$$\text{Or, } 3.15s^2 + (1 + 0.02625 \cdot 388.57) = 0$$

$$\text{Or, } s^2 = -11.199/3.15 = -3.556$$

$$\text{Or, } (jW)^2 = -3.556$$

$$\text{Or, } W = 1.8841 \text{ rad/sec}$$

$$\text{So, } K_P = K_{\text{Critical}}$$

$$\&P_U = \frac{2 \times 3.14}{1.8841} = 3.33 \text{ min/cycle}$$

6. Simulink Design

Here we design the tuned controller block with plant transfer function, shown in Figure 3:

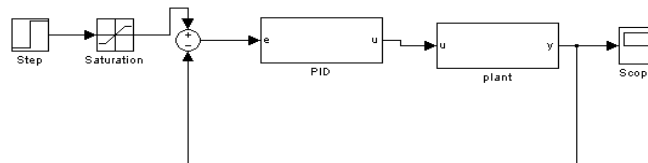


Figure 3. Simulink block diagram

7. Different Process Performance Indices

Table 1 indicate different performance indices that we consider for this particular experiment is defined as:

Performance Indexes	PID
ISE	1.24
ITSE	0.92
IAE	1.12
ITAE	2.05

8. Tuning Parameter Evaluation

Table 2 is defined the different parameter value for designing the system that we can get from closed loop tuning technique-

Sl. no.	K_P	K_P/T_I	$K_P T_D$
1	233.142	110.025	136
2	233.142	75	124

9. Simulation Result

After simulation we get three step responses for different parameter & tuning values. These step responses are shown in Figure 4(a), Figure 4(b) & Figure 4(c).

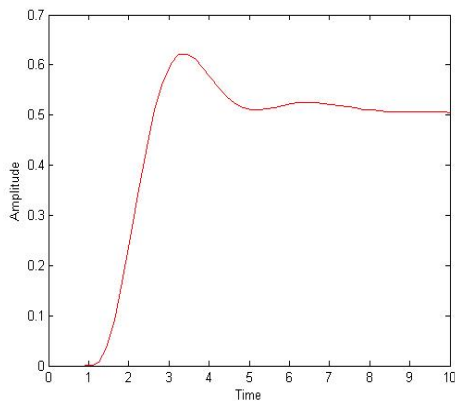


Figure 4(a). Step responses of plant for $K_P=233.142$, $K_P/T_I=110.025$, $K_P T_D=136$

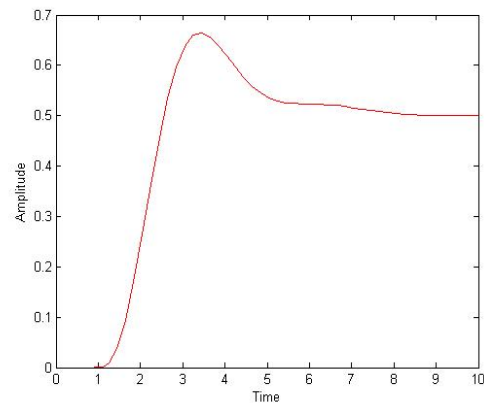


Figure 4(b). Step responses of plant for $K_P=233.142$, $K_P/T_I=75$, $K_P T_D=124$

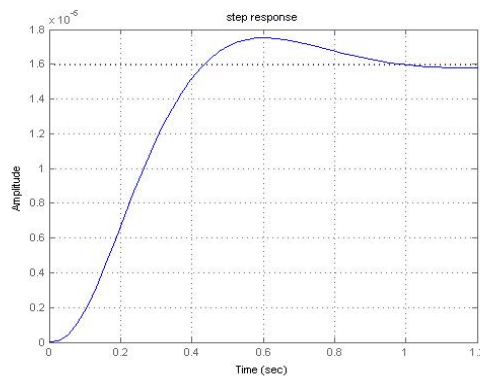


Figure 4(c). Step responses of overall system

10. Conclusion

Modeling of three tank temperature measuring system shows that system is unstable for a certain range. That's why we tried to design a conventional controller strategy process so that we can minimize the steady state error & maximize the settling time. In future we may use Genetic Algorithm for designing some advanced controlling strategy.

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