

# A Novel Technique for Vector Control of Single-Phase Induction Motors Based on Rotor Field-Oriented Control

Mohammad Jannati\*, Sajad Abdollahzadeh Anbaran, Nik Rumzi Nik Idris, Mohd Junaidi Abdul Aziz

UTM-PROTON Future Drive Laboratory, Faculty of Electrical Engineering,  
Universiti Teknologi Malaysia, UTM Skudai, 81310 Johor, Malaysia

\*Corresponding author, e-mail: m\_jannatyy@yahoo.com

## Abstract

The Variable Frequency Control (VFC) techniques such as scalar and vector control methods are recommended in numerous industrial and domestic applications. They can provide the saving energy and cost economy for Induction Motors (IMs). Due to asymmetrical characteristics of single-phase IMs, conventional vector control methods for three-phase IMs cannot be directly applied to single-phase IMs. This study aims to improve performance of the single-phase IMs drive system. Hence, a novel control approach based on Rotor Field-Oriented control (RFOC) was developed for single-phase IMs. For this purpose, transformation matrices are applied to the machine equations. It is shown by applying these transformation matrices to the unbalanced equations of single-phase IM results in set of balanced equations with backward and forward components. Therefore, by some modifications in the conventional RFOC method for three-phase IM, vector control of single-phase IM can be done. The proposed method demonstrates reasonably good speed and torque responses with satisfactory tracking capability. In contrast to previous studies on FOC for single-phase IMs or unbalanced two-phase IMs, the supposition of  $L_{qs}/L_{ds}=(M_q/M_d)^2$  has not been taken into account in the proposed technique. It is shown the effect of using this supposition is reflected on the motor torque and hence speed oscillations. Due to good tracking capability of proposed drive system, this control technique can be a well-suited candidate for application where precise move of IM in required such as cutting, knurling and deformation processes.

**Keyword:** a novel technique, single-phase induction motor, rotor field-oriented control, transformation matrices

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## 1. Introduction

Single-phase Induction Motors (IMs) are one of the most common IMs which are broadly used in both industrial and household applications. They used in dryers, compressors, fans, washing machines and many other applications. Generally, single-phase IMs have two windings. These windings are in space quadrate and have different resistances and impedances (these motors can be considered as unbalanced two-phase IMs) [1]. Since both stator windings in the single-phase IMs are fed by the same power supply, they are unbalance operated and are very susceptible in the speed torque pulsations. If a conventional vector control strategy for three-phase IM is applied to the single-phase or unbalanced two-phase IMs, significant oscillations in the speed and torque output will be produced [2-6]. Therefore, studying single-phase or unbalanced two-phase machine drives have been aroused increasing interest among researchers over the past decades [2-20].

Field-Oriented Control (FOC) methods are well established at high performance vector control of IMs. This controlling technique has some advantages such as wide range of speed control, accurate speed regulation and very fast dynamic response. It has been suggested using Rotor Field-Oriented Control (RFOC) method based on hysteresis current controller in [4-5], [7]. Using hysteresis current controller has some drawback in a light load condition. In [8], FOC of single-phase IM using current double sequence controller to eliminate the torque pulsations has been presented. However, current double sequence controller is a complex controller due to using PI controllers and extensive on-line computation. To solve the associated problems in [7, 8], in [9, 10] decoupling vector control of single-phase IM based on RFOC

strategy with optimum operation of this motor under Max T/A (torque per ampere) has been proposed. It was shown in [2-5], [11-14] by using transformation matrices, the RFOC equations of the unbalanced two-phase IM can be transformed into a structure of equations, which are similar to the RFOC equations of the three-phase IM. However, in the process of calculation the RFOC equations, the backward components of the stator voltage equations have been neglected [2-3], [11]. Neglecting the backward components is reflected in the speed and the torque dynamic responses of the drive system [12-14]. General problems encountered in the vector control of single-phase IMs have been discussed in [15] and finally, a FOC method for a symmetrical two-phase IM instead of asymmetrical two-phase IM has been proposed. The proposed method in [15] can not be used for vector control of single-phase IM with two unequal main and auxiliary windings or unbalanced two phase IMs. In [16-19], feedforward decoupling scheme for vector control of single-phase IMs has been presented. In [16], Indirect Stator Field-Oriented Control (ISFOC) for single-phase IM using DS1104 was discussed and implemented. In [17], feedforward decoupling method for single-phase IM with estimation of rotor speed based on motor model has been presented. The presented method for speed sensorless vector control of single-phase IM in [17] has been developed with estimation of stator resistance in [18]. In [19], Speed sensorless FOC of single-phase IM using adaptive sliding mode-model reference adaptive system strategy has been studied.

In the majority of the previously proposed FOC control methods for controlling single-phase or unbalanced two-phase IMs, for simplifying of FOC equations, the assumption  $L_{qs}/L_{ds}=(M_q/M_d)^2$  has been used [7-10], [15-18], [20]. As mentioned before, using this assumption is reflected in the oscillations of speed and hence the torque responses of the drive system. In [12-14], an exact model for RFOC of the two-phase IM without assumption  $L_{qs}/L_{ds}=(M_q/M_d)^2$  has been proposed. The proposed drive system in [12, 13] is sensitive to regulation of PI controller coefficients. Moreover, the proposed method in [14], is very complex due to the using two parallel FOC algorithms. In this study, a novel scheme for vector control of single-phase IM based on the RFOC and without assumption  $L_{qs}/L_{ds}=(M_q/M_d)^2$  is proposed and checked by MATLAB simulations.

## 2. Model of IM (Three-Phase and Single-Phase IM)

The equations of the IM (three-phase and single-phase IM) in the stationary reference frame (superscript "s") can be described as following equations [3]:

*Stator and rotor voltage equations:*

$$\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_{ds} + L_{ds} \frac{d}{dt} & 0 & M_d \frac{d}{dt} & 0 \\ 0 & r_{qs} + L_{qs} \frac{d}{dt} & 0 & M_q \frac{d}{dt} \\ M_d \frac{d}{dt} & \omega_r M_q & r_r + L_r \frac{d}{dt} & \omega_r L_r \\ -\omega_r M_d & M_q \frac{d}{dt} & -\omega_r L_r & r_r + L_r \frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (1)$$

*Stator and rotor flux equations:*

$$\begin{bmatrix} \lambda_{ds}^s \\ \lambda_{qs}^s \\ \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} = \begin{bmatrix} L_{ds} & 0 & M_d & 0 \\ 0 & L_{qs} & 0 & M_q \\ M_d & 0 & L_r & 0 \\ 0 & M_q & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (2)$$

*Electromagnetic torque equations:*

$$\begin{aligned} \tau_e &= \frac{Pole}{2} (M_q i_{qs}^s i_{dr}^s - M_d i_{ds}^s i_{qr}^s) \\ \frac{Pole}{2} (\tau_e - \tau_l) &= J \frac{d\omega_r}{dt} + F\omega_r \end{aligned} \quad (3)$$

Where,  $V_{ds}^s, V_{qs}^s, i_{ds}^s, i_{qs}^s, i_{dr}^s, i_{qr}^s, \lambda_{ds}^s, \lambda_{qs}^s, \lambda_{dr}^s$  and  $\lambda_{qr}^s$  are the d-q axes voltages, currents, and fluxes of the stator and rotor,  $r_{ds}, r_{qs}$  and  $r_r$  denote the d-q axes stator and rotor resistances.  $L_{ds}, L_{qs}, L_r, M_d$  and  $M_q$  denote the stator, the rotor self and mutual inductances.  $\omega_r$  is the machine speed.  $\tau_e$  is the electromagnetic torque,  $\tau_l$  is the load torque,  $J$  is the moment of inertia and  $F$  is viscous friction coefficient. For three-phase IM we have:  $r_{ds}=r_{qs}=r_s, L_{ds}=L_{qs}=L_s=L_s+3/2L_{ms}, M_d=M_q=M=3/2L_{ms}$  and for single-phase IM we have:  $r_{ds}\neq r_{qs}, L_{ds}\neq L_{qs}, M_d\neq M_q$ . As can be seen from equations (1)-(3), in general, the structure of the three-phase and single phase machine equations are the same.

**3. Problem Statement in Vector Control of Single-Phase IMS**

In RFOC technique for vector control of IMs, by using following matrix (conventional or balanced transformation matrix), IM equations are transferred to the rotating reference frame [21]:

$$[T_s^e] \begin{bmatrix} \cos\theta_e & \sin\theta_e \\ -\sin\theta_e & \cos\theta_e \end{bmatrix} \tag{4}$$

Where,  $\theta_e$  is the angle between the stationary reference frame and rotating reference frame (in this paper superscript e indicated that the equations are in the rotating reference frame). By applying (4) to the single-phase IM equations, the following equations are obtained.

$$\begin{bmatrix} v_{ds}^e \\ v_{qs}^e \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} \left(\frac{r_{ds}+r_{qs}}{2} + \frac{(L_{ds}+L_{qs})}{2} \frac{d}{dt}\right) & -\omega_e \frac{(L_{ds}+L_{qs})}{2} & \frac{(M_d+M_q)}{2} \frac{d}{dt} & -\omega_e \frac{(M_d+M_q)}{2} \\ \omega_e \frac{(L_{ds}+L_{qs})}{2} & \left(\frac{r_{ds}+r_{qs}}{2} + \frac{(L_{ds}+L_{qs})}{2} \frac{d}{dt}\right) & \frac{(M_d+M_q)}{2} \frac{d}{dt} & \frac{(M_d+M_q)}{2} \frac{d}{dt} \\ \frac{(M_d+M_q)}{2} \frac{d}{dt} & -(\omega_e - \alpha_r) \frac{(M_d+M_q)}{2} & r_r + L_r \frac{d}{dt} & -(\omega_e - \alpha_r)L_r \\ (\omega_e - \alpha_r) \frac{(M_d+M_q)}{2} & \frac{(M_d+M_q)}{2} \frac{d}{dt} & (\omega_e - \alpha_r)L_r & r_r + L_r \frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{ds}^+ \\ i_{qs}^+ \\ i_{dr}^+ \\ i_{qr}^+ \end{bmatrix}}_{\text{Forward Components}} + \underbrace{\begin{bmatrix} \left(\frac{r_{ds}-r_{qs}}{2} + \frac{(L_{ds}-L_{qs})}{2} \frac{d}{dt}\right) & \omega_e \frac{(L_{ds}-L_{qs})}{2} & \frac{(M_d-M_q)}{2} \frac{d}{dt} & \omega_e \frac{(M_d-M_q)}{2} \\ \omega_e \frac{(L_{ds}-L_{qs})}{2} & -\left(\frac{r_{ds}-r_{qs}}{2} + \frac{(L_{ds}-L_{qs})}{2} \frac{d}{dt}\right) & \frac{(M_d-M_q)}{2} \frac{d}{dt} & -\frac{(M_d-M_q)}{2} \frac{d}{dt} \\ \frac{(M_d-M_q)}{2} \frac{d}{dt} & (\omega_e - \alpha_r) \frac{(M_d-M_q)}{2} & 0 & 0 \\ (\omega_e - \alpha_r) \frac{(M_d-M_q)}{2} & -\frac{(M_d-M_q)}{2} \frac{d}{dt} & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{ds}^- \\ i_{qs}^- \\ i_{dr}^- \\ i_{qr}^- \end{bmatrix}}_{\text{Backward Components}} \tag{5}$$

Where:

$$\begin{bmatrix} i_{ds}^{+e} \\ i_{qs}^{+e} \\ i_{dr}^{+e} \\ i_{qr}^{+e} \end{bmatrix} = \begin{bmatrix} \cos\theta_e & \sin\theta_e \\ -\sin\theta_e & \cos\theta_e \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ i_{dr}^s \\ i_{qr}^s \end{bmatrix} = \begin{bmatrix} \cos\theta_e & \sin\theta_e \\ -\sin\theta_e & \cos\theta_e \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{dr}^s \\ i_{qs}^s \\ i_{qr}^s \end{bmatrix} \tag{6}$$

As can be seen, in general, Equation (5) includes forward and backward components (forward: +e and backward: -e). Generation of backward components in the single-phase IM equations is because of different parameters in the single-phase IM model ( $r_{ds}\neq r_{qs}, L_{ds}\neq L_{qs}$  and  $M_d\neq M_q$ ; there are not these terms in the three-phase IM). To solve this problem, in this study, transformation matrices as proposed in [12-14], are used. The goal of using these matrices is removing backward components.

#### 4. Proposed Method for Vector Control of Single-Phase IM Based on RFOC

As mentioned before, due to the asymmetrical characteristics of single-phase IM, the conventional transformation matrix for three-phase IM can not be used for single-phase IM. In this paper, based on proposed transformation matrices in [12-14], an exact model for RFOC of single-phase IM is presented. The transformation matrices for stator voltage and current variables are as following equations [12-14]:

*Transformation matrix for stator voltages:*

$$\begin{bmatrix} V_{ds}^e \\ V_{qs}^e \end{bmatrix} = [T_{vs}^e] \begin{bmatrix} V_{ds}^s \\ V_{qs}^s \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{L_{qs}}{L_{ds}}} \cos \theta_e & \sin \theta_e \\ -\sqrt{\frac{L_{qs}}{L_{ds}}} \sin \theta_e & \cos \theta_e \end{bmatrix} \begin{bmatrix} V_{ds}^s \\ V_{qs}^s \end{bmatrix} \quad (7)$$

*Transformation matrix for stator currents:*

$$\begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} = [T_{is}^e] \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{L_{ds}}{L_{qs}}} \cos \theta_e & \sin \theta_e \\ -\sqrt{\frac{L_{ds}}{L_{qs}}} \sin \theta_e & \cos \theta_e \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \quad (8)$$

Using proposed transformation matrices (Equation (7) and (8)) the unbalanced equations of single-phase IM (Equation (1)-(3)) change into balanced equations [3]. After simplifying, the RFOC equations of single-phase IM using proposed transformation matrices are gained as following equations (in RFOC method, the rotor flux vector is aligned with d-axis;  $\lambda_{dr}^e = |\lambda_r|$  and  $\lambda_{qr}^e = 0$ ):

$$\begin{aligned} |\lambda_r| &= \frac{M_g i_{ds}^e}{1 + T_r d/dt} \\ \omega_e &= \omega_r + \frac{M_g i_{qs}^e}{T_r |\lambda_r|} \\ \tau_e &= \frac{Pole M_g}{2 L_r} |\lambda_r| i_{qs}^e \end{aligned} \quad (9)$$

In Equation (9),  $T_r$  ( $T_r = L_r/r_r$ ) is the rotor time constant. Moreover, the stator voltage equations are obtained as:

$$\begin{aligned} [T_{vs}^e] \begin{bmatrix} V_{ds}^e \\ V_{qs}^e \end{bmatrix} - [T_{vs}^e] \begin{bmatrix} r_{ds} + L_{ds} \frac{d}{dt} & 0 \\ 0 & r_{qs} + L_{qs} \frac{d}{dt} \end{bmatrix} [T_{is}^e]^{-1} [T_{is}^e] \begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} \\ + [T_{vs}^e] \begin{bmatrix} M_d \frac{d}{dt} & 0 \\ 0 & M_q \frac{d}{dt} \end{bmatrix} [T_{is}^e]^{-1} [T_{is}^e] \begin{bmatrix} i_{dr}^e \\ i_{qr}^e \end{bmatrix} &= \end{aligned} \quad (10)$$

Where:

$$\begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} = \begin{bmatrix} \cos^2 \theta_e & -\sin \theta_e \cos \theta_e \\ \sin \theta_e \cos \theta_e & \sin^2 \theta_e \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \quad (11)$$

As can be seen from (9) and (10), using proposed transformation matrices, the RFOC equations of single-phase IM are obtained as RFOC equations of three-phase IM. The equations of RFOC for three-phase IM and block diagram of IRFOC of three-phase IM are shown in Equation (12), (13) and Figure 1 respectively [21].

$$\begin{aligned}
 |\lambda_r| &= \frac{M i_{ds}^e}{1 + T_r \frac{d}{dt}} \quad , \quad \omega_e = \omega_r + \frac{M i_{qs}^e}{T_r |\lambda_r|} \quad , \quad \tau_e = \frac{Pole M}{2 L_r} |\lambda_r| i_{qs}^e \\
 v_{ds}^e &= r_s i_{ds}^e + \hat{L}_s \frac{d}{dt} i_{ds}^e - \omega_e \hat{L}_s i_{qs}^e + \frac{M^2}{L_r} \frac{d}{dt} \left( \frac{|\lambda_r|}{M} \right) \\
 v_{qs}^e &= r_s i_{qs}^e + \hat{L}_s \frac{d}{dt} i_{qs}^e + \omega_e \hat{L}_s i_{ds}^e + \omega_e \frac{M}{L_r} |\lambda_r|
 \end{aligned} \tag{12}$$

Where:

$$M = \frac{3}{2} L_{ms} \quad \hat{L}_s = L_s - \frac{M^2}{L_r} \tag{13}$$

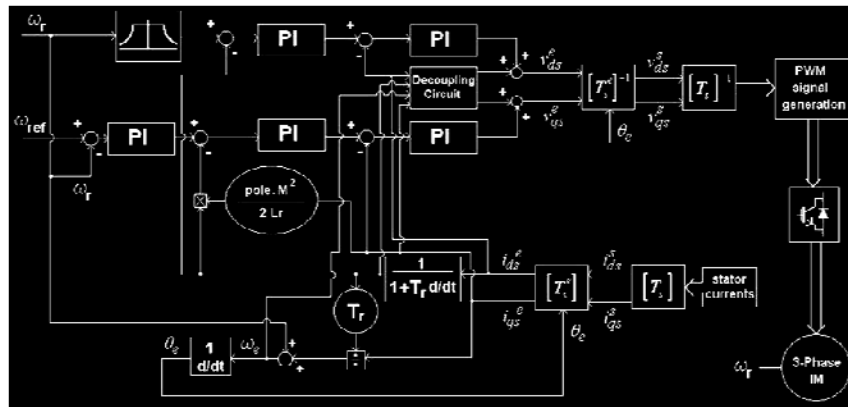


Figure 1. Block diagram of IRFOC for three-phase IM

The difference between Equation (9) and (10) and RFOC equations of three-phase IM (Equation (12)) is that in Equation (9) and (10) we have:  $r_{qs}$ ,  $M_q$ ,  $L_{qs}$  and backward components (superscript -e) but in the RFOC equations of three-phase IM, we have:  $r_s$ ,  $M=3/2L_{ms}$ ,  $L_s=L_{is}+3/2L_{ms}$  and we do not have backward components. Therefore, it is possible by some changes in the block diagram of the RFOC for three-phase IM (Figure 1), we can do vector control of single-phase IM. Equation (10) can be re-written as following equations:

*Stator d-axis voltage:*

$$\begin{aligned}
 v_{ds}^e &= r_{qs} i_{ds}^e + \hat{L}_{qs} \frac{d}{dt} i_{ds}^e - \omega_e \hat{L}_{qs} i_{qs}^e + \frac{M_q^2}{L_r} \frac{d}{dt} \left( \frac{|\lambda_r|}{M_q} \right) \\
 &+ \underbrace{\cos^2 \theta_e \left( \frac{M_q^2}{M_d^2} r_{ds} - r_{qs} \right) i_{ds}^e + \cos^2 \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) \frac{d}{dt} i_{ds}^e}_{\text{Backward}} \\
 &- \underbrace{\sin \theta_e \cos \theta_e \left( \frac{M_q^2}{M_d^2} r_{ds} - r_{qs} \right) i_{qs}^e - \sin \theta_e \cos \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) \frac{d}{dt} i_{qs}^e}_{\text{Backward}} \\
 &+ \underbrace{\omega_e \cos \theta_e \sin \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) i_{ds}^e - \omega_e \sin^2 \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) i_{qs}^e}_{\text{Backward}}
 \end{aligned} \tag{14}$$

Stator q-axis voltage:

$$\begin{aligned}
 v_{qs}^e &= r_{qs} i_{qs}^e + \hat{L}_{qs} \frac{d}{dt} i_{qs}^e + \omega_e \hat{L}_{qs} i_{ds}^e + \omega_e \frac{M_q}{L_r} |\lambda_r| \\
 &+ \omega_e \cos^2 \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) i_{ds}^e - \omega_e \sin \theta_e \cos \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) i_{qs}^e \\
 &- \underbrace{\sin \theta_e \cos \theta_e \left( \frac{M_q^2}{M_d^2} r_{ds} - r_{qs} \right) i_{ds}^e + \sin^2 \theta_e \left( \frac{M_q^2}{M_d^2} r_{ds} - r_{qs} \right) i_{qs}^e}_{\text{Backward}} \\
 &- \underbrace{\sin \theta_e \cos \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) \frac{d}{dt} i_{ds}^e + \sin^2 \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) \frac{d}{dt} i_{qs}^e}_{\text{Backward}}
 \end{aligned} \tag{15}$$

Based on Equation (9), (14) and (15), Figure 2 can be proposed for IRFOC of single-phase IM. In this Figure, the necessary voltages for vector control of single-phase IM are produced by PI controllers and Decoupling Circuit as follows (in (16),  $v_{ds}^{ref-d}$ ,  $v_{ds}^{ref-q}$ ,  $v_{qs}^{ref-d}$ ,  $v_{qs}^{ref-q}$  are produced by using PI controllers and  $v_{ds}^{D-C}$ ,  $v_{qs}^{D-C}$  are produced by using Decoupling Circuit as shown in Figure 1):

$$\begin{aligned}
 v_{ds}^e &= v_{ds}^{ref-d} + v_{ds}^{ref-q} + v_{ds}^{D-C} \\
 v_{qs}^e &= v_{qs}^{ref-q} + v_{qs}^{ref-d} + v_{qs}^{D-C}
 \end{aligned} \tag{16}$$

Where:

$$\begin{aligned}
 v_{ds}^{ref-d} &= (r_{qs} + \cos^2 \theta_e \left( \frac{M_q^2}{M_d^2} r_{ds} - r_{qs} \right)) i_{ds}^e + (\hat{L}_{qs} + \cos^2 \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right)) \frac{d}{dt} i_{ds}^e \\
 v_{ds}^{ref-q} &= -\sin \theta_e \cos \theta_e \left( \frac{M_q^2}{M_d^2} r_{ds} - r_{qs} \right) i_{qs}^e - \sin \theta_e \cos \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) \frac{d}{dt} i_{qs}^e \\
 v_{ds}^{D-C} &= \omega_e \cos \theta_e \sin \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) i_{ds}^e - \omega_e \sin^2 \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) i_{qs}^e - \omega_e \hat{L}_{qs} i_{qs}^e + \frac{M_q}{L_r} \frac{d}{dt} \left( \frac{|\lambda_r|}{M_q} \right) \\
 v_{qs}^{ref-q} &= (r_{qs} + \sin^2 \theta_e \left( \frac{M_q^2}{M_d^2} r_{ds} - r_{qs} \right)) i_{qs}^e + (\hat{L}_{qs} + \sin^2 \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right)) \frac{d}{dt} i_{qs}^e \\
 v_{qs}^{ref-d} &= -\sin \theta_e \cos \theta_e \left( \frac{M_q^2}{M_d^2} r_{ds} - r_{qs} \right) i_{ds}^e - \sin \theta_e \cos \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) \frac{d}{dt} i_{ds}^e \\
 v_{qs}^{D-C} &= -\omega_e \sin \theta_e \cos \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) i_{qs}^e + \omega_e \cos^2 \theta_e \left( \frac{M_q^2}{M_d^2} L_{ds} - L_{qs} \right) i_{ds}^e + \omega_e \hat{L}_{qs} i_{ds}^e + \omega_e \frac{M_q}{L_r} |\lambda_r| \\
 &\hat{L}_{qs} - L_{qs} - \frac{M_q^2}{L_r}
 \end{aligned} \tag{17}$$

The matrix  $[T_s]$ , in Figure 1 and Figure 2 for three-phase and single-phase IM is as follows [21]:

$$\text{Three-phase IM: } [T_s] = \frac{\sqrt{2}}{\sqrt{3}} \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad \text{Single-phase IM: } [T_s] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

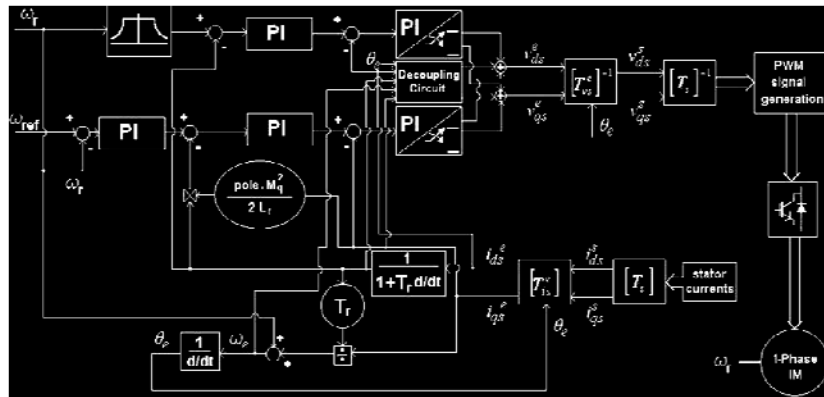


Figure 2. Block diagram of the proposed IRFOC for single-phase IM

Figure 2, depicts the proposed control scheme for vector control of single-phase IM. It composes a single RFOC algorithm which is adopted from conventional RFOC with some modifications and two switches. In each sampling time switches consecutively operate.

## 5. Simulations and Results

In this section, MATLAB simulation results based on proposed drive system as shown in Figure 2 for a single-phase IM have been presented. A single-phase IM is fed from a Sine Pulse Width Modulation (SPWM) two-leg Voltage Source Inverter (VSI) as shown in Figure 3. The Ratings and parameters of the simulated single-phase IM are as follows:

Voltage:110V,  $f=60\text{Hz}$ , No. of poles=4,  $r_{ds}=7.14\Omega$ ,  $r_{qs}=2.02\Omega$ ,  $r_r=4.12\Omega$ ,  $L_{ds}=0.1885\text{H}$ ,  $L_{qs}=0.1844\text{H}$ ,  $L_r=0.1826\text{H}$ ,  $M_q=0.1772\text{H}$ ,  $J=0.0146\text{kg.m}^2$

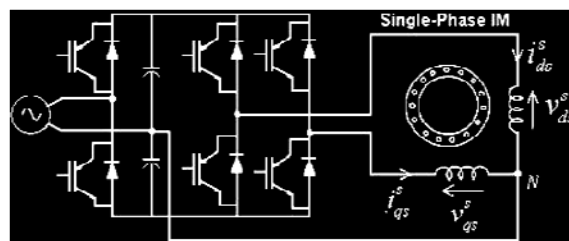


Figure 3. Single-phase IM drive system

Figure 4 shows the simulation results of the proposed vector controller when the speed reference changes from zero to the nominal value (zero to 1800rpm). At time  $t=10\text{s}$ , a load torque equal to  $1\text{N.m}$  is introduced (step load). As shown in Figure 4(a), the motor speed follows the command speed accuracy without any overshoot and steady-state error even at very low speed and after applying load torque (the oscillation of speed is about  $0.2\text{rpm}$  in steady-state and after applying load torque). Figure 4(b) and Figure 4(c) illustrated the currents of stator main and auxiliary windings and electromagnetic torque respectively. It can be seen from Figure 4(c) that the electromagnetic torque has a quick response with no pulsations (the oscillation of electromagnetic torque is about  $0.25\text{N.m}$  in steady-state and after applying load torque). Figure 4 illustrates the performance of the proposed drive system is extremely satisfactory.

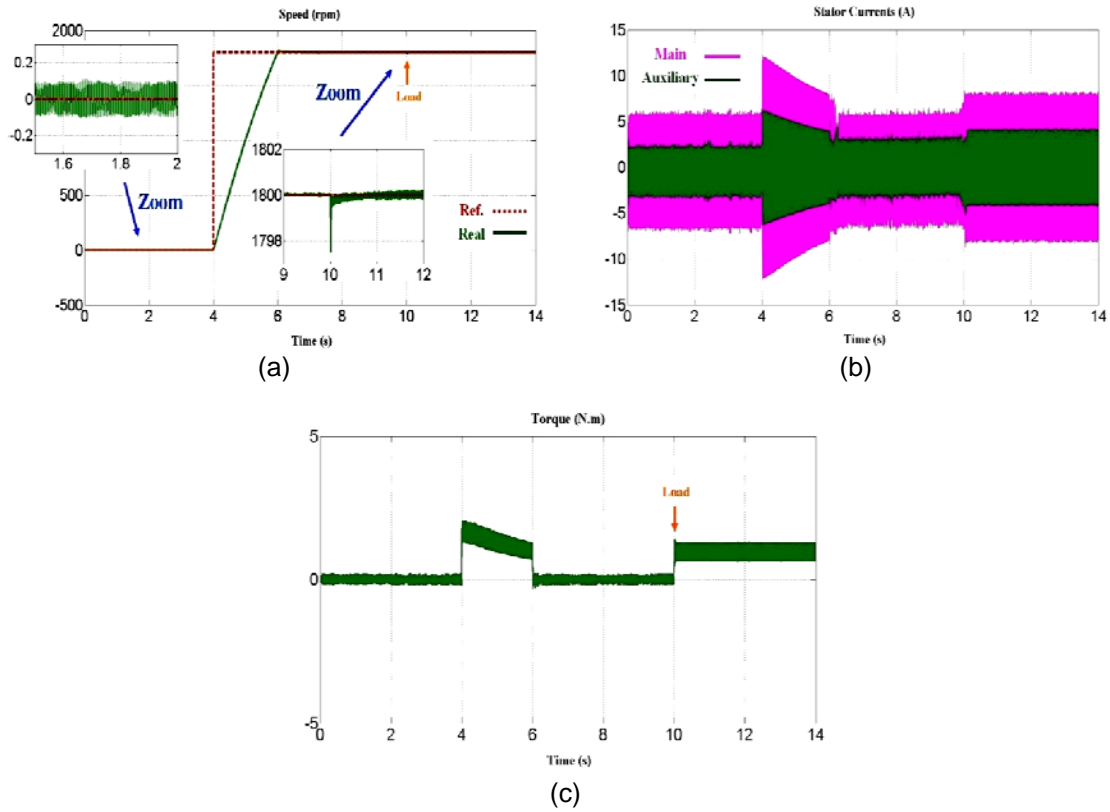


Figure 4. Simulation results of the proposed IRFOC method for a single-phase IM at nominal and zero speed; (a) speed, (b) stator currents, (c) electromagnetic torque

Figure 5 and Figure 6 show the good dynamic behavior of the proposed RFOC technique for a single-phase IM in the difference values of reference speed and for a trapezoidal reference speed respectively. In Figure 5, at  $t=9.5s$ , a load torque equal to  $0.5N.m$  is applied and removed at  $t=11.5s$ . It is evident from Figure 4-6 that the proposed scheme can operate stably at high, low and zero speed and provide the acceptable dynamic response.

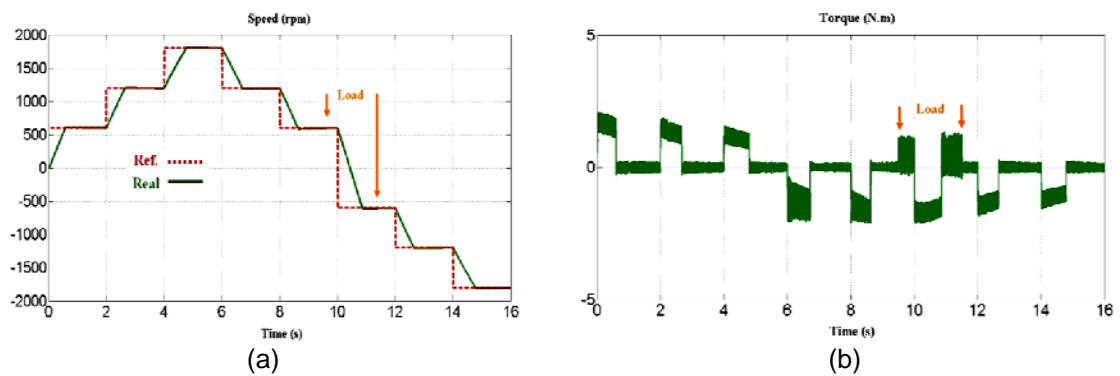


Figure 5. Simulation results of the proposed IRFOC method for a single-phase IM in the difference values of speed; (a) speed, (b) electromagnetic torque



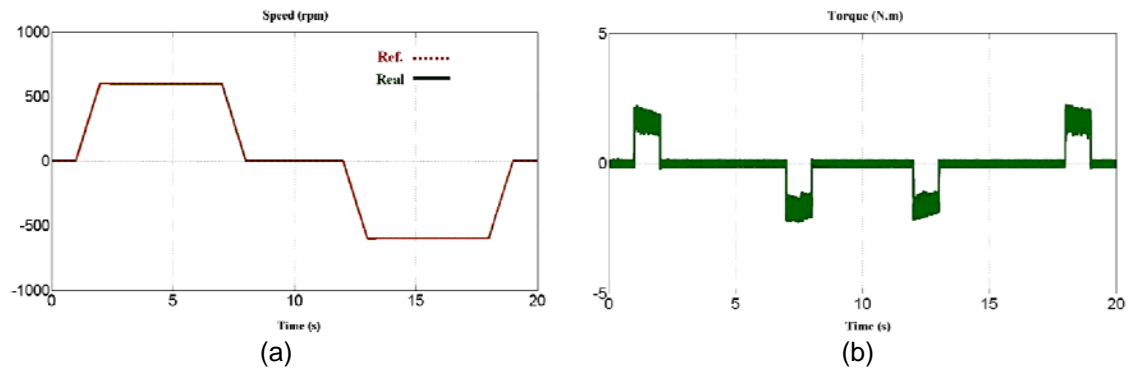


Figure 6. Simulation results of the proposed IRFOC method for a trapezoidal reference speed; (a) speed, (b) electromagnetic torque

Figure 7 shows a comparison between the steady-state electromagnetic torque response of the proposed scheme based on Figure 2 with considering of  $(M_q/M_d)^2=L_{qs}/L_{ds}$  (e.g, [7-10], [15-18], [20], [22-24]) and without considering of  $(M_q/M_d)^2=L_{qs}/L_{ds}$ . As can be seen from Figure 7, there is a small magnitude of oscillations at the nominal command speed in the electromagnetic torque response when the assumption  $(M_q/M_d)^2=L_{qs}/L_{ds}$  is used (the oscillations of electromagnetic torque with considering of  $(M_q/M_d)^2=L_{qs}/L_{ds}$  is about 0.25N.m around the average amount of 1N.m. Moreover, the oscillations of electromagnetic torque without considering of  $(M_q/M_d)^2=L_{qs}/L_{ds}$  is about 0.4N.m around the average amount of 1N.m). It is concluded in comparison with the previous control schemes for FOC of single-phase or two-phase IM (e.g, [7-10], [15-18], [20], [22-24]), the proposed FOC technique in this paper is more accurate.

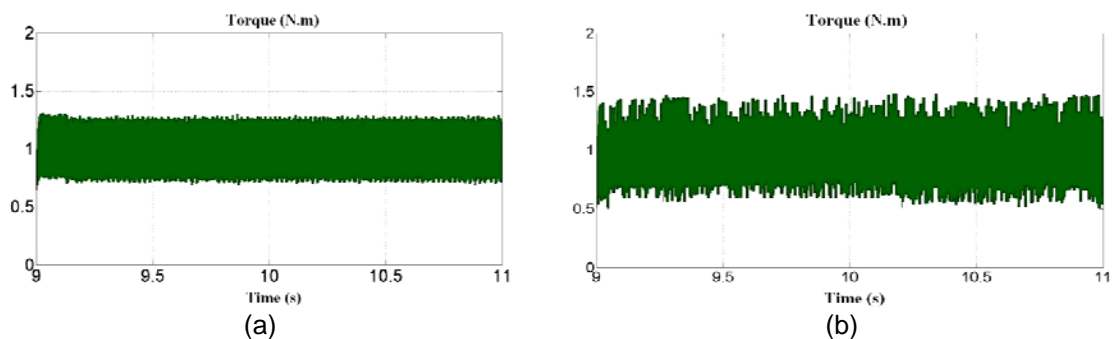


Figure 7. Simulation results of comparison between torque response in single-phase IM; (a) not assuming  $(M_q/M_d)^2=L_{qs}/L_{ds}$ , (b) assuming  $(M_q/M_d)^2=L_{qs}/L_{ds}$

## 6. Conclusion

In this paper, a novel technique for speed control of single-phase or unbalanced two-phase IMs based on RFOC has been proposed. It is shown using unbalanced transformation matrices the unbalanced equations of single-phase IM can be transferred into the equations that have the same structure as the three-phase IM. Using this similarity a novel vector control method for single-phase IMs which is obtained from the conventional vector control method for three-phase IMs is proposed. Unlike other FOC method for single-phase IMs, the proposed scheme in this paper does not use the supposition  $(M_q/M_d)^2=L_{qs}/L_{ds}$ . The performance of the proposed RFOC technique is highly satisfactory for controlling single-phase IM in wide range of speed.

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